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RESEARCH ARTICLE

ON THE HOMOGENEOUS BIQUADRATIC EQUATION WITH FIVE UNKNOWNS $x^4 - y^4 = 40(z + w)p^3$

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ABSTRACT

The bi-quadratic equation with 5 unknowns given by $x^4 - y^4 = 40(z + w)p^3$ is analysed for its patterns of non-zero distinct integral solutions. Four different patterens of integer solutions to the above bi-quadratic equation are presented. A few interesting relations between the solutions and special numbers, namely, polygonal number and pyramidal number are exhibited.

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INTRODUCTION

Biquadratic Diophantine equations, homogeneous and non-homogeneous, have aroused the interest of numerous mathematicians since antiquity as can be seen as from (Gopalan *et al*., 2007, Gopalan and Anbuselvi 2008, Gopalan and pandichelvi 2008, Gopalan and Janaki 2008, Gopalan *et al*., 2008, 2010, 2015). In this context, one may refer (Gopalan and pandichelvi 2009, Gopalan and Shanmukanandham 2010, Gopalan and Padma 2010, Gopalan *et al*., 2012, Gopalan and sivakami, 2013) for varieties of problems on the biquadratic diophantine equations with three and four variables.

Further, in (Gopalan and Kalingarani, 2009, Gopalan and Kalingarani, 2011, Gopalan *et al*., 2013, 2014) biquadratic equations with five variables are analysed. In this paper, biquadratic equations with five variables given by $x^4 - y^4 = 40(z + w)p^3$ is analysed for its non zero distinct integer solutions. A few interesting relations between the solutions and special polygonal numbers, pyramidal numbers are exhibited.

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Notations

$$t_{m,n} = n\left(1 + \frac{(n-1)(m-2)}{2}\right),$$

$$P_n^5 = \frac{n^2(n+1)}{2},$$

$$Pr_n = n(n+1)$$

MATERIALS AND METHODS

The diophantine equation representing the biquadratic equation with five unknowns under consideration is given by

$$x^4 - y^4 = 40(z + w)p^3 \qquad \text{..............................(1)}$$

Introducing the transformations

$$x = u + v;\ y = u - v;\quad z = uv + 1;\ w = uv - 1 \qquad \text{..............................(2)}$$

in (1), it simplifies to

$$u^2 + v^2 = 10p^3 \qquad \text{..........................(3)}$$

The above equations (3) is solved through different methods and thus, one obtains distinct patterns of integer solutions of (1)

PATTERN: 1

Let p= $p(a,b) = a^2 + b^2$ ………………….(4)

Write 10 as

$10 = (3+i)(3-i)$ ……………………………….(5)

Substituting (4) & (5) in (3) and using the method of factorization, define

$$u + iv = (3+i)(a+ib)^3$$

Equating real and imaginary parts, we have

$$u = 3a^3 + b^3 - 9ab^2 - 3a^2b$$

$$v = a^3 - 3b^3 - 3ab^2 + 9a^2b$$

Substituting the values of u and v in (2), the non zero distinct integral solutions of (1) are given by

$$x(a,b) = 4a^3 - 2b^3 - 12ab^2 + 6a^2b$$

$$y(a,b) = 2a^3 + 4b^3 - 6ab^2 - 12a^2b$$

$$z(a,b) = 3a^6 - 3b^6 + 24a^5b + 24ab^5 - 45a^4b^2 + 45a^2b^4 - 80a^3b^3 + 1$$

$$w(a,b) = 3a^6 - 3b^6 + 24a^5b + 24ab^5 - 45a^4b^2 + 45a^2b^4 - 80a^3b^3 - 1$$

………… (6)

PROPERTIES

- $x(a,1) - 2\,y(a,1) - 30t_{4,a} + 10 = 0$
- $6[\,2p(a,-a)]$ is a nasty number
- $y(a,1) - 3p(a,1) - 4p_a^5 + 17t_{4,a} \equiv 1 \pmod 6$
- $x(a,1) + y(a,1) - 6p(a,1) - 12p_a^5 + 18pr_a + 4 = 0$
- $y(1,b) - 8p_b^5 + 10pr_b = 0 \pmod 2$
- $y(a,1) + 6ap(a,1) - 16p_a^5 + 20t_{4,a} - 4 = 0$

PATTERN: 2

Write 10 as

$10 = (-3+i)(-3-i)$ ……………………… (7)

Substituting (4) & (7) in (3) and using the method of factorization, define

$$u + iv = (-3+i)(a+ib)^3$$

Equating real and imaginary parts, we have

$$u = -3a^3 + b^3 + 9ab^2 - 3a^2b$$

$$v = a^3 + 3b^3 - 3ab^2 - 9a^2b$$

Substituting the values of u and v in (2), the non-zero distinct integral solutions of (1) are given by

$$x(a,b) = -2a^3 + 4b^3 + 6ab^2 - 12a^2b$$
$$y(a,b) = -4a^3 - 2b^3 + 12ab^2 + 6a^2b$$
$$z(a,b) = -3a^6 + 3b^6 + 24a^5b + 24ab^5 + 45a^4b^2 - 45a^2b^4 - 80a^3b^3 + 1$$
$$w(a,b) = -3a^6 + 3b^6 + 24a^5b + 24ab^5 + 45a^4b^2 - 45a^2b^4 - 80a^3b^3 - 1$$

……….. (8)

PROPERTIES

- $2y(1,a) + x(1,a) - 30t_{4,a} + 10 = 0$
- $2[\,4(p(a,a)) - \text{nasty number}] = \text{perfect square}$
- $y(1,a) - 4\,p(1,a) + 4p_a^5 - 4t_{4,a} - 6pr_a + 8 = 0$
- $3y(a,1) + 12ap(a,1) - 18t_{4,a} \equiv 0 \pmod 4$
- $x(a,1) - y(a,1) + p(a,1) - 4p_a^5 + 19t_{4,a} \equiv 7 \pmod 6$

PATTERN 3

Consider 10 as

$$10=(1+3i)(1-3i) \quad \ldots\ldots\ldots\ldots(9)$$

Substituting (4) & (9) in (3) and using the method of factorization, define

$$u+iv=(1+3i)(a+ib)^3$$

Equating real and imaginary parts, we have

$$u=a^3+3b^3-3ab^2-9a^2b$$
$$v=3a^3-b^3-9ab^2+3a^2b$$

Substituting the values of u and v in (2), the non zero distinct integral solutions of (1) are given by

$$x(a,b)=4a^3+2b^3-12ab^2-6a^2b$$
$$y(a,b)=-2a^3+4b^3+6ab^2-12a^2b$$
$$z(a,b)=3a^6-3b^6-24a^5b-24ab^5-45a^4b^2+45a^2b^4+80a^3b^3+1$$
$$w(a,b)=3a^6-3b^6-24a^5b-24ab^5-45a^4b^2+45a^2b^4+80a^3b^3-1 \quad \ldots\ldots(10)$$

PROPERTIES

- $2x(a,1)+4y(a,1)+60t_{4,a}-20=0$
- $[-x(a,a)-2ap(a,a)]$ is a cubical integer
- $x(1,a)-4p_a^5+14t_{4,a} \equiv 4(\mathrm{mod}\,6)$
- $y(b,1)-p(b,1)+4p_b^5+11t_{4,b} \equiv 0(\mathrm{mod}\,3)$
- $x(a,1)+2y(a,1)+p(a,1)+29t_{4,a}-11=0$

PATTERN 4

Consider 10 as

$$10=(-1+3i)(-1-3i) \quad \ldots\ldots\ldots\ldots(11)$$

Substituting (4) & (11) in (3) and using the method of factorization, define

$$u+iv=(-1+3i)(a+ib)^3$$

Equating real and imaginary parts, we have

$$u=-a^3+3b^3+3ab^2-9a^2b$$
$$v=3a^3+b^3-9ab^2-3a^2b$$

Substituting the values of u and v in (2), the non zero distinct integral solutions of (1) are given by

$$x(a,b)=2a^3+4b^3-6ab^2-12a^2b$$
$$y(a,b)=-4a^3+2b^3+12ab^2-6a^2b$$
$$z(a,b)=-3a^6+3b^6-24a^5b-24ab^5+45a^4b^2-45a^2b^4+80a^3b^3+1$$
$$w(a,b)=-3a^6+3b^6-24a^5b-24ab^5+45a^4b^2-45a^2b^4+80a^3b^3-1 \quad ..(12)$$

PROPERTIES

- $x(a,1)-4p_a^5+6pr_a+8t_{4,a}-4=0$
- $y(a,1)+8p_a^5-10t_{4,a}-2=0$
- $2x(a,1)+y(a,1)+30t_{4,a}-10=0$
- $5p(2a+1,a+1)-25t_{4,a} \equiv 0(\mathrm{mod}\,10)$
- $y(1,a)+p(1,a)-4p_a^5-11t_{4,a} \equiv -3(\mathrm{mod}\,6)$
- $3p(a+2,2a)-3t_{4,a}-12pr_a-12=0$

Conclusion

In this paper, we have presented infinitely many non-zero integral solutions for the non-homogeneous biquadratic equation with five unknowns $x^4-y^4=40(z+w)p^3$. As biquadratic equations are rich in variety, one may consider the other forms of equations and search for their corresponding integer solutions.

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