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RESEARCH ARTICLE

AN ANALYTICAL SOLUTION OF THE DUAL PHASE LAG BIOHEAT TRANSFER MODEL OF HUMAN DENTIN SUB-JECTED TO SHORT-PULSED ER: YAG LASER IRRADIATION

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ABSTRACT

A dual phase lag mathematical model for the temperature evolution in the dentin layer of a human tooth exposed to Er: YAG solid-state laser is presented by including the phase lag in heat flux and the phase lag in the temperature gradient in the Fourier heat transfer model and a laser source term, described by the Beer-Lambert law. The Laplace transformation technique is used to obtain an analytical solution to the model and the computational results are presented through the graphs. The influence of various model parameters on the temperature distribution and difference in the dentin are investigated and illustrated. It displayed and described how several common characteristics, such as the heat flux phase lag and the number of pulses, affect temperature distribution and changes.

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INTRODUCTION

A frequent chronic infectious condition is dental caries caused by cariogenic bacteria that stick to teeth and metabolize sugars to produce acid,which eventually demineralizes the tooth structure. With this disease, bacterial activities harm the hard tooth structures. Dental caries is the result of the gradual breakdown of these tissues. Cavities begin with microscopic openings or holes in the hard, outer enamel of the tooth caused by the acids in plaque. As portions of the enamel are worn away, germs and acid can enter the dentin, the next layer of the tooth. If the disease is not treated, caries can reach the pulp layer of the tooth. Once the germs get to the pulp, they can annihilate the essential organic components of the tooth. As a result, dental caries must be treated immediately once they are found. Dental caries, which cause dentin colouration because different chromophores from bacteria present are more usually discovered once they reach the dentin layer because the demineralization of enamel is scarcely evident. As a result, the dentin layer of the tooth is the site of most studies on caries therapy. Lasers have also been researched as an alternative to remove dentin cavities due to the need for minimally invasive dental caries ablation techniques. The operating wavelengths of the Er: YAG, Er: YSGG, and Er, Cr: YSGG lasers which are 2940, 2790, and 2780 nm, respectively, are three examples of laser systems with similar wavelengths that are currently utilized often for cavity preparation and caries removal. A photomechanical mechanism predominates in the dentin ablation process. The composition and structure of dentin are not greatly impacted by laser exposure because of this ablation mechanism. Dentin's cohesiveness is reduced by the photochemical ablation of collagen, enabling the removal of the tissue with little thermal harm to neighbouring tissues. The cohesive strength of the tissue is decreased because of the ablation of dentin, which is affected by the ablation of collagen fibrils at low fluences. Findings (1-3) have demonstrated that the application of the Er: YAG laser to dental hard tissues is both secure and efficient for the removal of caries. Everyone appears to agree that Er: YAG is one of the most effective lasers because its wavelength, which is 2.940 nm, is greatly absorbed by water and hydroxyapatite. In the experimental investigations of the laser-dentin tissue interaction, a need for theoretical studies of the interaction was realized and a few mathematical models for the temperature distribution in the laser-irradiated dentin tissue were developed and analyzed. The increase in dental tissue temperature caused by irradiation with an ultrashort laser was addressed by Pavlina Pike *et al*. (4) in their study using algebraic and numerical solutions in cylindrical coordinates. They assumed a steady source of heat and consistent thermophysical characteristics.

The computation of the tissue temperature increase caused by the interaction of ultrafast laser pulses with dentin was the main objective of this study. The interaction of femtosecond laser pulses with dental tissue was described using the analytical solution to the Pennes Bioheat Equation. A mathematical model for the dentin subjected to Nd: YAG laser irradiation was developed by Moriyama *et al*. (5). Since dentin has a heterogeneous structure, it has been difficult to fully understand how laser therapy for hypersensitivity works. The finite difference approach developed by Crank and Nicolson was used to solve the mathematical model and result computations. In this study, the dentin surface was subjected to laser-induced perturbations and the mathematical model was used to simulate heat diffusion on the surface and within the dentin. To determine the temperature distribution in the dentin under Er: YAG pulse laser radiation, P. Elahi and B. Farsi (6) developed a mathematical model. Analytical solutions to the Pennes bioheat equation and the heat deposited from laser radiation were applied to obtain the temperature distribution. The graphs for temperature distribution and profiles of dentin were plotted and they compared their results with the experimental findings. A 2D experimental and theoretical research for thermal transfer in a thin piece of human tooth induced by a pulsed laser was presented by K. P. Chang *et al*. (7). They solved the classical heat conduction equation in a dental tissue subjected to ultra short pulsed lasers. The results were explained and validated with the experimental results. The credibility of this simplified thermal model is examined when the numerical and experimental data are compared. However, it was demonstrated that the 1D model simulation greatly deviates from the 2D axisymmetric one, indicating that caution should be exercised when a 1D thermal model is taken into consideration for calculating temperature response. Matsumoto *et al*. (8) assessed the efficacy of the Er: YAG laser under some clinical conditions and in different types of cavities. This minimizes the duration of the surgery and is effective, safe, efficient, and suited for removing caries. Only if careful attention is paid to the proper selection of laser settings can the appropriate interaction of lasers with dental hard tissue result in the effective and safe removal of the affected structures. Contrarily, if improper settings are picked, there could be unintended consequences including heat and mechanical tissue injury. As a result, choosing the best irradiation conditions is a crucial stage in laser research. Almost all of the models, which are based on Fourier's law and infinite heat diffusion velocity, have been reviewed over the previous ten years. When transitory heat flows for short periods, with high heat flux and non-homogeneous inner structure, Fourier law fails in anticipating the correct temperature distribution. There is a delay between the heat flux and the temperature gradient and a delay between the temperature gradient and energy transport because the inner structure of the dentin is complex and heterogeneous. To develop a mathematical model for temperature evolutionto ablate caries in dentin**.** Pennes bioheat equation should be integrated with the Fourier law of heat conduction, which takes the relaxation time of heat flux and energy transport into account. The temperature evolution in dentin tissue exposed to Er: YAG laser irradiation for the ablation of dentin caries is determined in the current article using an analytical method in conjunction with the Laplace transform. Due to their reduced cost and more precise estimation when compared to experimental and numerical computations, analytical solutions are particularly intriguing. A contemporary mathematical model is a useful tool for assessing the heat transmission of human dentin for the ablation of cavities in the dentin, as shown by the comparison between the numerical.

Physical Model and Mathematical Formulation: Human dentin is represented as a homogeneous, finite zone of thickness 'l' with constant thermal characteristics and a thermally isolated bottom boundary when it is exposed to pulsed laser irradiation at its anterior surface under Er: YAG solid-state laser for dentin caries ablation. The upper surface of the tissue is exposed to the laser beam as usual. The origin of a cylindrical coordinate system and the centre of the incident laser beam are congruent (Fig.1). The remaining boundary surfaces are thermally isolated from the surroundings, and the tissue is initially at a constant temperature T_0.Er: YAG laser systems that are used to remove caries from dentin integrate water cooling, which allows them to ablate the surrounding tissue with few thermal adverse effects. The coolant temperature has been reported 300 K (6)which is the temperature of the environment. The axisymmetric and axial heat transfer is considered in the dentin model. The laser heat source is described using the Beer-Lambert law which asserts that radiation intensity falls exponentially with the depth of penetration. The laser beam energy is assumed to be absorbed in the dentin.

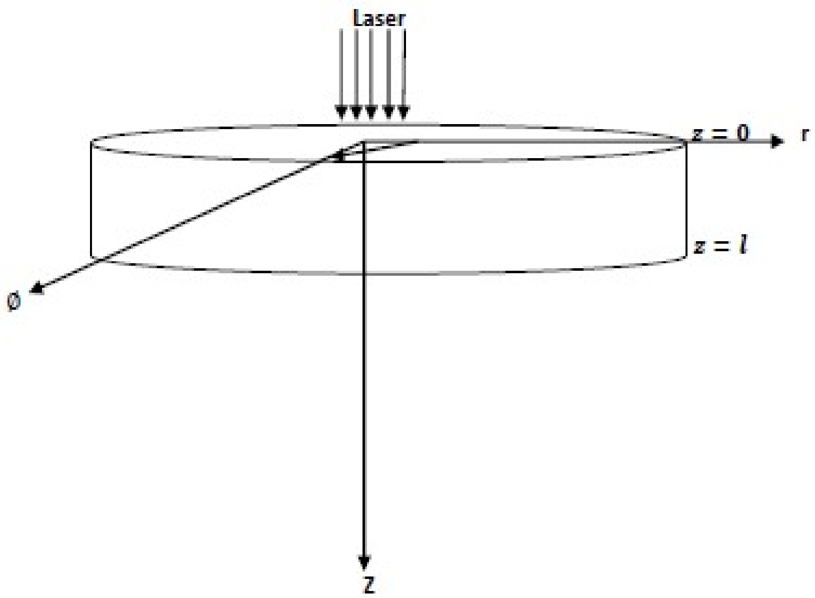

Fig. 1. Dentin exposed to Er: YAG short-pulsed laser, shown schematically as a finite domain

The heat transport in living biological tissues is governed by the Pennes equation (9),

$$\rho c\frac{\partial T}{\partial t} = -\nabla . q + \rho_b c_b \omega_b (T_b - T) + q_{mb} + Q \quad (1)$$

where ρ (Kgm^{-3}) is the density of tissue, $c(JKg^{-1}K^{-1})$ the specific heat of tissue, T (°C) the tissue temperature, $q(\,Wm^{-2}\,)$ the heat flux, $\rho_b(\,Kgm^{-3})$ the blood density, $c_b(JKg^{-1}K^{-1})$ the blood specific heat, $\omega_b(s^{-1})$ the volumetric blood perfusion rate per unit volume, T_b (°C)the blood temperature, $q_{mb}(\,Wm^{-3})$the heat generation due to metabolism and $Q(\,Wm^{-3})$ the heat generation due to the laser.

The laser volumetric heat source Q is described by:

$$Q = \mu_a \emptyset(z) f(t)$$

where $\mu_a(m^{-1})$is the absorption coefficient of the tissue at the radiation wavelength, the laser irradiance and $f(t)$the laser intensity. To calculate the laser energy deposition in scattering tissues, the laser irradiance $\emptyset(x)$ is expressed as:

$$\emptyset(z)= [C_1\, exp(-k_1 z/\delta) - C_2\, exp(-k_2 z/\delta)]$$

Where C_1, k_1, C_2, andk_2 are parameters that are determined based on the Monte Carlo solutions, depending on the diffuse reflectance; z is the effective optical penetration depth, which is defined from the diffusion theory as:

$$\delta = \frac{1}{\sqrt{3\mu_a[\mu_a + \mu_s(1-g)]}}$$

where $\mu_s$is the scattering coefficient; g is the anisotropy factor. A reduced scattering coefficient is usually introduced as $\mu_s = \mu_s(1-g)$for convenience.

The dentin layer is a hard, avascular and nonperfused tissue, the blood perfusion and metabolic heat at the targeted region are almost zero in contrast to the heat produced by laser accidents (10).

$$q_{mb} = 0, \quad \omega_b = 0$$

Therefore, Eq. (1) reduces to:

$$\rho c\frac{\partial T}{\partial t} = -\nabla . q + Q \quad (2)$$

The simple linear empirical relationship between the heat flux vector and the temperature gradient given by the Fourier heat conduction equation is

$$q(z,t) = -k\nabla T \quad (3)$$

where $k(Wm^{-1}°C^{-1})$ is the thermal conductivity of dentin, Introducing a time delay between the heat flux and the temperature gradient, τ_q to time variable 't' in $q$and a delay between the temperature gradient and energy transport, τ_T to time variable 't' in T of Eq. (3), we get

$$q(z, t+\tau_q) = -k\nabla T(z, t+\tau_T) \quad (4)$$

Taylor's series expansion of first-order approximation of q and $\nabla T$with respect to time 't' yields:

$$q(z,t) + \tau_q \frac{\partial q(z,t)}{\partial t} = -k\left(\frac{\partial T(z,t)}{\partial z} + \tau_T \frac{\partial^2 T(z,t)}{\partial z\, \partial t}\right) \quad (5)$$

Taking the divergence of Eq. (5) and substituting the value of $\nabla . q$ from Eq. (2) in the resulting equation, we get a dual phase lag model of heat conduction in the dentin exposed to the short-pulsed Er: YAG laser for the dentin caries ablation.

$$\tau_q \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} = \alpha\left[\frac{\partial^2 T}{\partial z^2} + \tau_T \frac{\partial^3 T}{\partial z^2\, \partial t}\right] + \frac{1}{\rho c}\left[Q + \tau_q \frac{\partial Q}{\partial t}\right] \quad (6)$$

where $\alpha = \frac{k}{\rho c}(cm^2 s^{-1})$ is the thermal diffusivity of the dentin.

The following conditions are specified as being physically realistic and mathematically consistent:

Initial Conditions

$$T(z,t) = T_0 \qquad \frac{\partial T(z,t)}{\partial t} = 0 \qquad at\ t = 07(\text{a, b}) \tag{7}$$

Boundary Conditions

$$-k\frac{\partial T(z,t)}{\partial z} = q_0 \qquad \text{at} \quad z = 0, \tag{8}$$

$$\frac{\partial T(z,t)}{\partial z} = 0 \qquad \text{at} \quad z = l, \tag{9}$$

Where q_0 (Wm^{-2})is the constant heat flux on the dentin surface.

The governing Eq. (6) is normalized using the following scheme:

$$Z = \frac{\omega z}{2\alpha}, \tau = \frac{t}{2\tau_q}, \psi = \frac{Q\tau_q}{\rho c T_0}, \theta = \frac{T}{T_0}$$

where ω is the thermal wave speed. The dimensionless form of Eq. (6) is given by

$$\frac{\partial^2\theta}{\partial\tau^2} + 2\frac{\partial\theta}{\partial\tau} = \frac{\partial^2\theta}{\partial Z^2} + \frac{\tau_T}{2\tau_q}\frac{\partial^3\theta}{\partial Z^2\partial\tau} + 2\psi_0\left[\,2\,\eta(\tau) + \frac{\partial\,\boldsymbol{\eta}(\tau)}{\partial\tau}\,\right]$$

where $\Psi\ (Z,\tau) = \psi_0\ \eta(\tau)\ \emptyset(\beta Z)$, $\psi_0 = \frac{\beta\ f_r}{2\omega\rho c T_0}$, $\beta =$

$2\ \omega\tau_q\ \mu_a$

The laser heat source term is represented in terms of arbitrary reference laser intensity f_r as $f(\tau) = f_r\ \eta(\tau)$, while $\eta(\tau)$represents the dimensionless rate of energy absorbed in the tissue.

The dimensionless forms of the initial and boundary conditions are

$$\theta(Z,\tau) = 1,\ \frac{\partial\theta(z,\tau)}{\partial\tau} = 0 \qquad \text{at}\ \tau = 0 \tag{11(a,b)}$$

$$-B\frac{\partial\theta}{\partial Z} = G \qquad \text{at} \quad Z = 0, \tag{12(a)}$$

$$\frac{\partial\theta}{\partial Z} = 0 \quad \text{at} \quad Z = \frac{\omega l}{2\alpha} \tag{(12)b}$$

where $G = \frac{q_0}{f_r}$, $B = \frac{\omega\rho c T_0}{2 f_r}$

Analytical solution to the Mathematical Model: The partial differential equation (6) subject to the initial conditions 7 (a, b) and boundary conditions 8 (a, b) is solved using the Laplace transform technique and the analytical solution for the temperature is obtained as follows:

$$\theta(Z,\tau) = \psi_0[\,\beta \exp(-\beta l')\, h(l' - Z,\tau) + \, exp(-\beta Z) f_H + \beta \exp(-\beta \boldsymbol{l'}) \sum_{n=0}^{\infty} h(\,(1-2n)l' - Z,\tau) + \boldsymbol{\beta} \sum_{n=0}^{\infty} h(-(2nl' + Z)\,,\tau) \; + \boldsymbol{\beta} \sum_{n=0}^{\infty} h(-(2l' + 2nl' - Z),\tau) + \,\beta \exp(-\beta \boldsymbol{l'}) \; \sum_{n=0}^{\infty} h(-(l' + 2nl' - Z),\tau)] - \frac{\boldsymbol{G}}{\boldsymbol{B}} \sum_{n=0}^{\infty} h_1\,(-(2l' + 2nl' - Z),\tau) - \frac{\boldsymbol{G}}{\boldsymbol{B}} \sum_{n=0}^{\infty} h_2(-(2nl' + Z),\tau) + 1$$

$$I_H(\tau) = \frac{\gamma_p^2 \exp(\gamma_m \tau) - \gamma_m^2 \exp(-\gamma_p \tau) - 4\gamma}{\gamma \gamma_p \gamma_m}$$

Where,

$$\gamma = \frac{[(2-\beta^2 D)^2 + 4\beta^2]^{1/2}}{2}$$

$$\gamma_p = \gamma - \frac{[2-\beta^2 D]}{2} \quad , \quad \gamma_m = \gamma + \frac{[2-\beta^2 D]}{2}$$

$$\mathrm{h}\,(\mathrm{p},\tau) = 0 \;\; \text{for} \quad p \geq \tau \geq 0$$

$$= \int_p^{\tau} \exp(-u)\, I_0[\,(u^2 - p^2)]^{\frac{1}{2}} I_H(\tau - u)\; du \;\; for\; \tau > p$$

$$h_i\,(\mathrm{p},\tau) = 0 \;\; \text{for} \quad p \geq \tau \geq 0$$

$$= \int_0^{\tau} \exp[-(\tau - u)] I_0[\,(\tau - u)^2 - p^{\,2})]^{\frac{1}{2}} \; du \;\; for \;\; \tau > p \qquad \mathrm{i} = 1,2$$

RESULTS AND DISCUSSION

The temperature profile and temperature variation over time in the dentin irradiated by Er: YAG laser used for the caries ablation as computed by the dual phase lag model are obtained and displayed by graphs using the typical values of the parameters listed in the following table.

Table 1.Computational Parameters

Parameter	Symbols	Magnitude	Units
Dentin density	ρ	2100	$Kg\, m^{-3}$
Dentin thermal conductivity	k	0.63	$Wm^{-1}{}^{\circ}C^{-1}$
Absorption coefficient of dentin	μ	813	m^{-1}
Radiation Wavelength	λ	2.94	μm
Dentin specific heat	c	1170	$Kg^{-1}K^{-1}$
Initial temperature	T_0	37	℃
Phase lag in heat flux	τ_q	16	sec.
Phase lag in temperature gradient	τ_T	2	sec.
Thickness of dentin	l	1.5	mm
Dentin radius	r	3	mm
Constant heat flux	q_0	19×10^3	Wm^{-2}

The distribution of dentin temperature differential along its depth after the deposition of laser heat at the surface of the dentin tissue with one laser pulse irradiation is illustrated in Fig.2. It is evident from the characteristic curve that the dentin temperature difference drops sharply along the depth and it approaches zero beyond approximately z = 1.0 mm. The model anticipated a similar outcome presented by P. Elahi, and B. Farsi (6). This result suggests/ supports that the use of laser Er: YAG with the values of optical parameters listed in the table for dentin caries removal avoids thermal damage to the pulp layer of the tooth, beneath the dentin.

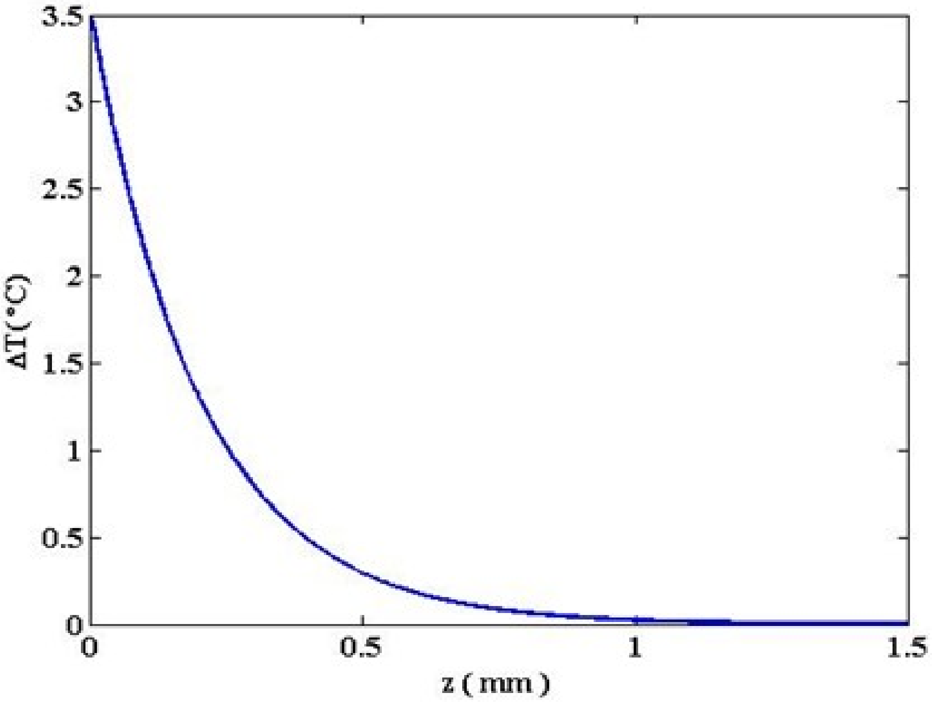

Fig. 2. Temperature difference profile after one laser pulse

In Fig. 3 the curve indicates the variation in the dentin temperature difference at the centre of dentin i.e. r = z = 0 with the time after the irradiation of one laser pulse in the treatment with water spray. It is seen that the greatest temperature differential in the dentin is predicted approximately 4 ℃ by the hyperbolic model during the pulse duration and it starts to decrease after the pulse duration. The temperature difference in the dentin increases with the number of pulses used under the process of dentin cavity ablation.

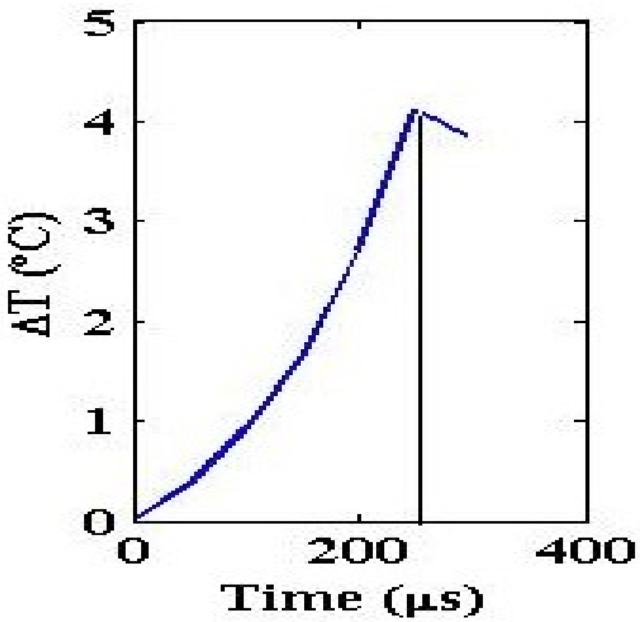

Fig. 3. The variation in the temperature difference with time for a single pulse

The effect of the heat flux phase lag vector on the temperature difference distribution during the cooling phase is in Figure 4. It is observed from the curves that the temperature difference increases as the phase lag of the heat flux increases and in the case of larger τ_q, the wavefront is more visible. Sinceτ_q is typically read as the non-zero time that accounts for the effect of "thermal inertia," τ_q causes a delay in the establishment of heat flux and the related conduction across the medium, which causes the temperature to rise.

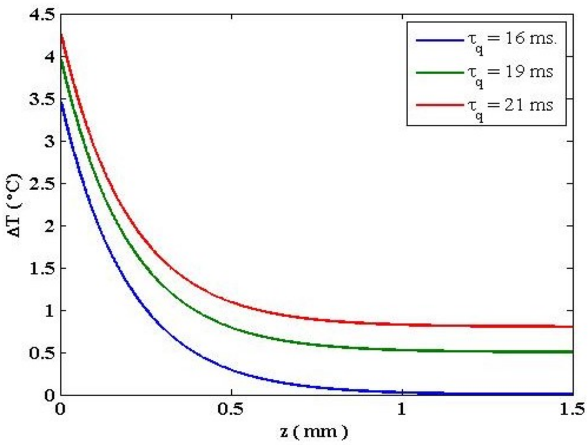

Fig. 4. The influence of relaxation time on the temperature difference distribution

The temperature evaluation at various depths of the dentin with time anticipated by the model is shown in Figure 5. The curves indicate that the dentin temperature rises with time and the maximum temperature is predicted at t = 750 μs thereafter the temperature begins to decrease with time also the temperature decreases along the depth of the dentin.

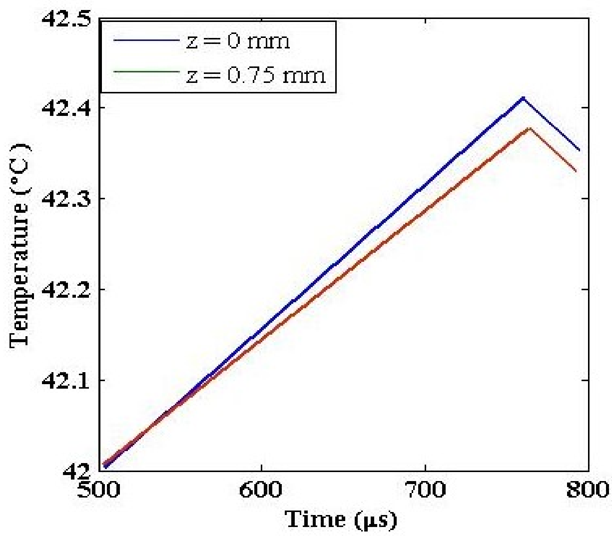

Fig. 5. The temperature variation with time at the laser spot centre on the irradiated surface and the mid-depth of the dentin tissue after two pulses

The effect of the phaselag in the temperature gradient on the dentin temperature difference variation along the dentin depth predicted by the DPL model is shown in Figure 6. It is observed that an increase in the phase-lag temperature gradient reduces the temperature. Also, the temperature difference reduces to zeroabove the 1 mm depth of the human dentin.

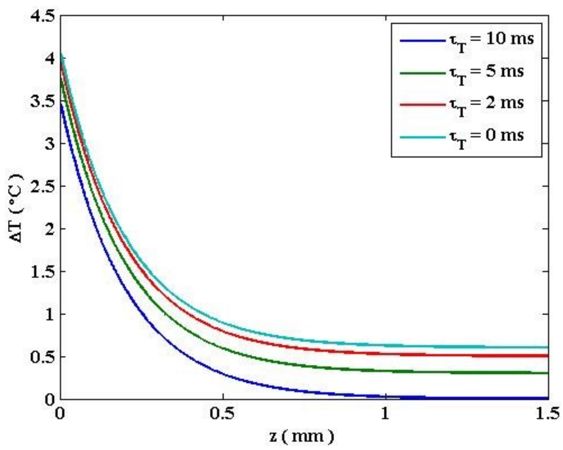

Fig. 6. The effect of the phaselag in a temperature gradient, predicted by the DPL model

CONCLUSION

In this study, temperature variations in dentin heated by short-pulsed Er: YAG lasers are described using the dual phase lag heat conduction model, and the effects of phase lag in the heat flux and the effects of phase lag in the temperature gradient on temperature distribution and changes over time are evaluated. The temperature of the dentin rises as the heat flux phase-lag lengthens. The computational results predicted by the dual-phase model show that the temperature in the dentin is induced by the Er: YAG laserand an increase in the phase-lag temperature gradient decreases the temperature. By comparing the analytical findings with the experimental data, it is vital to confirm the validity of the current findings. If the model is validated later, it could be applied to enhance the design of laser surgery for the treatment of caries ablation. A 3W (watts) Er: YAG laser operating on a pulse of 230 s (microseconds) demonstrated significant potential for the selective ablation of dental cavities in a study evaluating the rate of ablation and selectivity of healthy and demineralized dentin. Recent research demonstrates that using the Er: YAG laser to remove caries is a safe and comfortable procedure for the patient, requiring less local anaesthetic and preserving pulp vitality.

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Declarations

Conflict of interest: The authors declare that they have no conflict of interest.

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