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RESEARCHARTICLE

SOME MODIFIED EXPONENTIAL TYPE UNBIASED ESTIMATORS USING AUXILIARY ATTRIBUTE IN SIMPLE RANDOM SAMPLING

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ABSTRACT

The main concentration of this paper is to suggestmodified exponential type unbiased ratio and product estimators using auxiliary attribute in simple random sampling. A comparative analysis of efficiency is carried out between suggested and existing estimators theoretically as well as numerically.

INTRODUCTION

The estimation of the population parameters is a major challenge in sampling theory and many researchers in this field make serious efforts in the direction of efficiency and precision improvement of estimators of unknown population parameter of interest when study variable is highly correlated with the auxiliary variable. When the correlation between study variable and auxiliary variable is positive we use the ratio estimator and if correlation is negative we use the product estimator.In many situations auxiliary information is ignored when information is qualitative in nature such as sex and height of the person, amount of milk produced and a particular breed of the cow, amount of yield of wheat crop and a particular variety of wheat etc.(Shabbir and Gupta, 2010). In such situations, taking the advantage of point bi-serial correlation between the study variable y and the auxiliary attributes the estimators of population parameter of interest can be constructed by using prior knowledge of the population parameter of auxiliary attribute.The concentration of this paper is to suggest modified exponential type unbiased ratio and product estimators for estimating the population meanusing auxiliary attribute in simple random sampling. The unbiasedness and variance of the suggested estimators have been obtained. A comparative analysis of efficiency is carried out between suggested and existing estimators theoretically as well as numerically.

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Suggested Sampling Scheme and Notation

By consideringa finite population which hasN identifiable units $\Theta_i\,(1 \le i \le N)$ and using simple random sampling without replacement (SRSWOR)a sample of size n is drawn from a population of size N. Here Φ_i and y_i denote the observations on the variable Φ and y respectively for i^{th} unit (i=1, 2,……, N). Suppose there is a complete dichotomy in the population with respect to the presence or absence of an attribute, say Φ, and it is assumed that attribute Φ takes only two values 0 and 1 according as

$\Phi_i = 1$ if i^{th} unit of the population possesses attribute Φ

= 0 otherwise.

Here $P = \sum_{i=1}^{N}\Phi_i \Big/ N$ and $p = \sum_{i=1}^{n}\Phi_i \Big/ n$ denote the proportion of units in the population and sample respectively possessing attribute Φ. Let $\bar{y} = \frac{1}{n}\sum_{i}^{n} y_i$ and $p = \frac{1}{n}\sum_{i}^{n}\Phi_i$ be the sample means of variable of interest y and auxiliary attribute Φ, $\bar{Y} = \frac{1}{N}\sum_{i}^{N} y_i$ and $P = \frac{1}{N}\sum_{i}^{N}\Phi_i$ be the corresponding population means. Let $s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$ and $s_\Phi^2 = \frac{1}{n-1}\sum_{i=1}^{n}(\Phi_i - p)^2$ be the sample variance and $S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{Y})^2$ and $S_\Phi^2 = \frac{1}{N-1}\sum_{i=1}^{N}(\Phi_i - P)^2$ be the corresponding population variance. Let $C_y = S_y/\bar{Y}$ and $C_\Phi = S_\Phi/P$ .Let $\rho_{y\Phi} = S_{y\Phi}/S_y S_\Phi$ be the point bi-serial correlation coefficient between y and υ. To determine the characteristic of the suggested estimators and existing estimators considered here, we define the following terms, $\Upsilon_y = \frac{\bar{y} - \bar{Y}}{\bar{Y}}$ *and* $\Upsilon_\Phi = \frac{p - P}{P}$ such that $E\,[\Upsilon_i] = 0 \ \ for \ \ (i = y, \Phi)$, $E(\Upsilon_y^2) = \Psi \ S_y^2/\bar{Y}^2$, $E(\Upsilon_\Phi^2) = \Psi \ S_\Phi^2/P^2$ and $E(\Upsilon_y \Upsilon_\Phi) = \Psi \ \rho_{y\Phi} \ S_y/\bar{Y} \ S_\Phi/P$ where $\Psi = \left(\frac{1}{n} - \frac{1}{N}\right)$.

Existing Estimators

In this section, we consider the several existing estimators which are used for the estimation of population mean.

Suggested Estimators and its Properties

In this section, we propose the modified exponential type ratio and product estimatorsby using sampling design defined in section 2 for estimation of population mean as:

Modifiedexponential type ratio estimator

$$\theta_5 = [\,\bar{y} - (e^{A_1} - 1)] \qquad \text{...(1)}$$

Modifiedexponential type product estimator

$$\theta_6 = [\,\bar{y} - (e^{A_2} - 1)] \qquad \text{...................................} \quad (2)$$

where $A_1 = \left[P - \frac{NP - np}{N - n}\right]$ and $A_2 = \left[\frac{NP - np}{N - n} - P\right]$

To obtain the unbiasedness and variance of θ_5 and θ_6 up to the first order of approximation, equation (1) and (2) we expend the in terms of $\Upsilon's$.
The right hand side of equation (1) expand up to the first order of approximation in terms of $\Upsilon's$

Table 1. Exiting ratio and product estimators using auxiliary attribute in simple random sampling.

S.No.	Estimators	Mean Square Error [MSE(■)]
1.	$\theta_1 = \bar{y}\exp\left(\frac{P-p}{P+p}\right)$ [Bahl&Tuteja (1991) - Ratio Estimator]	$MSE(\theta_1) = \Psi \ \bar{Y}^2\left[C_y^2 + \frac{1}{4}C_\Phi^2 - \rho_{y\Phi} \ C_y C_\Phi\right]$
2.	$\theta_2 = \bar{y}\exp\left(\frac{p-P}{p+P}\right)$ [Bahl&Tuteja (1991)–Product Estimator]	$MSE(\theta_2) = \Psi \ \bar{Y}^2\left[C_y^2 + \frac{1}{4}C_\Phi^2 + \rho_{y\Phi} \ C_y C_\Phi\right]$
3.	$\theta_3 = \left(\frac{\bar{y}}{p}\right)P$ [Naik& Gupta (1996) - Ratio Estimator]	$MSE(\theta_3) = \Psi \ \left[S_y^2 + R^2 S_\Phi^2 - 2R\rho_{y\Phi} S_y \ S_\Phi\right]$
4.	$\theta_4 = \left(\frac{\bar{y}}{P}\right)p$ [Naik& Gupta (1996) -Product Estimator]	$MSE(\theta_4) = \Psi \ \left[S_y^2 + R^2 S_\Phi^2 - 2R\rho_{y\Phi} S_y \ S_\Phi\right]$

Table 2. Values of Parameters

$N = 89$	$\bar{Y} = 3.36$	$P = 0.124$	$\rho_{y\Phi} = 0.766$	$n = 23$	$C_y = 0.604$	$C_\Phi = 2.19$

Table 3. MSEs and Percentage relative efficiency

Estimator	MSEs	Percent Relative Efficiency			
		w.r.t θ_1	w.r.t. θ_2	w.r.t. θ_3	w.r.t. θ_4
θ_1	0.2841	100	■	■	■
θ_2	0.8696	■	100	■	■
θ_3	1.0836	■	■	100	■
θ_4	1.9511	■	■	■	100
θ_5	0.1960	144	443	552	995
θ_6	0.2132	133	407	508	915

Rewriting θ_5 as

$$\theta_5 = \bar{Y}\left[1+\Upsilon_y\right]-\left[\frac{nP\Upsilon_\Phi}{N-n}\right] \qquad (3)$$

it is clear that $E(\theta_5)=\bar{Y}$, after taking expectation on both sides of equation (3), i.e. up to the first order of approximation θ_5 is an unbiased estimator of the population mean $\bar{Y}$.

Now variance of θ_5 can be calculated as

$$Var(\theta_5) = E\left[\theta_5 - E(\theta_5)\right]^2$$

After neglecting the higher order terms

$$Var(\theta_5) = E\left[\bar{Y}\Upsilon_y - \frac{n}{N-n}P\Upsilon_\Phi\right]$$

$$Var(\theta_5) = \bar{Y}^2 E(\Upsilon_y^2) + P^2(\frac{n}{N-n})^2 E(\Upsilon_\Phi^2) - \frac{2n}{N-n}\bar{Y}P E(\Upsilon_y\Upsilon_\Phi)$$

After simplification variance of θ_5 is

$$Var(\theta_5) = \Psi\left[S_y^2 + (\frac{n}{N-n})^2 S_\Phi^2 - \frac{2n}{N-n}\rho_{y\Phi}S_y S_\Phi\right] \qquad (4)$$

Same as , right hand side of equation (2), expend up to the first order of approximation in terms of $\Upsilon's$

$$\theta_6 = \bar{Y}\left[1+\Upsilon_y\right]-\left[\{1+\frac{NP-nP(1+\Upsilon_\Phi)}{N-n}-P\}-1\right]$$

Rewriting θ_6 as

$$\theta_6 = \bar{Y}\left[1+\Upsilon_y\right]+\left[\frac{nP\Upsilon_\Phi}{N-n}\right] \qquad (5)$$

We can easily prove that $E(\theta_6)=\bar{Y}$ i.e. θ_6 is an unbiased estimator of the population mean $\bar{Y}$.

The variance of θ_6 can be calculated as

$$Var(\theta_6) = E\left[\theta_6 - E(\theta_6)\right]^2$$

$$= E\left[\bar{Y}\Upsilon_y + \frac{n}{N-n}P\Upsilon_\Phi\right] \text{ (neglecting the higher order terms)}$$

$$= \bar{Y}^2 E(\Upsilon_y^2) + P^2(\frac{n}{N-n})^2 E(\Upsilon_\Phi^2) + \frac{2n}{N-n}\bar{Y}P E(\Upsilon_y\Upsilon_\Phi)$$

After simplification variance of θ_6 is

$$Var(\theta_6) = \Psi\left[S_y^2 + (\frac{n}{N-n})^2 S_\Phi^2 + \frac{2n}{N-n}\rho_{y\Phi}S_y S_\Phi\right] \qquad (6)$$

Efficiency Comparison

In this section we compare efficiency of the suggested estimators with existing estimators defined in section 2 and we derive the conditions in which suggested estimators are better than the existing estimators:

Observation(1)

$$MSE(\theta_1)-Var(\theta_5) = [-\rho_{y\Phi}RS_yS_\Phi + \frac{1}{4}R^2S_\Phi^2 - (\frac{n}{N-n})^2S_\Phi^2 + \frac{2n}{N-n}\rho_{y\Phi}S_yS_\Phi] > 0$$

$$\text{if} \quad \rho_{y\Phi} > \frac{S_\Phi}{S_y}\left[(\frac{n}{N-n})^2 - \frac{R^2}{4}\Big/\frac{2n}{N-n} - R\right]$$

Observation (2)

$$MSE(\theta_2)-Var(\theta_6) = [-\rho_{y\Phi}RS_yS_\Phi + \frac{1}{4}R^2S_\Phi^2 - (\frac{n}{N-n})^2S_\Phi^2 - \frac{2n}{N-n}\rho_{y\Phi}S_yS_\Phi] > 0$$

$$\text{if} \quad \rho_{y\Phi} < -\frac{S_\Phi}{S_y}\left[(\frac{n}{N-n})^2 - \frac{R^2}{4}\Big/\frac{2n}{N-n} - R\right]$$

Observation (3)

$$MSE(\theta_3)-Var(\theta_5) = [R^2S_\Phi^2 - 2R\rho_{y\Phi}S_yS_\Phi - (\frac{n}{N-n})^2S_\Phi^2 + \frac{2n}{N-n}\rho_{y\Phi}S_yS_\Phi] > 0$$

$$\text{if} \quad \rho_{y\Phi} > \frac{S_\Phi}{2S_y}\left(\frac{n}{N-n}+R\right)$$

Observation(4)

$$MSE(\theta_4)-Var(\theta_6) = [R^2S_\Phi^2 - 2R\rho_{y\Phi}S_yS_\Phi - (\frac{n}{N-n})^2S_\Phi^2 - \frac{2n}{N-n}\rho_{y\Phi}S_yS_\Phi] > 0$$

$$\text{if} \quad \rho_{y\Phi} < -\frac{S_\Phi}{2S_y}\left(\frac{n}{N-n}+R\right)$$

Numerical Study

Now we compare the performance of various estimators considered here using the data sets as previously used by Shabbir and Gupta (2010).

Population: (Source: Sukhatme and Sukhatme (1970), pp. 256).

y = Number of villages in the circles.

Φ = A circle consisting more than five villages.

Conclusion

In section 4, we suggested modified exponential type unbiased ratio and product estimators and alsoobtained the characteristic of the suggested estimators. In section 5, we derive the general conditions in which we can say the suggested estimators are always more efficient than the existing estimators. In section 6, numerical study is done using the data used by Shabbir and

Gupta (2010). In numerical study, we see that the suggested estimators have highest percent relative efficiency w.r.t to all existing estimators defined in section 2.

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