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RESEARCH ARTICLE

UNSTEADY FREE CONVECTIVE FLOW PAST AN EXPONENTIALLY ACCELERATED VERTICAL PLATE WITH RAMPED WALL HEAT FLUX

Chinmoy. Mandal[1], Sanatan Das[2] and Rabindra Nath Jana[3]

Department of Applied Mathematics, Vidyasagar University, Midnapore 721102, India

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ABSTRACT

The effects of heat transfer on the flow past an exponentially accelerated vertical plate with ramped wall heat flux has been studied. It is found that both the fluid velocity as well as the fluid temperature decrease with an increase in Prandtl number. It is also found that the velocity increases with an increase in either Grashof number or accelerated parameter. Further, it is found that both the fluid velocity as well as the fluid temperature increase when time progresses. The absolute value of the shear stress at the plate reduces with an increase in Prandtl number while it increases with an increase in Grashof number.

INTRODUCTION

The heat transfer is the area that deals with the mechanism responsible for transferring energy from one place to another when a temperature difference exists. Natural convection is one of the most economical and practical methods of cooling and heating. Natural convection is caused by temperature or concentration induced density gradient within the fluid. Natural convection flow occurs as a result of influence of gravity forces on fluids in which density gradients have been thermally established. With the growing sophistication in technology and with the increasing concern with energy and the environment, the study of heat transfer has, over the past several years, been related to a very wide variety of problems, each with its own demands of precision and elaboration in the understanding of the particular processes of interest. Areas of study range from atmospheric, geophysical and environmental problems to those in heat rejection, space research and manufacturing systems.

In a wide class of natural convection processes, heat transfer occurs from a heated vertical surface placed in a quiescent medium at a uniform temperature. If the plate surface temperature is greater than the ambient temperature, the fluid adjacent to the vertical surface gets heated, becomes light and rises. Heavier fluid from the neighboring areas rushes into to take the place of the rising fluid; similarly the flow for a cooled surface is downwards. Isachenko *et al*.(1980) reviewed the problems of heat transfer. Hossain and Shayo (1986) studied analytically the skin friction in the unsteady free convection flow past an accelerated plate. The mass transfer effects on the flow past an exponentially accelerated vertical plate with constant heat flux was considered by Jha *et al*.(1991). Chandran *et al*.(1998) studied the unsteady hydromagnetic free convective flow with heat flux and accelerated boundary motion. Barletta (1999) presented an analysis on the heat transfer by fully developed flow and viscous heating in a vertical channel with prescribed wall heat fluxes. The transient free convection flow past an infinite vertical plate with periodic temperature variation was discussed by Das *et al*. (1999). Narahari *et al*.(2002) considered the transient free convection flow between infinitely long vertical parallel plates with constant heat flux at one boundary. Chandran *et al*.(2005) studied the natural convection near a vertical plate with ramped wall temperature. The developing flow near a semi-infinite vertical wall with ramped temperature was investigated by Singh *et al*. (2008). Muthucumaraswamy *et al*.(2008) studied the heat transfer effects on the flow past an exponentially accelerated vertical plate with variable temperature.

Singh and Singh(2010) presented the transient MHD free convective flow near a semi infinite vertical wall having ramped temperature. The effects of heat transfer and viscous dissipation on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature was investigated by Kishore *et al*.(2010). The motivation of our present investigation is to study the unsteady free convective flow of a viscous incompressible fluid past an exponentially accelerated vertical plate with ramped wall heat flux. Initially, at time $t \le 0$, the plate and the fluid are at the same constant temperature T_∞ in a stationary condition. At time $t > 0$, the plate starts to move with exponential accelerated velocity $u_0 e^{\lambda^* t}$, where u_0 and λ^* are constants. The heat flux at the plate changes rampedly with time. It is found that the velocity decreases with increase in Prandtl number Pr. An increase in Grashof number Gr leads to rise the fluid velocity u_1. It is

***Corresponding author:** jana261171@yahoo.co.in

seen that the fluid velocity u_1 increases with an increase in accelerated parameter λ. It is also seen that the fluid velocity u_1 increases with an increase in time τ. Further, it is seen that the fluid temperature θ decreases with an increase in Prandtl number Pr while it increases with an increase in time.

Formulation of the problem and its solutions

Consider the unsteady flow of a viscous incompressible fluid past an exponentially accelerated vertical plate with ramped wall heat flux. Choose a Cartesian co-ordinates system in such a way that x-axis is taken along the wall in a vertically upward direction and y-axis is normal to it into the fluid. At time $t \le 0$, both the fluid and the plate are at rest with the constant temperature T_∞. At time $t > 0$, the plate starts to move with an exponential accelerated velocity $u_0 e^{\lambda^* t}$ in its own plane and the heat flux at the plate changes rampedly with time. Since the plate is infinitely long in the x- direction, all the physical variables are the functions of y and t only. Under Boussinesq approximation, the unsteady flow is governed by the following system of equations

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta (T - T_\infty), \tag{1}$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2}, \tag{2}$$

where u is the velocity in the x-direction, T the temperature of the fluid, g the acceleration due to gravity, β the coefficient of thermal expansion, ν the kinematic coefficient of viscosity, ρ the fluid density, k the thermal conductivity, c_p the specific heat at constant pressure. The initial and boundary conditions are

$$u = 0,\ T = T_\infty \ \text{ for } y \text{ and } t \le 0,$$

$$u = u_0 e^{\lambda^* t},\ \frac{\partial T}{\partial y} = \begin{cases} -\dfrac{q}{k}\cdot\dfrac{t}{t_0} & \text{for } 0 < t \le t_0 \\ -\dfrac{q}{k} & \text{for } t > t_0 \end{cases} \quad \text{at } y = 0, \tag{3}$$

$$u \to 0,\ T \to T_\infty \ \text{ as } y \to \infty \ \text{ for } t > 0,$$

where q is the constant heat flux. Introducing the non-dimensional variables

$$\eta = \frac{y u_0}{\nu},\ \tau = \frac{t}{t_0}, t_0 = \frac{\nu}{u_0^2}, u_1 = \frac{u}{u_0}, \theta = \frac{u_0 k (T - T_\infty)}{q\nu}, \tag{4}$$

equations (1) and (2) become

$$\frac{\partial u_1}{\partial \tau} = \frac{\partial^2 u_1}{\partial \eta^2} + Gr\theta, \tag{5}$$

$$Pr \frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \eta^2}, \tag{6}$$

where $Pr = \dfrac{\rho \nu c_p}{k}$ is the Prandtl number and $Gr = \dfrac{g\beta \nu^2 q}{k u_0^4}$, the Grashof number. The initial and the boundary conditions given by equation (3) become

$$u_1 = 0, \theta = 0 \ \text{ for } \eta \text{ and } \tau \le 0,$$

$$u_1 = e^{\lambda\tau},\ \frac{\partial \theta}{\partial \eta} = \begin{cases} -\tau & \text{for } 0 < \tau \le 1 \\ -1 & \text{for } \tau > 1 \end{cases} \quad \text{at } \eta = 0, \tag{7}$$

$$u_1 \to 0, \theta \to 0 \ \text{ as } \eta \to \infty \ \text{ for } \tau > 0,$$

where $\lambda = \dfrac{\lambda^* \nu}{u_0^2}$ is the non-dimensional accelerated parameter. On the use of Laplace transformation (1) technique, equations (6) and (5) become

$$Pr s\bar{\theta} = \frac{d^2 \bar{\theta}}{d\eta^2}, \tag{8}$$

$$s\bar{u}_1 = \frac{d^2 \bar{u}_1}{d\eta^2} + Gr\bar{\theta}, \tag{9}$$

where

$$\bar{u}_1(\eta, s) = \int_0^\infty u_1(\eta,\tau) e^{-s\tau} d\tau \ \text{ and } \ \bar{\theta}(\eta, s) = \int_0^\infty \theta(\eta,\tau) e^{-s\tau} d\tau \tag{10}$$

and s is the Laplace transform variable.

The corresponding boundary conditions for $\bar{u}_1$ and $\bar{\theta}$ are

$$\bar{u}_1 = \frac{1}{s - \lambda},\ \frac{d\bar{\theta}}{d\eta} = -\frac{1 - e^{-s}}{s^2} \ \text{ at } \eta = 0,$$

$$\bar{u}_1 \to 0,\ \bar{\theta} \to 0 \ \text{ as } \eta \to \infty. \tag{11}$$

The solution of equations (8) and (9) subject to the boundary conditions (11) can easily be obtained and are given by

$$\bar{\theta}(\eta, s) = \frac{(1 - e^{-s})}{s^2 \sqrt{sPr}} e^{-\sqrt{sPr}\,\eta}, \tag{12}$$

$$\bar{u}_1(\eta, s) = \frac{1}{s - \lambda} e^{-\sqrt{s}\,\eta} + \frac{Gr(1 - e^{-s})}{s^{\frac{7}{2}} \sqrt{Pr}\,(Pr - 1)} \left(e^{-\sqrt{s}\,\eta} - e^{-\sqrt{sPr}\,\eta} \right). \tag{13}$$

The inverse Laplace transform of equations (12) and (13) give the temperature and the velocity distributions as

$$\theta(\eta,\tau) = F_1(\xi,\tau) - H(\tau-1)F_1(\xi,\tau-1), \tag{14}$$

$$u_1(\eta,\tau) = \frac{1}{2}e^{\lambda\tau}\left[e^{2\xi\sqrt{\lambda\tau}}\, erfc(\xi+\sqrt{\lambda\,\tau}) + e^{-2\xi\sqrt{\lambda\tau}}\, erfc(\xi-\sqrt{\lambda\,\tau})\right]$$
$$+\frac{Gr}{\sqrt{Pr}\,(Pr-1)}\left[\{G_2(\xi,\tau)-G_1(\xi,\tau)\}\right.$$

$$+\frac{Gr}{\sqrt{Pr}\,(Pr-1)}\left[\{G_2(\xi,\tau)-G_1(\xi,\tau)\}\right. \tag{15}$$

where

$$F_1(\xi,\tau) = \sqrt{\frac{\tau^3}{\pi Pr}}\left[\frac{4}{3}\left(1+Pr\xi^2\right)e^{-Pr\xi^2} - \frac{1}{6}\sqrt{Pr}\,\xi\left(6+4Pr\xi^2\right)\mathrm{erfc}\left(\sqrt{Pr}\,\xi\right)\right], \tag{16}$$

$$G_1(\xi,\tau) = \frac{\tau^{5/2}}{15}\left[\left\{\frac{1}{\sqrt{\pi}}\left(16+36Pr\xi^2+8Pr^2\xi^4\right)e^{-Pr\xi^2}\right.\right.$$
$$\left.\left.-2\xi\sqrt{Pr}(15+20Pr\xi^2+4Pr^2\xi^4)\,\mathrm{erfc}(\sqrt{Pr}\,\xi)\right\}\right], \tag{17}$$

$$G_2(\xi,\tau) = \frac{\tau^{\frac{5}{2}}}{15}\left[\left\{\frac{1}{\sqrt{\pi}}\left(16+36\xi^2+8\xi^4\right)e^{-\xi^2} - 2\xi(15+20\xi^2+4\xi^4)\,\mathrm{erfc}(\xi)\right\}\right], \tag{18}$$

$$\xi = \frac{\eta}{2\sqrt{\tau}}, \tag{19}$$

and erfc (x) being complementary error function and $H(\tau-1)$ is the unit step function.

Solution for $Pr = 1$:

The solutions of equations (6) and (5) subject to the boundary conditions (11) are

$$\bar{\theta}(\eta,s) = \frac{\left(1-e^{-s}\right)}{s^2\sqrt{s}}e^{-\sqrt{s}\,\eta}, \tag{20}$$

$$\bar{u}_1(\eta,s) = \frac{1}{s-\lambda}e^{-\sqrt{s}\,\eta} + \frac{Gr(1-e^{-s})}{2s^3}\eta e^{-\sqrt{s}\,\eta}. \tag{21}$$

The inverse Laplace transform of equations (20) and (21)give the temperature and velocity distributions as

$$\theta(\eta,\tau) = F_2(\xi,\tau) - H(\tau-1)F_2(\xi,\tau-1), \tag{22}$$

$$u_1(\eta,\tau) = \frac{1}{2}e^{\lambda\tau}\left[e^{2\xi\sqrt{\lambda\tau}}\, erfc(\xi+\sqrt{\lambda\,\tau}) + e^{-2\xi\sqrt{\lambda\tau}}\, erfc(\xi-\sqrt{\lambda\,\tau})\right]$$

$$+\frac{Gr}{2}\left[G_3(\xi,\tau) - H(\tau-1)G_3(\xi,\tau-1)\right], \tag{23}$$

where

$$F_2(\xi,\tau) = \sqrt{\frac{\tau^3}{\pi}}\left[\frac{4}{3}\left(1+\xi^2\right)e^{-\xi^2} - \frac{1}{6}\xi\left(6+4\xi^2\right)erfc\left(\xi\right)\right], \tag{24}$$

$$G_3(\xi,\tau) = 2\xi\left[\tau^{\frac{5}{2}}\left\{erfc(\xi) - 2\xi\left[\frac{1}{\sqrt{\pi}}\xi^2 - \frac{1}{2}\xi erfc(\xi)\right]\right\}\right.$$
$$\left.+\frac{1}{3}\tau^{3/2}\left\{\frac{1}{\sqrt{\pi}}\left(1-2\xi^2\right)e^{-\xi^2} + 2\xi^3 erfc\xi\left(\xi\right)\right\}\right], \tag{25}$$

where ξ is given by (19).

RESULTS AND DISCUSSION

The effect of heat transfer on the flow of a viscous incompressible fluid along exponentially accelerated moving vertical infinitely long plate has been considered. The governing non-dimensional linear coupled partial differential equations (5) and (6) have been solved analytically by the use of Laplace Transform technique. The numerical result of the non-dimensional velocity u_1 and the temperature distribution θ for several values of Prandtl number Pr, Grashof number Gr, accelerated parameter λ and time τ are presented in Figs.2-7. It is observed from Fig.2 that the velocity u_1 decreases with an increase in Prandtl number Pr. This is consistent with the physical point of view that the fluids with high Prandtl number have greater viscosity, which makes the fluid thick and hence move slowly. It is also observed that the temperature is maximum near the plate and gradually decreases away from the plate and finally tends to zero for all values of Prandtl number Pr. Fig.3 reveals that an increase in Grashof number Gr leads to rise in the fluid velocity u_1. This is due to the contribution from the buoyancy force near the plate because as the Grashof number increases the buoyancy force becomes significant and hence a rise in the velocity near the plate is observed. It is seen from Figs.4 and 5 that the velocity u_1 increases with an increase in either accelerated parameter λ or time τ. It means that the accelerated parameter and time accelerate the fluid motion. It may be noted that an increase of Prandtl number causes the decrease of thermal boundary layer thickness that is why the temperature distribution across the thermal boundary layer decreases. Fig.6 displays that the temperature decreases with an increase in Prandtl number Pr.

Table 1. Shear stress $-\tau_x$ at the moving plate $\eta = 0$ for $\lambda = 0.5$

	Pr with $Gr = 5$				Gr with $Pr = 0.71$			
τ	0.71	2	3	7	5	10	15	20
0.1	4.54490	3.14018	2.81390	2.38329	4.54490	7.12120	9.69750	12.27380
0.2	3.46362	2.41007	2.16537	1.84241	3.46362	5.39584	7.32806	9.26029
0.3	2.66007	1.95771	1.79457	1.57927	2.66007	3.94822	5.23637	6.52452
0.4	1.94457	1.59339	1.51182	1.40417	1.94457	2.58865	3.23272	3.87680

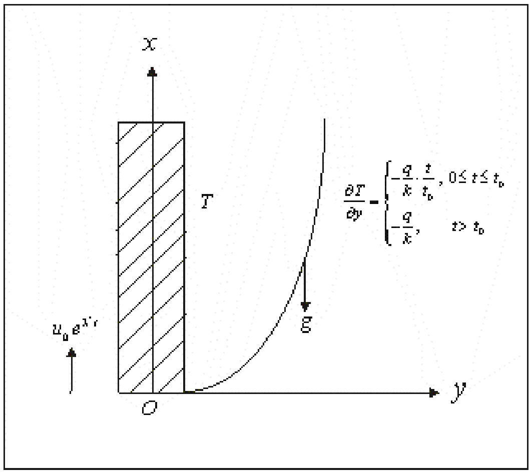

Fig. 1. Geometry of the problem

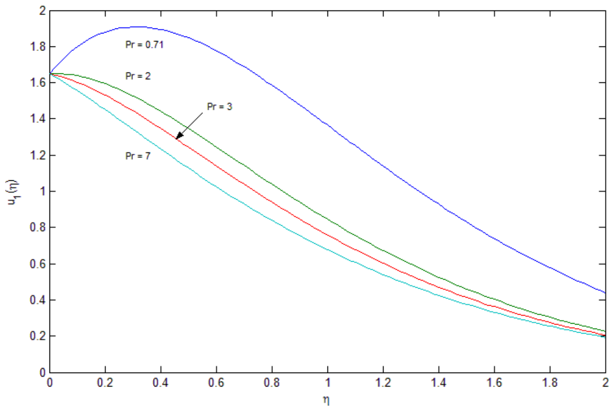

Fig. 2. Velocity profiles for Prandtl number Pr when $Gr = 5$, $\lambda = 0.5$ and $\tau = 0.2$

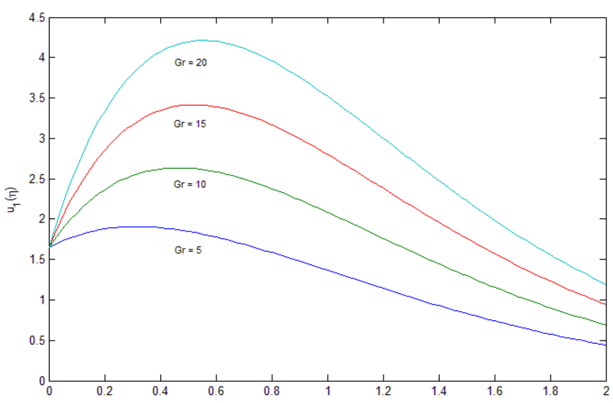

Fig. 3. Velocity profiles for Grashof number Gr when $Pr = 0.71$, $\lambda = 0.5$ and $\tau = 0.2$

It is observed from Fig.7 that the fluid temperature increases when time τ progresses.

The plate temperature $\theta(0,\tau)$ and shear stress τ_x at the plate $(\eta = 0)$ are given by

$$\theta(0,\tau) = F_1(0,\tau) - H(\tau-1)F_1(0,\tau-1) \tag{26}$$

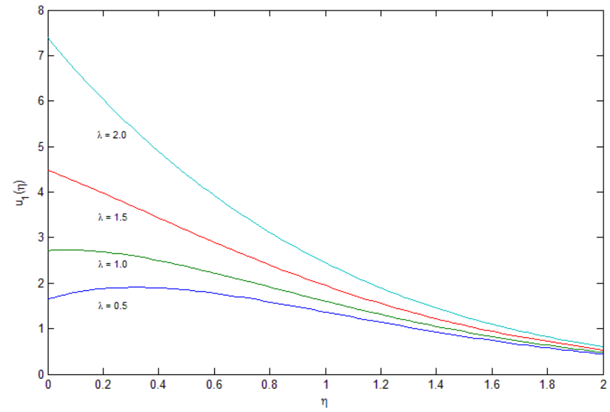

Fig. 4. Velocity profiles for accelerated parameter λ when $Pr = 0.71$, $Gr = 5$ and $\tau = 0.2$

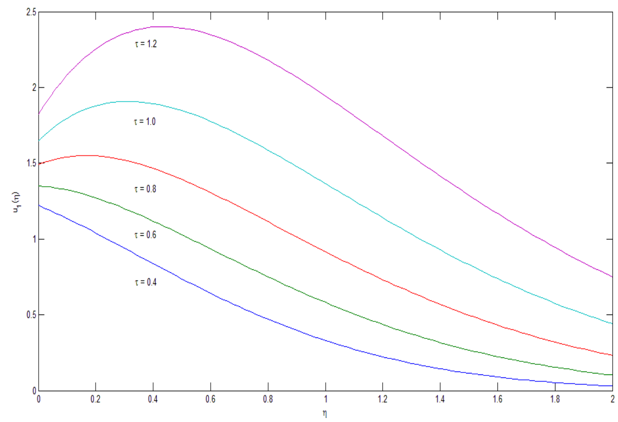

Fig. 5. Velocity profiles for time τ when $Pr = 0.71$, $Gr = 5$, $\lambda = 0.5$

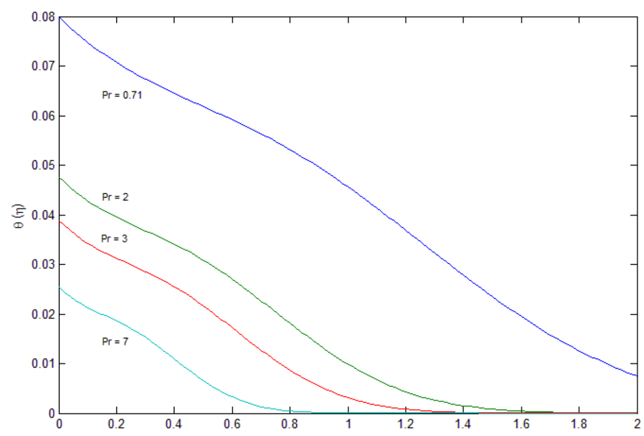

Fig. 6. Temperature profiles for Prandtl number Pr when $\lambda = 0.5$ and $\tau = 0.2$

and

$$-\tau_x = -\left(\frac{\partial u_1}{\partial \eta}\right)_{\eta=0} = \begin{cases} \frac{1}{\sqrt{\pi\tau}}\left[1+\sqrt{\pi\lambda\tau}\,e^{\lambda\tau}\,\mathrm{erf}\,(\sqrt{\lambda\tau})\right] \\ +\frac{Gr}{\sqrt{Pr}\,(\sqrt{Pr}+1)}\left[\tau^2-(\tau-1)^2\right] & \text{for } Pr \neq 1, \\ \frac{1}{\sqrt{\pi\tau}}\left[1+\sqrt{\pi\lambda\tau}\,e^{\lambda\tau}\,\mathrm{erf}\,(\sqrt{\lambda\tau})\right] \\ +\frac{3Gr}{2\sqrt{\pi}}\left[\tau^{3/2}-(\tau-1)^{3/2}\right] & \text{for } Pr = 1, \end{cases} \tag{27}$$

where

$$F_1(0,\tau) = \frac{4}{3}\tau\sqrt{\frac{\tau}{\pi Pr}}. \tag{28}$$

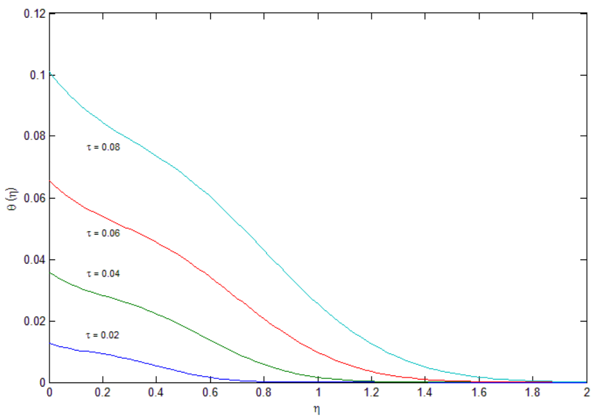

Fig. 7. Temperature profiles for time τ when $Pr = 0.71$, $\lambda = 0.5$

Numerical values of the shear stress τ_x at the plate $(\eta = 0)$ due to the flow are presented in Table 1 for several values of Prandtl number Pr, Grashof number Gr and time τ with $\lambda = 0.5$. Table 1 shows that the absolute value of the shear stress τ_x at the plate $(\eta = 0)$ decreases with an increase in Prandtl number Pr while it increases with an increase in Gr for fixed values of time τ as it expected since the fluid velocity decreases with an increase in Prandtl number Pr and it increases with an increase in time τ. Further, it is seen that for fixed values of Pr and Gr, the absolute value of the shear stress τ_x decreases with an increase in time τ. It is seen from (26) and (27) that the temperature at the plate $(\eta = 0)$ varies with $\frac{1}{\sqrt{Pr}}$.

Conclusion

The effects of heat transfer on the flow past an exponentially accelerated vertical plate with ramped wall heat flux have been investigated. It is found that the fluid velocity decreases with an increase in Prandtl number Pr. An increase in Grashof number Gr leads to rise the fluid velocity u_1. The fluid velocity u_1 increases with an increase in either accelerated parameter λ or time τ. The fluid temperature θ decreases with an increase in Prandtl number Pr whereas it increases with an increase in time τ. Further, it is found that the magnitude of shear stress τ_x at the plate $(\eta = 0)$ decreases with an increase in Prandtl number Pr while it increases with an increase in Gr.

The absolute value of the shear stress τ_x reduces when time τ progresses. It is noted that the plate temperature varies with $\frac{1}{\sqrt{Pr}}$ for fixed value of time τ.

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