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RESEARCH ARTICLE

A SINGLE COMMODITY INVENTORY MODEL WITH A GENERAL POLICY

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ABSTRACT

A simple computational method (SCM) to analyze a class of (s,S) type inventory problem is developed. Under this (s, S) policy, (i) the number of units demanded where d = 1,2,... a (≤ s) at successive demand epochs form a Markov chain (MC) with one step transition probability matrix (TPM) P and (ii) the replenishments are instantaneous. This method gives the algorithm for computation of stationary probabilities of inventory process, joint probability function of number of transitions and quantities of replenishments per cycle, conditional and unconditional average costs. Illustrative example for a few special cases are provided, which strengthen the applicability of the SCM to practical.

INTRODUCTION

Numerous methods have been suggested by various authors for formulating and finding optimal inventory policies which have more number of practical applications in real life situations. Such eventualities with modifications have been increasing in the inventory policies of the (s,S) types also. One more possible way out is discussed below.

In this paper, a different method of approach for the analysis of (s, S) inventory models, wherein the number of units demanded at successive demand epochs are Markov dependent as introduced by Krishnamoothy and Lakshmi (1991) is discussed. For a detailed discussion about the systematic analysis of various types of (s, S) inventory system refer Srinivasan and Ravichandaran (1994), Hiller and Liberman, (1990) and Krishnamoorthy et al., (1995).

A MARKOV DEPENDENT [(s, S), d, P] INVENTORY MODEL:

For the present (s, S) inventory problem, it is assumed that the bulk quantity demanded, say 'd' at a demand epoch is a random variable with d = 1, 2, … , a ; a ≤ s such that the sequence of units demanded in the successive demand epochs forms a finite state Mc on the state space {1, 2, …, a } with TPM 'p' . To avoid the perpetual shortage, it is assumed that (S-s) > s. further if the inventory level is s+1 or greater at a demand epoch, then no order is placed. On the other hand , if the inventory level either falls to 's' or dips below 's' at an epoch, an order is placed for at least (S-s) units. The quantity ordered is subject to review at the epoch of replenishment so as to bring inventory to level s.

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Zero lead time is assumed. This kind of inventory model may be represented in symbols as [(s, S), a, P].

Further the inter occurrence times between successive demand epochs are assumed to be independent and identically distributed (i. i . d) random variables with distribution function G (.) and density function g(.). If the demand is for 'I' units at an epoch, it may be called as I - type demand for I = 1, 2, …, a.

Notations

{I(t)}	:	inventory level process
*	:	Convolution
E_1	:	{1, 2, ..., a – 1, a}
E_2	:	{s + 1, s + 2, ... s – 1, S}
E	:	{ (1, s + 1), (2, s + 1), (3, s + 1), ..., (a – 1, s + 1), (a, s + 1)
		(1, s + 2), (2, s + 2), (3, s + 2), ..., (a – 1, s + 2), (a, s + 2)
		(1, S – a), (2, S – a), (3, S – a), ..., (a – 1, S – a), (a, S – a)
		(1, S – a), (2, S – a), (3, S – a + 1), ..., (a – 1, S – a + 1)
		(1, S – a), (2, S – a), (3, S – a + 2), ...,
		(1, S – 3), (2, S – 3), (3, S – 3)
		(1, S – 2), (2, S – 2)
		(1, S – 1)
		(1, S), (2, S), (3, S), ..., (a, S)}
E_3	:	{S – s, S – s + 1, ..., S – s + (a – 1) }
E_4	:	{s – a + 1, s – a + 2, ..., s – 1, s}
E_5 (r)	:	{r, r + 1, ..., S – s}, where r is the smallest integer ≥ $\frac{(S-s)}{a}$
N	:	{1, 2, ...,}

Here, the set E is a proper subset of E_1 x E_2 i.e. E ⊂ E_2 x E_2. It is due to the fact that the continuous time, inventory process decreases from the levels S, S – 1, S – 2,... etc., with instantaneous reflecting barriers at each of its levels $i \in E_4$ depending upon the demands. Hence the inventory level never decreases to zero at any instant. Further E_3 forms the set of all possible number of units of replenishments. The set $E_5(r)$ is the collection of all possible number of transitions in a cycle of replenishments.

Markov Renewal Inventory Process

Let $0 = T_0 < T_1 < T_2$ ... be the successive demand epochs and X_0, X_1, X_2, ... be the number of units demanded at these epochs respectively, so that the sequence $X=\{X_n\}$ forms a MC on the states $i, j \in E_1$ with initial distribution p $\{X_0 = i) = p_i$, and transition probabilities

$P\{X_{n+1} = j / X_n = i\} = p_{ij} \qquad i, j \in E_1$ ... (1)

Assume that the TPM $P = (p_{ij})$ is irreducible and a periodic throughout the discussions to follow.

Let $(S =) Y_0, Y_1, Y_2$ ... represent the inventory levels just after meeting the demands at T_0, T_1, T_2, ..., so that time intervals between successive occurrences of bulk demands induce the triple sequence process $\{(X, Y), T\} = \{(X_n, Y_n), T_n ; n \in N\}$ as a Markov Renewal inventory Process (MRIP) on the state space E with the Semi-Markov kernel $Q(t) = [\{Q(i, I), (j, J) ; t\}]$ where $Q \{(i, I), (j, J) ; t)\} = P[\{(X_{n+1} = j, Y_{n+1} = J) ; T_{n+1} - T_n \le t\} / (X_n = i, y_n = I)] = P(i, I), (j, J)$. G(t) the TPM of the underlying (X, Y) double sequence MC is given by $Q(\infty) = \overline{Q}$ ((i, I), (j, J)) (say)

Stationary Distribution of the (X, Y) MC

Let the stationary probability vector of the (X, Y) MC denoted by the row vector $\Pi = \{\pi_{(i, j)}\}$ for i = 1, 2, ..., a, j = s + 1, s +2, ..., S – a, i – 1, 2, ..., n, j = S – n ; n = a – 1, a – 2, ..., 1 and

i = 1, 2, ..., a ; j = S ...(3)

Here the subscripts of π are ordered as in the E set. If e = (1, 1, ..., 1) denotes a column vector of unities then the stationary equations $\Pi\overline{Q} = \Pi$, $\Pi e = 1$ have unique solution vector as the state space E is finite dimensional and is irreducible, simple computational method (SCM) is capsuled below as an algorithm.

Step - I

Using the TPM $P = (P_{ij})$, $i, j \in E_1$, obtain its stationary probability vector, say $\nu = (\nu_1, \nu_2, ..., \nu_a)$ by solving $\nu p = \nu$, $\nu e = 1$

Step – II

Arrange the states as they are ordered in the set E of section 1.1 and identify the TPM $\overline{Q}$ of the (X, Y) MC in terms of P_{ij}'s and zeros.

Step – III

Omit the last set of 'a' equations co-responding to $\pi_{(j, s)}$, j = 1, 2, ... a, from $\Pi = \Pi\overline{Q}$. Then select the remaining stationary equations together with $\sum_{j \in E_2} \pi(i, j) = \nu_i$ so that the resulting independent set of simultaneous equations are solvable.

Step – IV

Make suitable changes such that one set of 'a' equations corresponding to $\pi_{(1, S-1)}$, $\pi_{(2, S-2)}$, ..., $\pi_{(a, S-a)}$ appearing on the left of the system $\Pi = \Pi\overline{Q}$ reduce to a system of 'a' equations of the form $\nu = (\pi_{(1, S-1)}, \pi_{(2, S-2)}, ... \pi_{(a, S-a)})$ A, where A is a full rank square matrix of order of 'a'. Hence obtain values for $\pi_{(1, S-1)}, \pi_{(2, S-2)}, ..., \pi_{(a, S-a)}$ by any one of the standard methods.

Step – V

Using the values of $\pi_{(1, S-1)}, \pi_{(2, S-2)}, ..., \pi_{(a, S-a)}$) and selecting expressions for $\pi_{(1, S-2)}, \pi_{(2, S-3)}, ..., \pi_{(a-1, S-a)}$ and $\pi_{(a, S-a-1)}$, compute the next set of 'a' components in the solution of $\Pi = \Pi\overline{Q}$. Continue it, until values for all components $\pi_{(i, j)}$, $(i, j) \in E$ of the Π vector are obtained. A special case of obtaining the Π vector is explained for the specific values (S = 2, S = 7), (S = 2, S = 6), (S = 2, S = 5) in appendices – I(a), II(a), III(a) respectively.

Mean Sojurn Times

For the MRIP {(X, Y), T}, let the mean sojourn time at state $(i, j) \in E$ be $\mu_{(i, j)}$. Since the inter occurrence times between transition epochs are assumed to be i.i.d. random variables, with df G (.), it is seen that

$$\mu_{(i,j)} = \int_0^\infty [1 - G(t)]\, dt = \mu$$

Inventory Level Process {I(t)}

Since I(t) gives the onhand stock level at time 't', i.e. $I_{(t)} = Y_n$ for $T_n \le t < T_{n+1}$, it is seen that $\{I(t) ; t \ge 0\}$ is a Semi-Markov Process on the state space E_2 with kernel Q. Let $P[I_{(t)} = n / Y_0 = i]$. Then by the application of Key-renewal theorem, it can be established that $\lim_{t \to \infty}$ p(n, i ; t) = P(n) exists. Thus using the results of (4), it can be shown that for $i \in E_1$.

$$P(n) = \frac{\sum_{j=1}^{a} \pi_{(j,n)}\, \mu_{(j,n)}}{\sum_{(1,k) \in E_1} \pi_{(1,k)}\, \mu_{(1,k)}} = \sum_{j=1}^{a} \pi_{(j,n)} \qquad \text{... (5a)}$$

where n = S + 1, S + 2, ..., S – a and n = S

$$P(n) = \frac{\sum_{j=1}^{S-n} \pi_{(j,n)}\, \mu_{(j,n)}}{\sum_{(1,k) \in E_1} \pi_{(1,k)}\, \mu_{(1,k)}} = \sum_{j=1}^{S-n} \pi_{(j,n)}$$

where n = S – a + 1, S – a + 2, ..., S – 1

Expected Total Costs

In this type of [(S, S), a, p] inventory situations, demand cum replenishment epochs play a very crucial role in obtaining cost expressions. Hence, to study the characteristics of the time durations of the successive replenishment epochs, consider any two consecutive demand cum replenishment epochs W_n and W_{n+1}, $n \in N$ under steady state conditions. Assume that an i-type demand occurs at W_n while a j-type demand takes place at W_{n+1}. Let it be called as j-cycle.

Further, assume that 'M' denotes the quantity of replenishment at W_{n+1}, '$Z = (W_{n+1} - W_n)$' denotes the random length of the cycle between the epochs W_n and W_{n+1} and K denotes the random number of demand epochs in it, with the k-th one being the last demand epoch of the cycle i.e at W_{n+1}.

Then materialization of the events (M=m), (K = k) and $Z \in (z, z + dz)$ implies that $m \in E_3$, $k \in E_5$ (r) and $z \in [0, \infty)$. Hence for a given i, $j \in E_1$, expression for the joint p.d.f., subject to

i. S_1 = Sum of the demands of the first (k-1) demands in the j-cycle is strictly less than (S-s),

ii. S_2 = Sum of the demands of all k-demand epochs is equal to m and

iii. $z \le Z \le z + dz$ is to take the following form

$$f_{i,j}(m, k ; z)\, dz = \frac{1}{B} p_{i,i_1}, p_{i_1,i_2}, \ldots, p_{i_{k-1},i_k}\, g^{*k}(z)dz$$

where $i, i_1, i_2, \ldots, i_k \in E_1$, $S_1 < (S - s)$
$S_2 = m$, $m \in E_3$, $k \in E_5$
and B is the normalizing constant and $i_k = j$.

From (6) with little effort, one could obtain the marginal distribution for M, K and Z. The joint probability function (pf) of M and K is sufficient to derive the cost per transition and is given by

$$\bar{f}_j(m,k) = \frac{1}{B} \Sigma\; p_{i,i_1}, p_{i_1,i_2}, \ldots, p_{i_{k-1},j} \qquad \ldots (7)$$

$i, i_1, i_2, \ldots, i_k \in E_1$, $S_1 < S - s$ and $S_2 = m$, $m \in E_3$, $k \in E_5$ since $i_k = j$. For the constructional details of $\bar{f}_j(.)$ refer appendices I(b), II(b), III(b) for some specific cases.

Marginal Distributions in a j-cycle

Let the marginal probability functions of quantity of replenishment M, number of transitions K in a j-cycle be respectively designated by $f^{(1)}_{j(m)}$ and $f^{(1)}_{j(k)}$.

Thus

$$f^{(1)}_{j(m)} = \sum_{K \in E_5} \bar{f}_j(m,k)\; ; \; j \in E_1$$

$$f^{(2)}_{j(k)} = \sum_{K \in E_5} \bar{f}_j(m,k)\; ; \; j \in E_1$$

Further let

f_j^* = probability that a cycle is terminated by a j-demand.

$$= \sum_{m \in E_3} \sum_{k \in E_5} \bar{f}_j(m,k)$$

Hence, the following are the conditional expected values of the random variables (M/K), K and M in the j-cycle.

$$E_j\left(\frac{M}{K}\right) = \Sigma_{m \in E_3} \Sigma_{j \in E_5} \left(\frac{m}{k}\right) \bar{f}_j(m,k) \qquad \ldots (8a)$$

$$E_j(K) = \sum_{K \in E_5} k\, f_j^{(2)}(k) \qquad \ldots (8b)$$

$$E_j(M) = \sum_{K \in E_5} m\, f_j^{(1)}(m)$$

Average Inventory Per Transition

In a j-cycle, choose a demand epoch T_n at random. Let $(j, i) \in E$ be the state of the (X_n, Y_n) process at T_n. The inventory level during the transition interval $(T_n - T_{n-1})$ is $(j + i)$, if i = S and it is equal to $[S - M + j]$ if i = S. Further, let $I^+ = I(T_n^+)$ be the inventory level just after the demand epoch T_n. Hence the conditional expected inventory level during the interval $(T_n - T_{n-1})$, given that $(X_n, Y_n) = (j, 1)$ is given by

$$E_j(I^+) = \sum_{(i \neq S)} (j+1)\, \pi_{(j,i)_{i \in E_2}} + [S - E_j(M) + j]\, \pi_{(j,S)} \qquad \ldots (9)$$

Cost Aspects

Consider the cost considerations for a j-cycle as

H_j the unit storage cost per transition
L_j fixed ordering cost
C_j unit variable cost

Then the conditional expected total 'cost per transition (TCT) in a j-cycle is given by

$$E_j(TCT) = \frac{L_j}{E_j(k)} + C_j E_j\left(\frac{M}{K}\right) + H_j E_j(I^+) = T_j, say \qquad \ldots (10a)$$

Thus the expected total cost per transition in any cycle is given by

$$E(TCT) = \sum_{j \in E_3} T_j\, \bar{f}_j^* \qquad .. (10b)$$

Expected Long Run Cost Per Unit Time

For a given $j \in E_1$, the expected long run total cost (TC) per unit time in a j-cycle is given by

$$E_j(TC) = \frac{L_j}{E_j(z)} + C_j E_j(M/Z) + h_j E(I) \qquad \ldots (11a)$$

Thus

$$E(TC) = \sum_{j=1}^{a} E_j(TC)\, \bar{f}_j \qquad \ldots (11b)$$

Table 1(c)

E (TCT) values for [(s, S), a, p] = [(2, S), 2, p_0] models when S = 5, 6, 7

S	j	ν_j	L_j/E_j (k)	C_j E_j (M/K)	H_j E_j (I)	T_j	f^*_j	$T_j f_j^*$	E(TCT)
	1	.36	244.10	21.08	17.83	283.01	.29	82.07	
5									242.72
	2	.64	134.18	62.26	29.83	226.27	.71	160.65	
	1	.36	265.02	12.21	19.45	296.68	.19	56.37	
6									219.46
	2	.64	96.36	72.69	32.29	201.34	.81	163.09	
*	1	.36	184.89	9.15	21.53	215.57	.24	51.74	
7									165.09
	2	.64	80.65	32.57	35.93	149.15	.76	113.35	

Table – 2
E (TEC) values for [(2, 7), a, p] = [(2, 7), 2, p] model when p varies

S	j	ν_j	Lj / E_j (K)	C_j E_j (M/K)	H_j E_j (I)	T_j	f_j	T_j f_j	E (TCT)
I*	1	.36	123.26	12.80	17.22	153.28	.24	36.79	
									139.2
	2	.64	60.49	45.60	28.74	134.83	.76	102.47	
	1	.50	78.03	16.59	22.99	117.61	.34	39.99	
II									123.4
	2	.50	65.81	37.24	23.47	126.52	.66	83.50	
	1	.80	33.06	26.81	35.20	95.07	.67	63.70	
III*									115.0
	2	.20	126.47	18.83	10.27	155.57	.33	51.34	

where $E(I) = \sum_{n \in E_2} n\, p(n)\, E_j\left(\frac{M}{Z}\right)$

$$= \int_0^\infty \sum_{m \in E_3} \sum_{k \in E_5} (m/z) f_j(m,k;z)\, dz$$

and

$$E_j(z) = \int_0^\infty \sum_{m \in E_3} \sum_{k \in E_5} z\, f_j(m,k;z)\, dz$$

Optimization Problems

The best policies under two different contexts are worked out assuming specific values for the parameters involved. These are supported by numerical illustrations.

Example 1

It is assumed that a retailer keeps stock pile according to the [(S, S), a, p] system. The best policy given the values s = 2, a = 2, p_{11} = .3, p_{21} = .4, L_1 = 150, L_2 = 200, C_1 = C_2 = 50, H_1 = H_2 = 10 when S = 5, 6 and 7 is determined by using the calculations of tables 1(a), 1(b) and 1(c) given below which are outlined in expressions (1) – (10) and appendices (I) through (III). Let $p_0 = \begin{pmatrix} .3 & .7 \\ .4 & .6 \end{pmatrix}$

The resulting calculation of E(TCT) for three values 5, 6, 7 of S is given in Table 1(c) which indicates that the over all minimum E(TCT) corresponds to S = 7. Hence [(2, 7), 2, p_0] is the most desirable policy, among the above three [(2, S), 2, p_0], S = 7, 6, 5 policies.

Example 2

It is assumed that the demand for the product is seasonal according to three different TPM's obtained on the basis of the historical data. Now the problem is to identify the season which has the minimum E(TCT). For demonstration, [(2, 7), 2, P) policy is again investigated at three different seasons I, II and III respectively with

$P = P_1 = \begin{pmatrix} .3 & .7 \\ .4 & .6 \end{pmatrix}$, $P = P_2 = \begin{pmatrix} .5 & .5 \\ .5 & .5 \end{pmatrix}$, $P = P_3 = \begin{pmatrix} .9 & .1 \\ .4 & .6 \end{pmatrix}$. Further, placing S = 2, a = 2, L_1 = 100, L_2 = 150, C_1 = C_2 = 35, H_1 = H_2 = 8 the corresponding results are computed, as reported in Table – 2.

From table 2, it may be observed that the best season (*) corresponds to the P matrix which yields the highest ν_1 value. It may be due to the fact that the number of transition in a cycle increases with increasing ν_1 values and this may minimize the E(TCT) = $\Sigma[L_j / E_j (K) + C_j E_j (M/K) + H_j E_j (I^+)] f_j^*$ because number of transitions 'K' in a 'j' cycle appears in the denominator of the first two terms in the above E(TCT) expression.

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