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RESEARCH ARTICLE

PROPOSED NEW BINARY OPERATIONS BETWEEN TWO NUMBERS SIMILAR TO DIVISION AND MODULUS DIVISION THAT MAKE FORMULA OF JOSEPHUS PROBLEM INTO SIMPLER FORM

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ABSTRACT

Division a/b is the number that how many times b is subtracted from a until reduced value is 0. Modulus division $a\%b$ or, $a \bmod b$ is the number that is obtained by subtracting from a the maximum multiple of b that is less than or equal to a. Similarly, we can define two binary operations between two numbers: power division and power modulus division (proposed names) with operators $\hat{\div}$ and $\hat{\%}$ (proposed symbols) respectively. Power division $a\hat{\div}b$ is the number that how many times a is divided by b until reduced value is 1. Power modulus division $a\hat{\%}b$is the number that is obtained by subtracting from a themaximum power of b that is less than or equal to a. This article proposes these two operations with names and symbols and their applications which makes the formula of Josephus problem into simpler form.

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INTRODUCTION

Division is the one of the four basic operations of arithmetic addition, subtraction, multiplication and division. Here, division is described in such a way by which we will define our proposed new operator. If a and b are two numbers, then a is divided by b is denoted by

$$a/b \ or, a \div b$$

and defined by the number how many times b is subtracted from a until reduced value is 0. How it works is shown Table 1 with an example 5/2- Since, $1 < 2$, we can't subtract full *2* from *1*. But we have to get 0 subtracting some scalar multiplication of *2* from *1*. If we subtract *0.5* scalar multiplication of *2* from *1*, then we get 0, that is, $1 - 2 \times 0.5 = 0$. Therefore, *2.5* times *2* is subtracted from *5* to get *0*.

The Floor Function$\lfloor x \rfloor$ (Ronald, 1994)

This function is used to define proposed operator. This function is also used in modulus division. For any real x, one signifies by the floor function $\lfloor x \rfloor$ the largest integer $\leq x$, that is, the unique integer such that $x - 1 < \lfloor x \rfloor \leq x$. The floor function $\lfloor x \rfloor$ is called "the integral part of x".

Example:$\lfloor 2.5 \rfloor = 2$ and $\lfloor 3 \rfloor = 3$.

Modulus Division (Victor Shoup, 2005)

Let a and b be two integers, then modulus division is denoted by $a\%b$ or $a \bmod b$ and defined by

$$a\%b = a - b \times \lfloor a/b \rfloor$$

Example:

$5\%2 = 5 - 2 \times \lfloor 2.5 \rfloor = 5 - 2 \times 2 = 5 - 4 = 1$

Now proposed operators with proposed names and symbols are described below those are similar to the above operators.

MATERIALS AND METHODS

Proposed Operator Name: Power Division: We may use the symbol $\hat{\div}$ or $\hat{\div}$as power division operator. The division sign with caret upon it.

$a\hat{\div}b$ means how many times a is divided by b until reduced value is 1. How it works is shown Table 2 with an example $20\hat{\div}2$-

Therefore, $20\hat{\div}2 = 4.3219280949$. In words, *20* is power divided by *2* is equal to *4.321980949*.

As like as, subtraction is inverse of addition, division is inverse of multiplication, similarly, power division is inverse of exponentiation operation with respect to base. Root extraction is inverse of exponentiation operation with respect to exponent.

Table 1. How division works

Operations	Reduced value	Times	Description
$5 - 2 \times 1$	3	1	We get 3 subtracting full 2 from 5
$3 - 2 \times 1$	1	1	We get 1 subtracting full 2 from 3
$1 - 2 \times 0.5$	0	0.5	We get 0 subtracting half 2 from 1
	Sum	2.5	

Table 2. How power division works

Operations	Reduced value	Times	Description
20/2^1	10	1	We get 10 dividing 20 by full power of 2
10/2^1	5	1	We get 5 dividing 10 by full power of 2
5/2^1	2.5	1	We get 2.5 dividing 5 by full power of 2
2.5/2^1	1.25	1	We get 1.25 dividing 2.5 by full power of 2
1.25/2^0.3219280949	1	0.3219280949	We get 1 dividing 1.25 by 0.32192809 power of 2
	Sum	4.3219280949	

Table 3. List of operations with corresponding inverse operations

Operation	Inverse Operation	Inverse Relation
Addition $+$	Subtraction $-$	$a + b = c$ $\Rightarrow a = c - b \ or, b = c - a$
Multiplication $\times$	Division $\div$ or, $/$	$a \times b = c$ $\Rightarrow a = \frac{c}{b}\ (b \neq 0)\ or, \quad b = \frac{c}{a}\ (a \neq 0)$
Exponentiation ^ (Since, exponentiation is not commutative; so, there are two inverse operations: one for exponent and other for base.)	For Exponent: Root Extraction $\sqrt{\ }$	$a^b = c$ $\Rightarrow a = c^{1/b} = \sqrt[b]{c}$ Exponent b is transferred from left to right.
	For Base: Power Division $\hat{\div}$ or, $\hat{/}$	$a^b = c$ $\Rightarrow b = \log_a c = c\hat{/}a$ Base a is transferred from left to right.

As like as, $20/2 = 10$ and $2 \times 10 = 20$.

Similarly, $20\hat{/}2 = 4.3219280949$ and $20 = 2$^4.321928.

We know that, logarithmic function is inverse of exponential function with respect to base. So, similar result we get from

$$\log_2 20 = \frac{\ln 20}{\ln 2} = 4.3219280949.$$

But our approach is to use an operator between two numbers as like as addition, subtraction, etc. instead of a function. So, we can define power division by

$$a\hat{/}b = \log_b a = \frac{\ln a}{\ln b} \qquad a > 0, b > 0 \text{ and } b \neq 1$$

In this case, base b is used as a divisor in the power division operation. By using power division operation, we can complete the following table-3

Proposed Operator Name: Power Modulus Division: We may use the symbol $\hat{\%}$ as the operator of the operation named "power modulus division". Power modulus division $a\hat{\%}b$ is the number that is obtained by subtracting from a the maximum power of b that is less than or equal to a. For any two positive real numbers and $b \neq 1$ this operation is defined by

$$a\hat{\%}b = a - b^{\lfloor a\hat{/}b \rfloor}$$

$$= a - b^{\left\lfloor \frac{\ln a}{\ln b} \right\rfloor}$$

Example:

$$20\hat{\%}2 = 20 - 2^{\lfloor 20\hat{/}2 \rfloor}$$
$$= 20 - 2^{\left\lfloor \frac{\ln 20}{\ln 2} \right\rfloor}$$
$$= 20 - 2^{\lfloor 4.3219280949 \rfloor}$$
$$= 20 - 2^4$$
$$= 4$$

Maximum power of 2 that is less than or equal to 20 is 16. Next power of 2 is 32 that is greater than 20. So, we will accept 16. Now if we subtract 16 from 20, we get 4 and that is the result of power modulus division of 20 by 2.

RESULTS AND DISCUSSION

In Josephus problem (Ronald, 1994), where a number of people are standing in a circle waiting to be executed. Counting begins at a specified point in the circle and proceeds around the circle in a specified direction. First person will execute the second person, the third person will execute the fourth person and so on. The procedure is repeated with the remaining people, until only one person remains, and is freed. If total number of people is $n$and they are numbered by 1, 2, 3, … , then the freed person number is evaluated by the following formula

$$J(n) = J(2^m + l) = 2l + 1, \quad for\ m \geq 0\ and\ 0 \leq l < 2^m$$

That means, if there are n persons arranged in a circle and first person kills second, third person kills fourth and so on. At last, one will survive. $J(n)$ function gives the serial no. of the person who will survive.

If we use power modulus division, then this formula turns into simpler form as follows where m, l are not used with their conditions that make the formula complicated.

$$J(n) = 2\left(n \widehat{\%} 2\right) + 1$$

We see that the above formula is simpler than previous one.

CONCLUSION

We introduce here two operations power division and power modulus division with symbols. Although power division is similar to logarithmic function, but there is no such function or operation like power modulus division. Power modulus division operation reduces the formula of Josephus problem into simpler form. Thus, these operations may be used to other places. Logarithmic function is the inverse of exponential function. But there is no inverse operation of exponentiation operation with respect to base. Root extraction is the inverse operation of exponentiation function with respect to exponent. For inverse operation of exponentiation function with respect to base we use logarithm. Here, we have used a binary operation "power division" for inverse operation of exponentiation with respect to base. Using power division, we have originated another binary operation "power modulus division" by which formula of Josephus problem turns into easy form.

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After delivering lectures on Josephus problem in a class, I see that the formula for Josephus problem may be easier by use a different operation that is similar to modulus division. From where I got this concept.

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