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RESEARCH ARTICLE

ATTENUATION OF MILLIMETRE WAVE BY SPHERICAL RAIN DROPS USING PARABOLIC WAVE EQUATION METHOD

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ABSTRACT

This letter explores the parabolic equation method used in describing millimetre wave propagation and challenges it encounters when it interacts with rain drops in Yenagoa, Bayelsa State. This propagation model can predict millimetre wave scattering for both finite surface conductivity and highly irregular terrain. The parabolic equation method is adopted to model millimetre wave diffraction and refraction effects by rain drops using Yenagoa climate weather averages for 2022 from weather.com.

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INTRODUCTION

This study entails numerical methods for modelling wave millimetre propagation in range-dependent environments using the parabolic wave equation (PWE). Leontovich (Fock, 1965; Ryan, 1991) was the first to purpose the parabolic wave equation method in 1944 as a solution to solving elliptic partial differential wave equation. The PE method was applied to solve electromagnetic wave propagation above plane earth problem. Leontovich and Fock (1991) in 1946, used the PE method to solve trans-horizon radio wave propagation above spherical earth problem which was a significant breakthrough in electromagnetic wave propagation modelling. After Leontovich and Fock (1946) developed the parabolic wave equation, it took approximately 30 years before a practical algorithm was reported. Hardin and Tappert (1959) solved the problem of modelling ionospheric radar propagation by developing the Split-step Fourier parabolic equation (SSFPE) algorithm. Tappert in 1977 introduced the SSFPE algorithm to solving underwater acoustic problems which became prominent for evaluating range-dependent underwater sound propagation (Malyuzhinets, 1959). The split-step Fourier PE gained prominence due to advances in computer adware technology and evolution of the fast Fourier transform (FFT) algorithm. This set up an efficient numerical solution to the Leontovich and Fock parabolic wave equation. Ko, Sari and Skura applied the SSFPE method for radar propagation to study anomalous microwave propagation in the troposphere (Fock, 1965; Ryan, 1991; Leontovich, 1946). The SSPFE was also applied by Dockery and Konstanzer to analyse phased radar performance. Recently, several authors have developed electromagnetic PE models (Ryan, 1991). The parabolic wave equation method has gained prominence in radio waver propagation over irregular terrain and underwater acoustics. Most of its application assumes low-grazing angle or near horizontal propagation of EM waves (Ryan, 1991; Malyuzhinets, 1959). Over the years, interest on the problem of large-scale surface undulations is on the increase. Ray theory is an efficient technique for high frequencies but become less accurate when large scale (comparable to wavelength) atmospheric irregular surfaces are encountered (Radder, 1979). The parabolic equation method is not constrained by asymptotic frequency restrictions of ray theory. As stated earlier, Hardin and Tappert developed the very efficient Fourier/split-step solution for acoustic problems and Claebout introduced finite-difference codes for geophysics application (Hardin, 1973; Radder, 1979). The development of terrain masking approximation was accredited to Tappert and Ngeim-Phu, it was used to advance the field of the boundary, which is like representing the boundary by a series of knife-edge diffractors (Radder, 1979).

The effect of ocean surface and bottom roughness on shallow water acoustic propagation was evaluated by Rouseff and Ewart (1973; Radder, 1979). Kuttler (Mireille Levy, 2000) utilize a global conformal map to evaluate scattering from a sinusoidal boundary with results been consistent with Bragg scattering theory. (8) developed a model by tilting and steering the field to counteract the flattening of the surface which seem similar to that of Beilis and Tappert technique without explicit coordinate transformation. Donohue et.al developed a hybrid scheme that alternates between piecewise linear shift map and terrain masking when slopes are encountered (9). The scope of this study is focussed on a detailed analysis of millimetre wave diffraction and refraction by rain drops using the Split-step Fourier PE algorithm. To achieve this, there is need to have significant knowledge of rain drops parameters such as rain drop size distribution (DSD), rain-drop diameters, effective dielectric constant, shape of rain drops, rain rate and temperature. We develop the desired model for propagation of electromagnetic radiation in rain media by making some important assumptions. Here we adopt Debye model for estimation of the effective dielectric constant of water and consider a linear, isotropic nonionized medium. The electrical properties of this medium were modelled as a lossy dielectric. We apply Maxwell's equations to derive the electromagnetic propagation model.

Maxwell's Equation: The source of EM radiation is assumed to emit linearly polarized, monochromatic radiation with harmonic time dependence t given by exp $(i\omega t)$. The source free monochromatic Maxwell's equation in rationalized mks units is given below.

$$\nabla \cdot E = {}^{\rho}/_{\epsilon} \tag{1}$$

$$\nabla \times E = -j\omega\mu H \tag{2}$$

$$\nabla \cdot H = 0 \tag{3}$$

$$\nabla \times H = J + j\omega\,\epsilon E \tag{4}$$

The Field components, $\bar{E}$, $\bar{D}$, B and H denotes the electric field, electric flux density, magnetic flux density, and the magnetic field. Their sources, the charge density ρ and current density J are functions of the spatial co-ordinate (x, y, z). The propagation media is characterized by The electric flux density $\bar{D}$ and electric field $\bar{E}$ are mathematically related by

$$D = \epsilon_r \epsilon_0 E \tag{5}$$

Similarly, magnetic flux density and the magnetic field are related by

$$B = \mu_r \mu_0 H \tag{6}$$

To describe the EM wave propagation in an inhomogeneous varying space, we use the vector identity expressed below to evaluate equation (2)

$$\nabla \times \nabla \times A = \nabla(\nabla \cdot A) - \nabla^2 A \tag{7}$$

This becomes

$$\nabla \times \nabla \times A = \nabla(\nabla \cdot E) - \nabla^2 E \tag{8}$$

Taking the left-hand side of equation (8) we obtain

$$\nabla \times (\nabla \times E) = -j\omega\mu(\nabla \times H) \tag{9}$$

Substituting equation (4) into (9) yields

$$\nabla \times (\nabla \times E) = -j\omega\mu(J + j\omega\varepsilon E) \tag{10}$$

Where $J = \sigma\bar{E}$, for non-conducting or charge free medium $(\sigma = 0)$the current density $J = 0$. Equation (10) reduces to the form

$$\nabla \times (\nabla \times \bar{E}) = -j\omega\mu(j\omega\varepsilon E)$$

$$\nabla \times (\nabla \times E) = \omega^2\mu\varepsilon E$$

For a lossless or non-conducting media, the propagation constant $\gamma^2 = -\omega^2\mu\varepsilon$, since the wave is not attenuated as it propagates, we introduce the wave number $k = \omega\sqrt{\mu\varepsilon}$. From these assumptions we can say that

$$-\gamma^2 = k^2$$

Taking the both the left-hand side and right-hand side of (8) yields

$$\omega^2\mu\varepsilon\bar{E} = \nabla(\nabla \cdot \bar{E}) - \nabla^2\bar{E} \tag{11}$$

For a homogeneous media this becomes

$$\nabla^2\bar{E} + \omega^2\mu\varepsilon\bar{E} = 0 \tag{12}$$

$$\nabla^2\bar{E} + k^2\bar{E} = 0 \tag{13}$$

Where $\nabla \cdot \bar{E} = 0$but for inhomogeneous media $\nabla \cdot \bar{D} = \rho$and$\nabla \cdot \bar{E} \neq 0$in a time varying space where $\varepsilon_r = n^2$.

Hence, equation (13) for inhomogeneous media can be written as

$$\nabla^2 \bar{E} + k^2 \bar{E} = \nabla(\nabla \cdot \bar{E}) \quad (14)$$

From equation (5) we can express the electric field $\bar{E}$ as

$$E = \frac{1}{\epsilon_0}\frac{D}{n^2} \quad (15)$$

Applying product rule of differentiation

$$(\nabla \cdot E) = \frac{1}{\epsilon_0}\nabla \cdot \left(\frac{D}{n^2}\right) = \frac{1}{\epsilon_0}\frac{1}{n^2}\nabla \cdot D + \frac{1}{\epsilon_0}\nabla \cdot D \cdot \nabla\left(\frac{1}{n^2}\right) \quad (16)$$

Substituting $(\nabla \cdot D) = 0$ in (16) yields

$$(\nabla \cdot E) = n^2 E \cdot \nabla\left(\frac{1}{n^2}\right) \quad (17)$$

$$\nabla^2 \bar{E} + k^2 \bar{E} = n^2 E \cdot \left(\frac{-2}{n^3}\right)\nabla(n) \quad (18)$$

$$(\nabla \cdot E) = -\frac{2}{n}E \cdot \nabla(n) = -2E\frac{1}{n}\nabla(n) \quad (19)$$

$$(\nabla \cdot E) = -2\, E \cdot \nabla(\ln(n)) \quad (20)$$

$$\nabla(\nabla \cdot E) = \nabla\{-2\, E \cdot \nabla(\ln(n))\} \quad (21)$$

Hence, the inhomogeneous Helmholtz equation can be expressed as

$$\nabla^2 \bar{E} + k^2 \bar{E} = \nabla\{-2\, E \cdot \nabla(\ln(n))\} \quad (22)$$

Derivation of the Parabolic Wave Equation

Using a simple model which describe the propagation of a reduced function

$$\psi(x,z) = u(x,z)e^{ikx} \quad (23)$$

associated with the direction of propagation x. Where $u(x,z)$ can be expressed as

$$u(x,z) = \Psi(x,z)e^{-ikx} \quad (24)$$

The Helmholtz equation of the reduced function is obtained by decoupling Maxwell's equations and can be expressed as

$$\nabla^2\Psi + k^2 n^2(x,z)\Psi = 0 \quad (25)$$

Where $n^2(x,z)$ is the refractive index and k is the wavenumber. The refractive index $n^2(x,z)$ is be assumed to possess smooth variations. The reduced function implies that the propagation energy varies slowly at angles close to the paraxial direction (6)

.

The Laplacian of $\Psi(x,z)$ can be expressed as

$$\nabla^2\Psi = (\nabla^2 u + 2ik\nabla u - k^2 u)e^{ikx} \quad (27)$$

Where Ψ is taken as the product of a plane wave solution, substituting equation (27) into equation (25) yields

$$(\nabla^2 u + 2ik\nabla u - k^2 u)e^{ikx} + k^2 n^2(x,z)\, u e^{ikx} = 0 \quad (28)$$

$$\nabla^2 u + 2ik\nabla u + k^2 n^2\{(x,z) - 1\}\, u = 0 \quad (29)$$

The Laplacian operator for 2D can be expressed as $(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2})$, in the atmosphere where $n - 1$ is small, we neglect $\frac{\partial^2}{\partial x^2}$ as small (paraxial approximation) and equation (29) becomes

$$\frac{\partial^2 u}{\partial z^2} + 2ik\frac{\partial u}{\partial x} + k^2(n^2 - 1)u = 0 \quad (30)$$

Where equation (1.31) is the standard parabolic equation. Equation (1.31) by method of separation of variables can be written as

$$2ik\frac{\partial u}{\partial x} = (1 - n^2)k^2 u - \frac{\partial^2 u}{\partial z^2} \quad (31)$$

Which can be expressed as

$$\frac{\partial u}{\partial x} = i\left\{(1-n^2)k^2 - \frac{p^2}{2k}\right\}u \tag{32}$$

Let consider $A = (1-n^2)k^2 - \frac{p^2}{2k}$ and substitute A into equation (32)

These yields

$$\frac{\partial u}{\partial x} = iAu \tag{33}$$

Taking like terms

$$\frac{\partial u}{\partial u} = iAx \tag{34}$$

The solution of the parabolic wave equation becomes

$$u(x_0 + \Delta x, z) = u(x_0)e^{iA\Delta x} \tag{35}$$

It is worthy of note that equation (30) can be factored out to obtain

$$\left\{\frac{\partial}{\partial x} + ik(1-Q)\right\}\left\{\frac{\partial}{\partial x} + ik(1+Q)\right\} \tag{36}$$

This gives us

$$\frac{\partial u}{\partial x} = -ik(1-Q) \tag{37}$$

$$\frac{\partial u}{\partial x} = -ik(1+Q) \tag{38}$$

Where Q is the pseudo-differential operator and is defined by

$$Q = \sqrt{\frac{1}{k^2}\frac{\partial^2}{\partial z^2} + n^2(x,z)} \tag{39}$$

Equation (37) is the outgoing parabolic equation and equation (38) is the incoming parabolic wave equation. Considering the propagation medium as homogeneous with refractive index n, the field component Ψ satisfies two-dimensional scalar wave equation

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial x^2} + k^2n^2 = 0 \tag{40}$$

The refractive index varies with the range x and height z and equation (40) is a good approximation provided n varies slowly with wavelength. It is worth noting that equation (40) is not exact (7). If the propagation medium is vacuum, the standard parabolic wave equation in (40) is expressed as

$$\frac{\partial^2 \psi}{\partial z^2} + 2ik\frac{\partial u}{\partial x} = 0 \tag{41}$$

The solution of equation (1.43) can be expressed as

$$u(x_0 + \Delta x, z) = u(x_0)e^{i\frac{p^2}{2k}\Delta x} \tag{42}$$

The Split Step Fourier Transform Solution

The split-step Fourier method is a very efficient PEM which separate the refractive effect from the diffractive part of the propagator. Considering a two-dimensional scalar wave equation for horizontally and vertically polarised wave. Hardin et.al introduced the split-step Fourier method which transforms the rough surface problem with propagation through a sequence of phase screens (7).

The standard parabolic equation (SPE) in equation (30) can be written as

$$\frac{\partial u}{\partial x} = \frac{ik}{2}\left\{\frac{1}{k^2}\frac{\partial^2}{\partial z^2} + (n^2(x,z) - 1)\right\}u \tag{43}$$

Let $A = \frac{1}{k^2}\frac{\partial^2}{\partial z^2}$ (44)

$$B = n^2(x,z) - 1 \tag{45}$$

Equation (43) becomes

$$\frac{\partial u}{\partial x} = \frac{ik}{2}\{A + B\}u \tag{46}$$

The analytic solution of the SPE is $u(x+\Delta x, z) = u(x,z)e^{\frac{ik}{2}\Delta x(A+B)}$ (47)

Using $\delta = \frac{ik\Delta x}{2}$ (48)

Equation (47) yields

$$u(x+\Delta x, z) = u(x,z)e^{\delta(A+B)} \quad (49)$$

Equation (47) is the split-step solution which represent the field propagating through series of phase screens. The field is first propagated through a slice of homogeneous medium characterised by the exponent of A. In this paper, we predict the attenuation by raindrops using climate weather averages of 2022 in Yenagoa (7), Bayelsa State (See Table 1). Attenuation by rain drops depend on physical and electrical properties of rain drops such as drop size distribution (DSD), diameter (D) of rain drops, permittivity of water at specific temperature and propagating frequency. Rain drops diameter range from $0.1mm - 8mm$ as drops with diameter larger than $8mm$ are unstable and breakup (7). Hence, high frequency approximation methods should be used at millimetre wavelength. From Table 1, we can see that the rain rate per hour is less than $1mm/hr$. With diameters less than $1.5mm - 2mm$, we assume that the shape of rain drops is spherical (7). In other cases, it is oblate ellipsoidal. The effective permittivity of water was calculated by Liebe's formula and the rain drop spectrum adopted is Marshall-Palmer spectrum (7).

Table 1. Yenagoa Climate Weather Averages for 2022 [7]

Month	Day Time Temperature	Night Time Temperature	Rain Days	Monthly Rain Rate (mm)	Hourly Rain Rate (mm/hr)
January	$306K$	$296K$	7	117.85	0.1584
February	$305K$	$296K$	11	176.29	0.2369
March	$304K$	$297K$	17	236.73	0.3182
April	$304K$	$297K$	18	258.69	0.3477
May	$303K$	$296K$	18	332.38	0.4467
June	$301K$	$295K$	22	452.29	0.6079
July	$299K$	$295K$	23	508.99	0.6841
August	$299K$	$294K$	22	526.86	0.7081
September	$300K$	$295K$	24	590.99	0.7943
October	$301K$	$295K$	22	478.90	0.6437
November	$303K$	$296K$	17	330.11	0.4437
December	$305K$	$296K$	6	104.68	0.1407

In this study, we shall investigate millimeter wave propagation on flat earth surface in a rain medium (7)-(8). Here, we assume that the rain drops are spherical in order to evaluate its interaction with an incident EM field by adopting split-step solution of the parabolic equation method (7)-(8). The aforementioned assumption is valid at low rain rate intensity. For high intensity rain, it is more realistic to model rain drops as oblate spheroids (7)-(8).

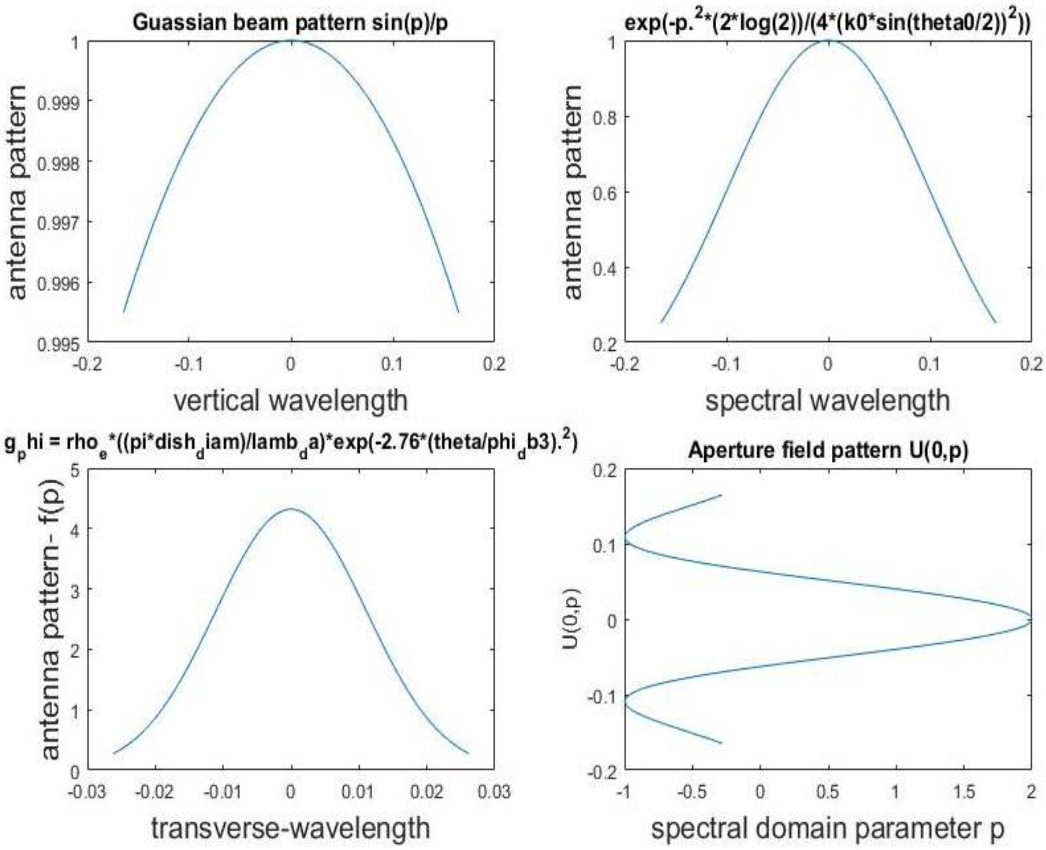

Figure 1. Plots of Antenna Patterns for Parabolic Equation (PE) Source Modelling Here we consider a PE source with antenna Gaussian beam pattern defined as

The initial field profile $U(0,p)$was obtained via inverse FFT of the far-field antenna pattern with specified height z_0, antenna beam width θ_{bw} and tilt angle θ_{tilt}. This is shown in the transverse-wavenumber (p) domain as

$$U\ (0,p)\ =\ f\ (p)exp(-ipz_0\ )\ -\ f\ (-p)exp(ipz_0\ ) \quad (50)$$

which obeys Dirichlet boundary condition and

$$U\ (0,p) =\ f\ (p)exp(-ipz_0\ ) +\ f\ (-p)exp \quad (51)$$

Neumann boundary condition, where $U(0,p)$ is the forward Fourier transform of $u(x_0,z)$. $u(x_0,z)$ is the initial field profile and incident propagating field of the PE which we can describe as the field at range $x\ =\ 0$. Introducing antenna tilt we can evaluate $u(0,z)$ by rewritting $f(p)$ as $f(p\ -\ k_0 sin\theta_{tilt})$. The standard PE in equation 43 can be solved by direct decomposition from its spatial form (z)–domainto spectral form (p)–domainvia Fourier transform. Real problems have refractive index n as a function of range x and height $zn(x,z)$, which is appropriate as the equation is solved at each small range-step size Δx, which is chosen small enough so that within any interval the refractive index can be assumed constant with respect to x.

The numerical split-step parabolic equation solution for $j\ =\ 1,2,...M$is given as

$$u(x_0\ + j\Delta x, z) = exp(i\frac{k_0}{2}(n^2\ - 1)\Delta x)F^{-1}(exp(-i\frac{p^2\Delta x}{2k_0}))F(u(x_0\ + (j-1)\Delta x, z))) \quad (53)$$

This equation can be used to calculate $u(x,z)$ along z with steps of Δx, for known initial source distribution $u(0,z)$. We can use an array to store the transverse -field profiles of N_Z vertical height points and N_X discrete ranges, with replacement. Here, the initial field $u(0,z)$ profile obtained from an antenna beam pattern is propagated longitudinally from $x_0$to $x_0\ +\ \Delta x$ for $j =\ 1$ using equation (53)until the solution $u(x_0\ +\ \Delta x, z)$ is obtained. This is used as the initial field profile for the next step $(j\ =\ 2)$ to obtain $u(x_0\ +\ 2\Delta x, z)$, this process is repeated for $j\ =\ 1,2,..M$ and the vertical field profiles are computed for each range step until the required range is reached.

RESULTS AND DISCUSSION

The PE method was used to model millimetre wave propagation in rain over a flat earth at temperature $T = 306K$ and rain rate $0.1584\ mm/hr$ in Yenagoa city, Bayelsa State. The simulation was implemented for both rain intensity and effective refractive index$n$ that are uniform through the rainfall region. A Gaussian source field that is horizontally and vertically polarized at $z_s = 100\ m$, with a beamwidth of 1°, elevation or tilt angle 1°, and frequency $300\ GHz$. Here we consider one obstacle that is modelled over the flat earth using trigonometric functions. The location of the obstacle is between $3-5\ km$ with height $50\ km$. Figure 2 and 3 illustrate variation of field profile over range as the field travels through flat earth with rain drops with rain rate $0.1584\ mm/hr$, temperature$T = 306K$and effective refractive index $n = 2.58125 + 1.1304i$ with maximum heights$5\ km$ and $10\ km$, earth radius $r_e = 6371\ km$. The figures below show diffraction and reflection effects through the range $0-1\ km$ for flat earth surfaces with maximum heights $5\ km$ and $10\ km$respectively.

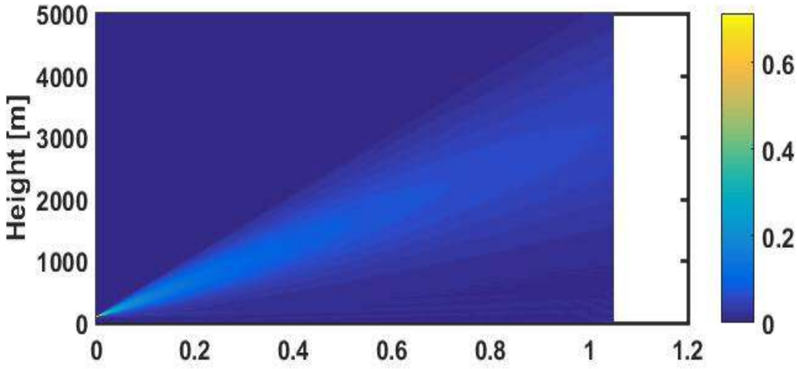

Figure 2. Field profile over a flat earth at maximum height $5km$

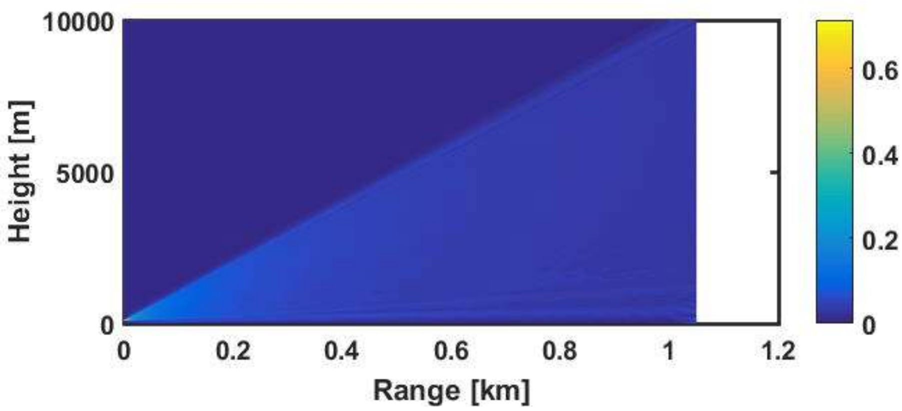

Figure 3. Field profile over a flat earth at maximum height $10k$

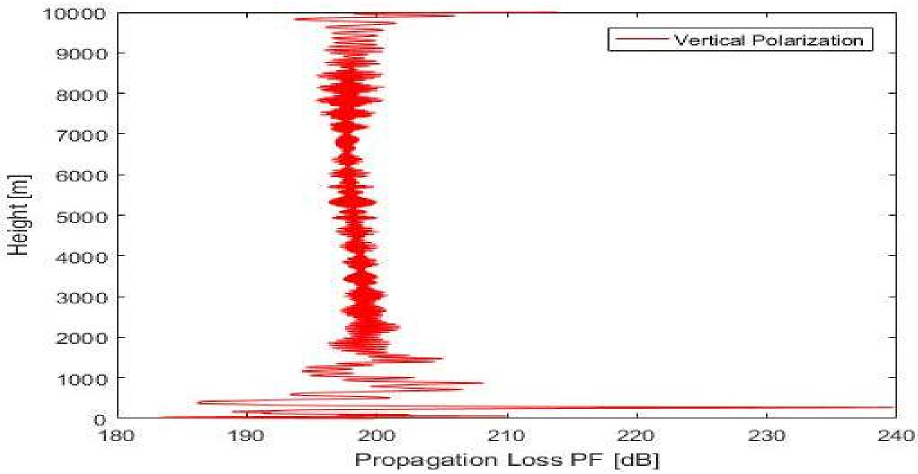

Figure 4. Propagation loss (PF) versus height (m) over a flat earth with rain rate $0.1584\ mm/hr$

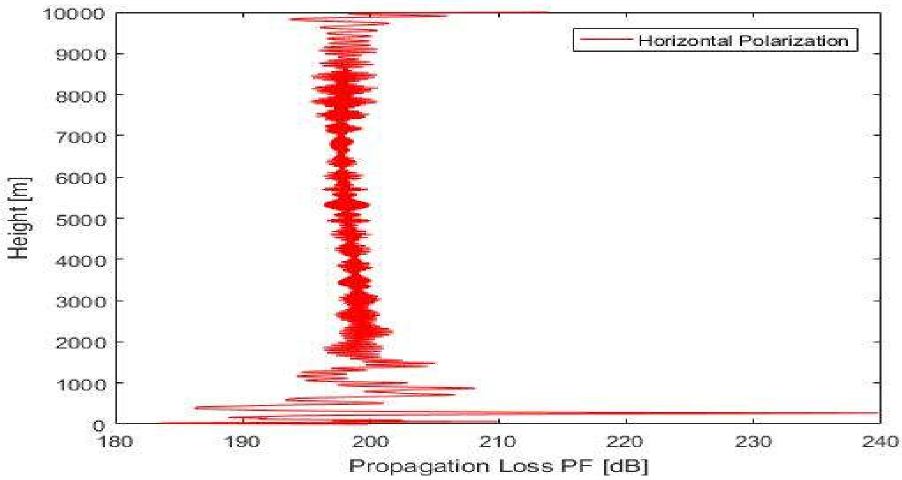

Figure 5. Propagation loss (PF) versus height (m) over a flat earth with rain rate $0.1584\ mm/hr$

We can see that diffuse reflections increases with increase in height of the earth surface. Figure 4 and 5 illustrate the propagation loss (PL) as a function of height in rain medium with maximum height $z = 10\ km$ for both horizontal and vertical polarizations.

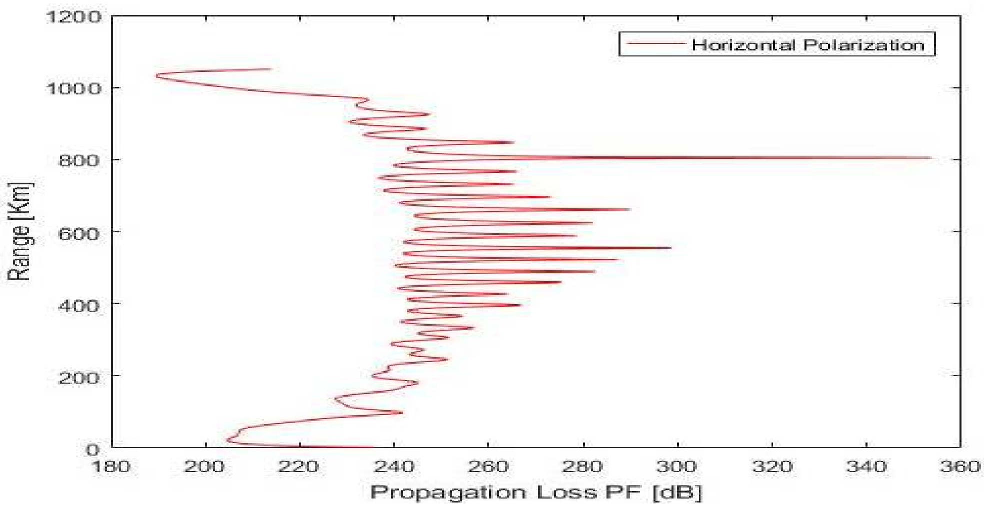

Figure 6. Propagation loss (PF) versus Range (km) over a flat earth with rain rate $0.1584\ mm/hr$

The field profiles vary similarly in both cases for maximum range $x = 50\ km$. The flat earth surfaces show less diffuse reflections than irregular terrains assumed to be wet as shown in both cases of horizontal and vertical polarizations. Figure 4 and 5 illustrate the propagation loss (PL) as a function of range in rain medium with maximum height$z = 10\ km$ for both horizontal and vertical polarizations. The field profiles vary similarly in both cases for maximum height $z\ = 10\ km$. The flat earth surfaces show less diffuse reflections than irregular terrains assumed to be wet as shown in both cases of horizontal and vertical polarizations. The propagation loss is highest at range $x = 800\ km$ and lowest at $= 0\ km$. This explains that low altitudes, multipath propagation effects are not significant.

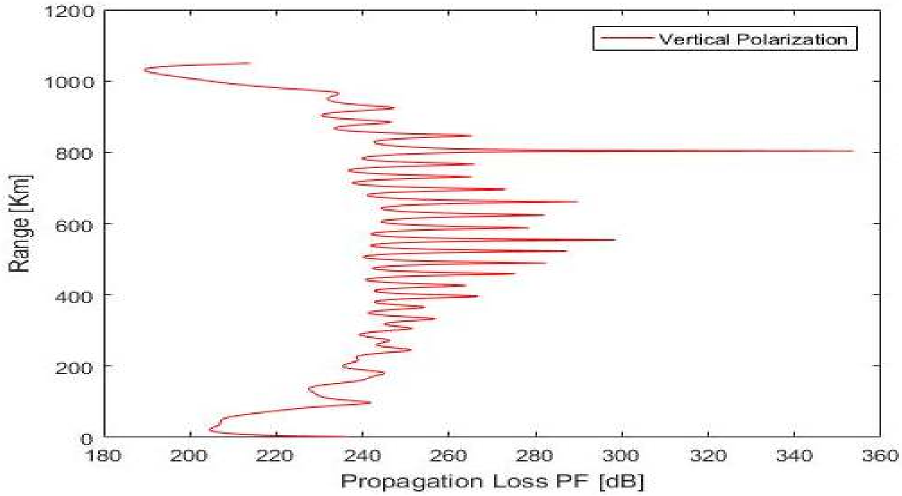

Figure 7. Propagation loss (PF) versus Range(km) over a flat earth with rain rate $0.1584\ mm/hr$

CONCLUSION

The parabolic equation method was applied to model millimetre wave propagation in rain medium with irregular terrain conditions. The results obtained illustrate that the PE model can predict multipath propagation effects as well as diffraction and refraction of millimetre wave by rain drops. The split step Fourier method provides an efficient numerical approach for computing millimetre propagation characteristics in variable terrain.

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