ISSN: 0975-833X

Available online at http://www.journalcra.com

International Journal of Current Research
Vol. 14, Issue, 06, pp.21690-21698, June, 2022
DOI: https://doi.org/10.24941/ijcr.43705.06.2022

INTERNATIONAL JOURNAL OF CURRENT RESEARCH

REVIEW ARTICLE

COMPLEX INTUTIONISTIC FUZZY SOFT MATRICES

*Chinnadurai, V., Madhanraj, S. and Thayalan, S.

Department of Mathematics, Annamalai University, Annamalainagar-608002, India

ARTICLE INFO

Article History:
Received 10th March, 2022
Received in revised form
09th April, 2022
Accepted 24th May, 2022
Published online 30th June, 2022

Key words:

Intuitionistic Fuzzy set, Intuitionistic fuzzy Matrices.Complex Fuzzy Set.

Corresponding Author:
Chinnadurai, V.,

ABSTRACT

In this article, we define the concept of complexintuitionistic fuzzy soft matrices (CIFSMs) and define some operations on these matrices. Also, we investigated some theoretical properties onCIFSMs. In final, we develop an algorithm for complexintuitionistic fuzzy soft matrices and apply it to a decision making problem using different types of T-norm operators.

Citation: ***Chinnadurai, V., Madhanraj, S. and Thayalan, S.*** **2022.** "Complex intutionistic fuzzy soft matrices". *International Journal of Current Research*, 14, (06), 21690-21698.

INTRODUCTION

The fuzzy set (FS) was introduced by Zadeh (Zadeh, 1965), also he discussed the notion of linguistic variables. Atanassov (Atanassov, 1986; Atanassov , 1989; Atanassov, 2005), developed the fuzzy set and intuitionist fuzzy set (IFS), also he discussed the operators on interval valued intuitionistic fuzzy sets (IVIFSs).Bustince (Bustinceand, 1995) introduced the concept of correlation of intervalvalued intuitionistic fuzzy sets. In 1999, Molodtsov (Molodtsov, 1999) approaches the theory of soft set(SS) which has a rich potential for uncertainty and vagueness. Maji *et al* (2001) expanded the fuzzy set (FS) to fuzzy soft sets (FSS). Fuzzy matrices (FM) were introduced for the first time by Thomason (1977), he discussed theconvergence of powers of fuzzy matrix. Fuzzy matrices engage in recreation to a vital role in scientific development. Intuitionistic fuzzy matrix was proposed by Pal and Khan (2002). Bhowmik (2008; Bhowmik, 2008) proposed the concept of generalized intuitionistic fuzzy matrices and also he investigated some results on intuitionistic fuzzy matrices and intuitionistic circulant fuzzy matrices.Cagman (Cagman, 2012) presented the theory of fuzzy soft matrix (FSM). Torra (2015) proposed the concept ofhesistant fuzzy sets can be used as a functional tool allowing many potential membership degrees of an element to a set, these fuzzy sets having several membership degrees of an element to be possible between zero.Szmidt (Szmidt, 2012; Szmidt, 2010) apply the concept of intuitionistic fuzzy sets in group decision making and also he approaches the concept of correlation of intuitionistic fuzzy sets.Ramote et al.,(2012) discussed complex fuzzy set ($\tilde{C}$FS), in which he presented new operations such as reflections and rotations.In recent years, Chinnadurai and Thayalan (Chinnadurai, 2021) studied the concept of complex interval-valued Pythagorean fuzzy set and its application. In this manuscript our intention is to define the concept ofcomplex intuitionistic fuzzy soft matrices (CIFSMs). Further, we investigated some theoretical properties onCIFSMs.

PRELIMINERIES

We present some of the basic concepts which are required for this study. Let us consider the following notations through out this study unless otherwise specified. Let U be the universe, $u \in U$, A be a set of parameters, E$E \subseteq A$ and P(U) represent the set of all subsets of U. C(0,1) denotes the set of all closed sub-interval of (0,1).

Definition 2.1. (17) A fuzzy set is a set of the form$F = \{(u, \alpha_f(u)) | u \in U\}$, where$\alpha_f: U \to (0,1)$defines thedegree of membership of the element $u \in U$.

Definition 2.2. (3) An intuitionistic fuzzy sets is an object of the form $T = \{(u, \alpha_T(u), \gamma_T(u) | u\epsilon U)\}$, where $\alpha_T: U \to (0,1)$and $\gamma_T: U \to (0,1)$define the degree of membership and degree of non-membership of the element u$\in U, 0 \leq \alpha_T(u) + \gamma_T(u) \leq 1$, where $\pi_T(u) = 1 - \alpha_T(u) - \gamma_T(u)$ represents the degree of hesitancy.

Definition 2.3. (12)

Let $\rho_S(x) = \vartheta_S(x)e^{i\omega_S(x)}$ is a complex fuzzy set, where $\rho_S(x)$ is a amplitude of grade of membership belongs to (0,1) and $\omega_S(x)$ is a real valued function.

Complex intuitionistic fuzzy soft matrix

In this section, we introduce a new approach to complex intuitionistic fuzzy soft matrices.

Definition 3.1.

Let U=$\{u_1, u_2, \dots, u_n\}$ be the universal set and E be the set of parameters given by E=$\{e_1, e_2, \dots, e_n\}$ and A$\subseteq E$. Acomplexintuitionistic fuzzy soft matrix over U is defined as a pair (ϕ_μ, A) where ϕ_μ is a mapping given by, $\phi_\mu: A \to P^U$ and P^U is a power set of U, then the complex intuitionistic fuzzy soft set (ϕ_μ, A) can be expressed as a matrix form as,

$(A_{m\times n}) = |A_{ij}|$ for i=1,2,…,m and j=1,2,…,n

where $|a_{ij}^P|$=$\{< |\mu_A(e_i), \gamma_A(e_i)|_j >\}$ and ($\{\mu_A(e_i), \gamma_A(e_i)\}_j$ is complex intuitionistic fuzzy soft set) represent the element of a_i corresponding to the element a_j of U, fori=1,2,…,n j=1,2,…,n and $< |\mu_A(e_i), \gamma_A(e_i)|_j >$ =ρ_{ij} such that $\rho_{ij} \in [0,1]$.i=1,2,…,m and j=1,2,…,n. Also satisfying the condition that,$0 \leq |\mu_A(e_i)|_j + |\gamma_A(e_i)|_j \leq 1$, then A is an (m$\times n$)complex intuitionistic fuzzy soft matrix (CIFSM).

Definition 3.2.
Let $A = (|\mu_A(e_i), \gamma_A(e_i)|_j)_{(m\times n)}$ be CIFSMs, then the complement of the CIFSMs is denoted by, $(A)^C = (|\gamma_A(e_i), \mu_A(e_i)|_j)_{(m\times n)}$ for all i,j.

3.3. Definition

The transposeof aCIFSM,$A_{(m\times n)}$ is obtained by interchanging rows and columns. It isdenoted by $[(A_{m\times n})^T]$.

Example 3.4. Suppose that there are three houses under consideration namely the universes U=$\{h_1, h_2, h_3\}$ and the parameter set E=$\{e_1, e_2, e_3\}$, where e_i stands for Price,Quality of construction and Location respectively. Consider the mapping ϕ_μ which describes the "outlook of the houses" that is considering for purchase. Then fuzzy soft set (ϕ_μ, A) is given as,

$(\phi_\mu, A) = (< |\mu_A(e_i), \gamma_A(e_i)|_j >)_{(m\times n)}$,where$\phi_\mu(e_1) = \{(R_{11}, |0.40e^{i\frac{\pi}{2}}, 0.60e^{i\frac{\pi}{3}}|, R_{12}, |0.50e^{i\frac{\pi}{6}}, 0.30e^{i\frac{\pi}{4}}|, R_{13}, |0.70e^{i2\pi}, 0.30e^{i2\pi} |)\}$
$\Phi_\mu(e_2) = \{(R_{21}, |0.10e^{i\frac{\pi}{4}}, 0.10e^{i\frac{\pi}{3}}|, R_{22}, |0.20e^{i2\pi}, 0.50e^{i\frac{\pi}{6}}|, R_{23}, |0.60e^{i\frac{\pi}{3}}, 0.30e^{i\frac{\pi}{4}}|)\}$
$\Phi_\mu(e_3) = \{(R_{31}, |0.30e^{i\frac{\pi}{6}}, 0.70e^{i2\pi}|, R_{22}, |0.40e^{i\frac{\pi}{3}}, 0.40e^{i\frac{\pi}{6}}|, R_{33}, |0.70e^{i\frac{\pi}{4}}, 0.10e^{i\frac{\pi}{3}}|)\}$
$0.40e^{i\frac{\pi}{2}}$=0.40(cos $\frac{\pi}{2}$ + i sin $\frac{\pi}{2}$)= 0.40(0+i) =|0.4i| =$\sqrt{0.16}$ =0.4.
$0.60e^{i\frac{\pi}{3}}$=0.60(cos $\frac{\pi}{3}$ + i sin $\frac{\pi}{3}$)= 0.60(0.5+0.86i) =|0.3+0.48i | =$\sqrt{0.09 + 0.23}$ =0.56.
$0.50e^{i\frac{\pi}{6}}$=0.50(cos $\frac{\pi}{6}$ + i sin $\frac{\pi}{6}$)= 0.50(0.86+i0.5) =|0.43+0.25i| =$\sqrt{0.1849 + 0.0625}$ =0.5.
$0.30e^{i\frac{\pi}{4}}$=0.30(cos $\frac{\pi}{4}$ + i sin $\frac{\pi}{4}$)= 0.30(0.70+i0.70) =|0.21+0.21i| =$\sqrt{0.04 + 0.04}$ =0.3.$0.70e^{i2\pi}$=0.70(cos 2π + i sin2π)= 0.70(0+i) =|0.70i|=$\sqrt{0.49}$ =0.7
$0.30e^{i2\pi}$=0.30(cos 2π + i sin2π)= 0.30(1+0i) =|0.30|=$\sqrt{0.09}$ =0.3.

proceeding in this manner, we can find $\Phi_\mu(e_2)$ and $\Phi_\mu(e_3)$.Now we represent this complex intuitionistic fuzzy soft set in matrix form as,

$$(A_{m\times n}) = |a_{ij}^P| = \begin{bmatrix} < [0.40,0.56] >< [0.50,0.30] >< [0.70,0.30] > \\ < [0.09,0.08] >< [0.20,0.46] >< [0.56,0.28] > \\ < [0.26,0.70] >< [0.35,0.31] >< [0.69,0.08] > \end{bmatrix}$$

$$((A_{m\times n})^C) = |(a_{ij}^P)^C| = \begin{bmatrix} <[0.56,0.40]><[0.30,0.50]><[0.30,0.70]> \\ <[0.08,0.09]><[0.46,0.20]><[0.28,0.56]> \\ <[0.70,0.26]><[0.31,0.35]><[0.08,0.69]> \end{bmatrix}$$

$$[(A_{m\times n})^T] = |(a_{ij}^P)^T| = \begin{bmatrix} <[0.40,0.56]><[0.09,0.08]><(0.26,0.70)> \\ <[0.50,0.30]><[0.20,0.46]><(0.35,0.31)> \\ <[0.70,0.30]><[0.56,0.28]><(0.69,0.08)> \end{bmatrix}$$

Definition 3.5. Let A = $|\mu_A(e_i), \gamma_A(e_i)|_j$ and B = $|\mu_B(e_i), \gamma_B(e_i)|_j$ be any two CIFSM, then the addition of A, B is followed by,
A + B = $|\mu_A(e_i), \gamma_A(e_i)|_j + |\mu_B(e_i), \gamma_B(e_i)|_j$
$\Rightarrow (|\mu_A(e_i) + \mu_A(e_i)|_j, |\gamma_B(e_i) + \gamma_B(e_i)|_j)$

Example 3.6.

$$A = \begin{bmatrix} <[0.21,0.18]> & <(0.22,0.12)> \\ <[0.19,0.31]> & <[0.26,0.24]> \end{bmatrix}$$

$$B = \begin{bmatrix} <[0.20,0.25]> & <(0.24,0.11)> \\ <[0.20,0.27]> & <[0.25,0.20]> \end{bmatrix},$$

$$\text{then} A + B = \begin{bmatrix} <[(0.21+0.20),(0.18+0.25)]> & <((0.22+0.24),(0.12+0.11))> \\ <[(0.19+0.20),(0.31+0.27)]> & <[(0.26+0.25),(0.24+0.20)]> \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} <[0.41,0.43]> & <(0.46,0.23)> \\ <[0.39,0.58]> & <[0.51,0.44]> \end{bmatrix}$$

Note: Let A and B are two CIFSM, then the addition of A, B is A + B if A + B > 1, then we using the following condition

$$\Rightarrow \frac{(A+B)}{2}$$

$$\Rightarrow \frac{|\mu_A(e_i), \gamma_A(e_i)|_j + |\mu_B(e_i), \gamma_B(e_i)|_j}{2}$$

$$\Rightarrow \left[\frac{|\mu_A(e_i) + \mu_A(e_i)|_j}{2}, \frac{|\gamma_B(e_i) + \gamma_B(e_i)|_j}{2}\right]$$

Similarly, we adding three CIFSMs, then the addition of A, B, C is A + B + C if A + B + C > 1, then we using the following condition

$\Rightarrow \left[\frac{|\mu_A(e_i)+\mu_B(e_i)+\mu_C(e_i)|_j}{2}, \frac{|\gamma_A(e_i)+\gamma_B(e_i)+\gamma_C(e_i)|_j}{2}\right]$ Matrices addition is generalized by 'n' terms

$$\Rightarrow \frac{(A+B+C+\cdots+Z)}{n}$$

Definition 3.7. Let A = $|\mu_A(e_i), \gamma_A(e_i)|_j$ and B = $|\mu_B(e_i), \gamma_B(e_i)|_j$ be any two CIFSM, then the multiplication of A, B is followed by,
A . B = $|\mu_A(e_i), \gamma_A(e_i)|_j . |\mu_B(e_i), \gamma_B(e_i)|_j$
$\Rightarrow (|\mu_A(e_i). \mu_A(e_i)|_j, |\gamma_B(e_i). \gamma_B(e_i)|_j)$

Example 3.8.

$$A = \begin{bmatrix} <[0.21,0.18]> & <(0.22,0.12)> \\ <[0.19,0.31]> & <[0.26,0.24]> \end{bmatrix}$$

$$B = \begin{bmatrix} <[0.20,0.25]> & <(0.24,0.11)> \\ <[0.20,0.27]> & <[0.25,0.20]> \end{bmatrix},$$

$$\text{then} A + B = \begin{bmatrix} <[(0.21\times 0.20),(0.18\times 0.25)]> & <((0.22\times 0.24),(0.12\times 0.11))> \\ <[(0.19\times 0.20),(0.31\times 0.27)]> & <[(0.26\times 0.25),(0.24\times 0.20)]> \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} <[0.042,0.045]> & <(0.052,0.013)> \\ <[0.038,0.083]> & <[0.065,0.048]> \end{bmatrix}$$

Definition 3.9. Let A = $|\mu_A(e_i), \gamma_A(e_i)|_j$ and B = $|\mu_B(e_i), \gamma_B(e_i)|_j$ be any two CIFSM. Then(1)Union of A and B is denoted by, A ∨B is defined as

A ∨B= max $\{|\mu_A(e_i), \mu_B(e_i)|_j\}$, max $\{|\gamma_A(e_i), \gamma_B(e_i)|_j\}$
(2)Intersection of A and B is denoted by, A ∧ B is defined as
A ∧ B = min $\{|\mu_A(e_i), \mu_B(e_i)|_j\}$, min $\{|\gamma_A(e_i), \gamma_B(e_i)|_j\}$

Definition3.10.

Let A = $(|\mu_P(e_i), \gamma_P(e_i)|_i)_{(n\times n)}$ be a CIFSM, then the trace of complex intuitionisticfuzzysoft matrix is the sum of the elements of the principal diagonalelementsof a squarematrix is known as the trace of matrix. It isdenoted by Tr(A). Where,

$$\text{Tr (A)} = \sum_{i,j=1}^{n} (|\mu_A(e_1), \gamma_A(e_1)|_1) + (|\mu_A(e_2), \gamma_A(e_2)|_2) + (|\mu_A(e_3), \gamma_A(e_3)|_3) + \cdots + (|\mu_A(e_n), \gamma_A(e_n)|_n)$$

Example 3.11.$(A_{n\times n}) = |a_{ij}^P|$=$\begin{bmatrix} < [0.21,0.19] > & < (0.23,0.12) > \\ < [0.20,0.32] > & < [0.27,0.25] > \end{bmatrix}$
Tr (A) = $(< [0.21,0.19] > + < [0.27,0.25] >) = < [0.48,0.44] >$

Propertiesof complex intuitionistic fuzzy soft matrices In this section, we investigate some properties for complex intuitionistic fuzzy soft matrices (CIFSMs).

Property 4.1.

Let A and Bbe twoCIFSM,

ConsiderA = $|\mu_A(e_i), \gamma_A(e_i)|_j$ and B $= |\mu_B(e_i), \gamma_B(e_i)|_j$
1.(A + B) =(B + A) commutative under '+'
ie., $|\mu_A(e_i), \gamma_A(e_i)|_j + |\mu_B(e_i), \gamma_B(e_i)|_j$= $|\mu_B(e_i), \gamma_B(e_i)|_j + |\mu_A(e_i), \gamma_A(e_i)|_j$ for all i,j.
2. (A + B) + C=A+(B + C) associative under '+' ie., $(|\mu_A(e_i), \gamma_A(e_i)|_j + |\mu_B(e_i), \gamma_B(e_i)|_j) + |\mu_C(e_i), \gamma_C(e_i)|_j$=$|\mu_A(e_i), \gamma_A(e_i)|_j + (|\mu_B(e_i), \gamma_B(e_i)|_j + |\mu_C(e_i), \gamma_C(e_i)|_j)$ for all i,j.
3. (A. B) =(B. A) commutative under '.'
ie., $|\mu_A(e_i), \gamma_A(e_i)|_j . |\mu_B(e_i), \gamma_B(e_i)|_j$= $|\mu_B(e_i), \gamma_B(e_i)|_j . |\mu_A(e_i), \gamma_A(e_i)|_j$ for all i,j.
4. (A . B). C= A.(B. C)associative under '.'
ie.,$(|\mu_A(e_i), \gamma_A(e_i)|_j . |\mu_B(e_i), \gamma_B(e_i)|_j) . |\mu_C(e_i), \gamma_C(e_i)|_j$=$|\mu_A(e_i), \gamma_A(e_i)|_j .$
$(|\mu_B(e_i), \gamma_B(e_i)|_j . |\mu_C(e_i), \gamma_C(e_i)|_j)$ for all i,j.
5.A.(B + C)= (A. B) + (A. C) distributive under (+,.)
$|\mu_A(e_i), \gamma_A(e_i)|_j . (|\mu_B(e_i), \gamma_B(e_i)|_j + |\mu_C(e_i), \gamma_C(e_i)|_j)$
$= (|\mu_A(e_i), \gamma_A(e_i)|_j . |\mu_B(e_i), \gamma_B(e_i)|_j) + (|\mu_A(e_i), \gamma_A(e_i)|_j . |\mu_C(e_i), \gamma_C(e_i)|_j)$ for all i,j.

Property 4.2.

Let A ,B∈ $(\text{CIFSM})_{m\times n}$
(i)A ∨B = B ∨A
(ii)A ∧B = B ∧A

Proof:

Let A = $|\mu_A(e_i), \gamma_A(e_i)|_j$, B = $|\mu_B(e_i), \gamma_B(e_i)|_j$
(i)A ∨ B = $|\mu_A(e_i), \gamma_A(e_i)|_j \vee |\mu_B(e_i), \gamma_B(e_i)|_j$
⇒ max $\{|\mu_A(e_i), \gamma_A(e_i)|_j, |\mu_B(e_i), \gamma_B(e_i)|_j\}$.
⇒ max $|\mu_A(e_i), \mu_B(e_i)|_j$, max $|\gamma_A(e_i), \gamma_B(e_i)|_j$
⇒ max $|\mu_B(e_i), \mu_A(e_i)|_j$, max $|\gamma_B(e_i), \gamma_A(e_i)|_j$
⇒ $|\mu_B(e_i), \gamma_B(e_i)|_j \vee |\mu_A(e_i), \gamma_A(e_i)|_j$
Hence A ∨B = B ∨A
(ii) A ∧ B = $|\mu_A(e_i), \gamma_A(e_i)|_j \wedge |\mu_B(e_i), \gamma_B(e_i)|_j$
⇒ min $\{|\mu_A(e_i), \gamma_A(e_i)|_j, |\mu_B(e_i), \gamma_B(e_i)|_j\}$.
⇒ min $|\mu_A(e_i), \mu_A(e_i)|_j$, min $|\gamma_B(e_i), \gamma_B(e_i)|_j$
⇒ min $|\mu_B(e_i), \mu_A(e_i)|_j$, min $|\gamma_B(e_i), \gamma_A(e_i)|_j$
⇒ $|\mu_B(e_i), \gamma_A(e_i)|_j \wedge |\mu_B(e_i), \gamma_A(e_i)|_j$
HenceA ∧B = B ∧A

Property 4.3.

LetA, B and C ∈ $(\text{CIFSM})_{m\times n}$. Then

(i) (A + B) ∨ (B + C) =(A + B) ∨(A + C), whenever C ≥ A ≥ B
(ii) (A + B) ∨ (B + C) =(A + C) ∨(B + C),whenever A ≥ B ≥ C
(iii)(A + B) ∧ (B + C) =(A + C) ∧(B + C),wheneverA ≤ C ≤ B
(iv) (A + B) ∧ (B + C) =(A + B) ∧(A + C),whenever C ≥ A ≥ B

Proof:

Let $A = |\mu_A(e_i), \gamma_A(e_i)|_j$,$B = |\mu_B(e_i), \gamma_B(e_i)|_j$, $C = |\mu_C(e_i), \gamma_C(e_i)|_j$(i)L.H.S$\Rightarrow$ $(A + B) \vee (B + C) = \left(|\mu_A(e_i), \gamma_A(e_i)|_j + |\mu_B(e_i), \gamma_B(e_i)|_j\right) \vee \left(|\mu_B(e_i), \gamma_B(e_i)|_j + |\mu_C(e_i), \gamma_C(e_i)|_j\right) \Rightarrow$ $\max\{(|\mu_A(e_i), \mu_B(e_i)|_j, |\gamma_A(e_i), \gamma_B(e_i)|_j)\}+\max\{(|\mu_B(e_i), \mu_C(e_i)|_j, |\gamma_B(e_i), \gamma_C(e_i)|_j)\}$From the hypothesis,$C \geq A \geq B$L.H.S $= |\mu_A(e_i), \gamma_A(e_i)|_j + |\mu_C(e_i), \gamma_C(e_i)|_j$Now, R.H.S$\Rightarrow(A + B) \vee (A + C)=\left(|\mu_A(e_i), \gamma_A(e_i)|_j + |\mu_B(e_i), \gamma_B(e_i)|_j\right) \vee \left(|\mu_A(e_i), \gamma_A(e_i)|_j + |\mu_C(e_i), \gamma_C(e_i)|_j\right) \Rightarrow \max\{(|\mu_A(e_i), \mu_B(e_i)|_j, |\gamma_A(e_i), \gamma_B(e_i)|_j)\} + \max\{(|\mu_A(e_i), \mu_C(e_i)|_j, |\gamma_A(e_i), \gamma_C(e_i)|_j)\}$Again from the hypothesis,$C \geq A \geq B$R.H.S $= |\mu_A(e_i), \gamma_A(e_i)|_j + |\mu_C(e_i), \gamma_C(e_i)|_j$Which implies that, L.H.S=R.H.SHence $(A + B) \vee (B + C) =(A + B) \vee (A + C)$.

(ii)L.H.S$\Rightarrow$ $(A + B) \vee (B + C) = \left(|\mu_A(e_i), \gamma_A(e_i)|_j + |\mu_B(e_i), \gamma_B(e_i)|_j\right) \vee \left(|\mu_B(e_i), \gamma_B(e_i)|_j + |\mu_C(e_i), \gamma_C(e_i)|_j\right) \Rightarrow$ $\max\{(|\mu_A(e_i), \mu_B(e_i)|_j, |\gamma_A(e_i), \gamma_B(e_i)|_j)\}+\max\{(|\mu_B(e_i), \mu_C(e_i)|_j, |\gamma_B(e_i), \gamma_C(e_i)|_j)\}$ From the hypothesis, $A \geq B \geq C$L.H.S$=|\mu_A(e_i), \gamma_A(e_i)|_j + |\mu_C(e_i), \gamma_C(e_i)|_j$ Now,R.H.S$\Rightarrow (A + C) \vee (B + C) = \max\{(|\mu_A(e_i), \mu_C(e_i)|_j, |\gamma_A(e_i), \gamma_C(e_i)|_j)\} + \max\{(|\mu_B(e_i), \mu_C(e_i)|_j, |\gamma_B(e_i), \gamma_C(e_i)|_j)\} \Rightarrow \max\{(|\mu_A(e_i), \mu_C(e_i)|_j, |\gamma_A(e_i), \gamma_C(e_i)|_j)\} + \max\{(|\mu_B(e_i), \mu_C(e_i)|_j, |\gamma_B(e_i), \gamma_C(e_i)|_j)\}$Again from the hypothesis, $A \geq B \geq C \Rightarrow |\mu_A(e_i), \gamma_A(e_i)|_j + |\mu_C(e_i), \gamma_C(e_i)|_j$Which implies that, L.H.S=R.H.SHence $(A + B) \vee (B + C) =(A + C) \vee (B + C)$.

(iii)L.H.S$\Rightarrow (A + B) \wedge (B + C) =$ $\left(|\mu_A(e_i), \gamma_A(e_i)|_j + |\mu_B(e_i), \gamma_B(e_i)|_j\right) \wedge \left(|\mu_B(e_i), \gamma_B(e_i)|_j + |\mu_C(e_i), \gamma_C(e_i)|_j\right) \Rightarrow \min\{(|\mu_A(e_i), \mu_B(e_i)|_j, |\gamma_A(e_i), \gamma_B(e_i)|_j)\}+\min\{(|\mu_B(e_i), \mu_C(e_i)|_j, |\gamma_B(e_i), \gamma_C(e_i)|_j)\}$From the hypothesis,$A \leq C \leq B \Rightarrow |\mu_A(e_i), \gamma_A(e_i)|_j + |\mu_C(e_i), \gamma_C(e_i)|_j$L.H.S$= |\mu_A(e_i), \gamma_A(e_i)|_j + |\mu_C(e_i), \gamma_C(e_i)|_j$Now, R.H.S$\Rightarrow (A + C) \wedge (B + C) = \min\{(|\mu_A(e_i), \mu_C(e_i)|_j, |\gamma_A(e_i), \gamma_C(e_i)|_j)\} + \min\{(|\mu_B(e_i), \mu_C(e_i)|_j, |\gamma_B(e_i), \gamma_C(e_i)|_j)\} \Rightarrow \min\{(|\mu_A(e_i), \mu_C(e_i)|_j, |\gamma_A(e_i), \gamma_C(e_i)|_j)\} + \min\{(|\mu_B(e_i), \mu_C(e_i)|_j, |\gamma_B(e_i), \gamma_C(e_i)|_j)\}$Again from the hypothesis,$A \leq C \leq B$ $\Rightarrow |\mu_A(e_i), \gamma_A(e_i)|_j + |\mu_C(e_i), \gamma_C(e_i)|_j$ Which implies that, L.H.S = R.H.S Hence $(A + B) \wedge (B + C) =(A + C) \wedge (B + C)$.

(iv)

L.H.S$\Rightarrow (A + B) \wedge (B + C) =$ $\left(|\mu_A(e_i), \gamma_A(e_i)|_j + |\mu_B(e_i), \gamma_B(e_i)|_j\right) \wedge \left(|\mu_B(e_i), \gamma_B(e_i)|_j + |\mu_C(e_i), \gamma_C(e_i)|_j\right) \Rightarrow \min\{(|\mu_A(e_i), \mu_B(e_i)|_j, |\gamma_A(e_i), \gamma_B(e_i)|_j)\}+\min\{(|\mu_B(e_i), \mu_C(e_i)|_j, |\gamma_B(e_i), \gamma_C(e_i)|_j)\}$ From the hypothesis,$C \geq A \geq B \Rightarrow |\mu_A(e_i), \gamma_A(e_i)|_j + |\mu_C(e_i), \gamma_C(e_i)|_j$ L.H.S$= |\mu_A(e_i), \gamma_A(e_i)|_j + |\mu_C(e_i), \gamma_C(e_i)|_j$ Now, R.H.S$\Rightarrow (A + B) \wedge (A + C) = \min\{(|\mu_A(e_i), \mu_B(e_i)|_j, |\gamma_A(e_i), \gamma_B(e_i)|_j)\} + \min\{(|\mu_A(e_i), \mu_C(e_i)|_j, |\gamma_A(e_i), \gamma_C(e_i)|_j)\} \Rightarrow \min\{(|\mu_A(e_i), \mu_B(e_i)|_j, |\gamma_A(e_i), \gamma_B(e_i)|_j)\} + \min\{(|\mu_A(e_i), \mu_C(e_i)|_j, |\gamma_A(e_i), \gamma_C(e_i)|_j)\}$ Again from the hypothesis,$C \geq A \geq B \Rightarrow |\mu_A(e_i), \gamma_A(e_i)|_j + |\mu_C(e_i), \gamma_C(e_i)|_j$ Which implies that, L.H.S = R.H.SHence $(A + B) \wedge (B + C) =(A + B) \wedge (A + C)$.

Property 4.4.

Let A ,B $\in (\mathrm{CIFSM})_{m\times n}$. Then the following conditions are holds
(i) $(A^T)^T = A$
(ii) $(A + B)^T=(A)^T+(B)^T$
(iii) $k(A)^T=(kA)^T$
(iv) $(AB)^T = (B)^T(A)^T$
Proof:
Straight forward

Property 4.5.

(i) Tr A $\neq$ Tr A^C
(ii) $(\mathrm{Tr}\ A)^C$ = Tr A^C
(iii) Tr ($A+A^C$) = Tr A + Tr A^C
(iv) Tr (λA) = λ Tr A
(v) Tr (A + B) = Tr A + Tr B
(vi) Tr (AB) = Tr (BA)
(vii) Tr (ABCD) = Tr (BCDA) = Tr (CDAB) = Tr (DABC)
Proof:
Straight forward

Application of complex intuitionisticfuzzy soft square matrices (CIFSSMs) in Decision Making Based on T-norm operators.

In this section, we constructcomplex intuitionisticfuzzy soft square matrices in decision making by using different t-norm operators.

Definition 5.1. Let us discuss the t-norm minimum operator of CIFSSMs

$\tilde{T}^M(|\mu_1(e_i),\gamma_1(e_i)|_j, |\mu_2(e_i),\gamma_2(e_i)|_j, \dots, |\mu_n(e_i),\gamma_n(e_i)|_j) = \min(|\mu_1(e_i),\gamma_1(e_i)|_j, |\mu_2(e_i),\gamma_2(e_i)|_j, \dots, |\mu_n(e_i),\gamma_n(e_i)|_j) \Rightarrow \min(|\mu_1(e_i),\mu_2(e_i),\dots,\mu_n(e_i)|_j), \min(|\gamma_1(e_i),\gamma_2(e_i),\dots,\gamma_n(e_i)|_j)$

Definition 5.2. Let us discuss the t-norm product operator ofCIFSSMs

$$\tilde{T}^P\left((|\mu_1(e_i),\gamma_1(e_i)|_j, |\mu_2(e_i),\gamma_2(e_i)|_j, \dots, |\mu_n(e_i),\gamma_n(e_i)|_j)\right) = \prod_{i,j=1}^{n}\{(|\mu_1(e_i),\gamma_1(e_i)|_j, |\mu_2(e_i),\gamma_2(e_i)|_j, \dots, |\mu_n(e_i),\gamma_n(e_i)|_j)\}$$

$$\Rightarrow \prod_{i,j=1}^{n} (|\mu_1(e_i),\mu_2(e_i),\dots,\mu_n(e_i)|_j), \prod_{i,j=1}^{n} (|\gamma_1(e_i),\gamma_2(e_i),\dots,\gamma_n(e_i)|_j)$$

Definition 5.3. Let us discuss the t-norm Bounded operator of CIFSSMs

$$\tilde{T}^B\left((|\mu_1(e_i),\gamma_1(e_i)|_j, |\mu_2(e_i),\gamma_2(e_i)|_j, \dots, |\mu_n(e_i),\gamma_n(e_i)|_j)\right) = \frac{1}{n}\left\{\sum_{i,j=1}^{n}\{(\{\mu_1(e_i),\gamma_1(e_i)\}_j, \{\mu_2(e_i),\gamma_2(e_i)\}_j, \dots, \{\mu_n(e_i),\gamma_n(e_i)\}_j\}\right\}^{\frac{1}{n}}$$

$$\Rightarrow \frac{1}{n}\left\{\sum_{i,j=1}^{n}\{\mu_1(e_i),\mu_2(e_i),\dots,\mu_n(e_i)\}_j^{\frac{1}{n}}, \sum_{i,j=1}^{n}\{\gamma_1(e_i),\gamma_2(e_i),\dots,\gamma_n(e_i)\}_j^{\frac{1}{n}}\right\}$$

Definition 5.4. Arithmetic mean (A.M) of CIFSSMs

$$\mathrm{A_{AM}} = \frac{|\mu_P(e_i) + \gamma_P(e_i)|_j}{2}$$

Definition 5.5. Geometric mean (G.M) of CIFSSMs

$$\mathrm{A_{GM}} = \{\{\mu_P(e_i).\gamma_P(e_i)\}_j\}^{\frac{1}{2}}$$

Algorithm

Step-1: Choose the set of parameters

Step-2: Construct the complexintuitionistic fuzzy softsquare matrices.

Step-3: Compute $\tilde{T}^M$, $\tilde{T}^P$ and $\tilde{T}^B$

Step-4: Compute the membership value of the complex intuitionistic fuzzy soft square matrix of thearithmetic mean and geometric mean as $\mathrm{A_{AM}}(\tilde{T}^M)$, $\mathrm{A_{AM}}(\tilde{T}^P)$, $\mathrm{A_{AM}}(\tilde{T}^L)$ and $\mathrm{A_{GM}}(\tilde{T}^M)$, $\mathrm{A_{GM}}(\tilde{T}^P)$, $\mathrm{A_{GM}}(\tilde{T}^L)$ respectively.

Step-5: Find the highest membership value.

Statement of the problem Suppose a mobile production company produces four types of mobilesm_1, m_2, m_3, m_4 such that M = $\{m_1, m_2, m_3, m_4\}$and S = $\{s_1, s_2, s_3, s_4\}$ be the set ofparameters. Here, we just explained the four types of parameters as followedby,s_1 = multiple windows, s_2 = infrared remote control, s_3 = wireless charging, s_4= near field communications.

(1) Form CIFSSMsas

$$A = \begin{bmatrix} <(0.25,0.64)> & <(0.34,0.53)> & <(0.36,0.64)> & <(0.39,0.48)> \\ <(0.61,0.38)> & <(0.36,0.49)> & <(0.32,0.50)> & <(0.41,0.47)> \\ <(0.43,0.06)> & <(0.73,0.09)> & <(0.64,0.21)> & <(0.48,0.38)> \\ <(0.42,0.25)> & <(0.45,0.40)> & <(0.28,0.72)> & <(0.52,0.25)> \end{bmatrix}$$

$$B = \begin{bmatrix} <(0.69,0.23)> & <(0.54,0.33)> & <(0.23,0.57)> & <(0.63,0.12)> \\ <(0.34,0.63)> & <(0.37,0.34)> & <(0.35,0.60)> & <(0.14,0.28)> \\ <(0.61,0.12)> & <(0.25,0.70)> & <(0.42,0.25)> & <(0.38,0.46)> \\ <(0.55,0.45)> & <(0.50,0.48)> & <(0.56,0.34)> & <(0.69,0.29)> \end{bmatrix}$$

$$C = \begin{bmatrix} <(0.51,0.34)> & <(0.33,0.34)> & <(0.61,0.36)> & <(0.91,0.09)> \\ <(0.66,0.25)> & <(0.56,0.41)> & <(0.33,0.43)> & <(0.79,0.21)> \\ <(0.42,0.44)> & <(0.46,0.37)> & <(0.68,0.25)> & <(0.40,0.33)> \\ <(0.44,0.45)> & <(0.70,0.30)> & <(0.37,0.43)> & <(0.25,0.61)> \end{bmatrix}$$

$$D = \begin{bmatrix} <(0.40,0.37)> & <(0.23,0.54)> & <(0.47,0.11)> & <(0.32,0.33)> \\ <(0.46,0.54)> & <(0.40,0.33)> & <(0.41,0.39)> & <(0.46,0.34)> \\ <(0.71,0.28)> & <(0.12,0.88)> & <(0.01,0.81)> & <(0.93,0.07)> \\ <(0.37,0.49)> & <(0.72,0.23)> & <(0.45,0.36)> & <(0.01,0.99)> \end{bmatrix}$$

(2) Using Definition 5.1, the computation of $\tilde{T}^M$ is as below:

$$\tilde{T}^M = \begin{bmatrix} <(0.40,0.23)> & <(0.23,0.33)>& <(0.23,0.11)> & <(0.32,0.09)> \\ <(0.46,0.25)> & <(0.36,0.33)>& <(0.32,0.39)> & <(0.14,0.21)> \\ <(0.42,0.06)> & <(0.12,0.09)>& <(0.01,0.21)> & <(0.38,0.07)> \\ <(0.37,0.25)> & <(0.45,0.23)>& <(0.28,0.34)> & <(0.01,0.25)> \end{bmatrix}$$

(3) Using Definition 5.4, the computation of AM($\tilde{T}^M$) is as below:

$$\text{AM}(\tilde{T}^M) = \begin{bmatrix} <0.32\ 0.28\ 0.17\ 0.20> \\ <0.35\ 0.34\ 0.35\ 0.17> \\ <0.24\ 0.10\ 0.11\ 0.22> \\ <0.31\ 0.34\ 0.31\ 0.13> \end{bmatrix}$$

(4) Add each entries and find the highest value for AM($\tilde{T}^M$)

$$= \begin{bmatrix} 0.97 \\ 1.21 \\ 0.67 \\ 1.09 \end{bmatrix}$$

(5) Using Definition 5.5, the computation of GM($\tilde{T}^M$) is as below:

$$\text{GM}(\tilde{T}^M) = \begin{bmatrix} <0.30\ 0.27\ 0.15\ 0.16> \\ <0.33\ 0.34\ 0.35\ 0.17> \\ <0.15\ 0.10\ 0.04\ 0.16> \\ <0.30\ 0.32\ 0.30\ 0.05> \end{bmatrix}$$

(6) Add each entries and find the highest value for GM($\tilde{T}^M$)

$$= \begin{bmatrix} 0.88 \\ 1.19 \\ 0.45 \\ 0.97 \end{bmatrix}$$

From AM ($\tilde{T}^M$) and GM ($\tilde{T}^M$), it is obvious that m_4 mobile will be preferred. If $\tilde{T}^P$ and $\tilde{T}^B$ are used instead of $\tilde{T}^M$, then we have

(7) Using Definition 5.2, the computation of $\tilde{T}^P$ is as below:

$$\tilde{T}^P = \begin{bmatrix} <(0.0351,0.0185)> & <(0.0139,0.0321)>& <(0.0237,0.0144)> & <(0.0715,0.0017)> \\ <(0.0629,0.0323)> & <(0.0298,0.0225)>& <(0.0715,0.0503)> & <(0.0208,0.0093)> \\ <(0.0782,0.0008)> & <(0.0100,0.0205)>& <(0.0018,0.0106)> & <(0.0678,0.0040)> \\ <(0.0376,0.0248)> & <(0.1134,0.0132)>& <(0.0261,0.0378)> & <(0.0008,0.0437)> \end{bmatrix}$$

(8) Using Definition 5.4, the computation of AM($\tilde{T}^P$) is as below:

$$\text{AM}(\tilde{T}^P) = \begin{bmatrix} <0.0268\ 0.0230\ 0.0190\ 0.0366> \\ <0.0476\ 0.0261\ 0.0609\ 0.0150> \\ <0.0395\ 0.0152\ 0.0062\ 0.0359> \\ <0.0312\ 0.0633\ 0.0319\ 0.0225> \end{bmatrix}$$

(9) Add each entries and find the highest value for AM($\tilde{T}^P$)

$$= \begin{bmatrix} 0.1054 \\ 0.1496 \\ 0.0968 \\ 0.1489 \end{bmatrix}$$

(10) Using Definition 5.5, the computation of GM($\tilde{T}^P$) is as below:

$$\text{GM}\,(\tilde{T}^P) = \begin{bmatrix} < 0.0254\ 0.0211\ 0.0184\ 0.0110 > \\ < 0.0450\ 0.0258\ 0.0599\ 0.0139 > \\ < 0.0079\ 0.0143\ 0.0043\ 0.0164 > \\ < 0.0305\ 0.0386\ 0.0314\ 0.0059 > \end{bmatrix}$$

(11) Add each entries and find the highest value for GM($\tilde{T}^P$)

$$= \begin{bmatrix} 0.0759 \\ 0.1446 \\ 0.0429 \\ 0.1064 \end{bmatrix}$$

From AM ($\tilde{T}^P$) and GM ($\tilde{T}^P$), it is obvious that m_2 mobile will be preferred.

(12) Using Definition 5.3, the computation of $\tilde{T}^B$ is as below:

$$\tilde{T}^B = \begin{bmatrix} < (0.29{,}0.28) > & < (0.27{,}0.28) >< (0.28{,}0.28) > & < (0.30{,}0.25) > \\ < (0.29{,}0.28) > & < (0.28{,}0.27) >< (0.27{,}0.29) > & < (0.28{,}0.26) > \\ < (0.30{,}0.24) > & < (0.27{,}0.29) >< (0.28{,}0.27) > & < (0.30{,}0.26) > \\ < (0.28{,}0.28) > & < (0.31{,}0.27) >< (0.28{,}0.29) > & < (0.27{,}0.30) > \end{bmatrix}$$

(13) Using Definition 5.4, the computation of AM($\tilde{T}^B$) is as below:

$$\text{AM}\,(\tilde{T}^B) = \begin{bmatrix} < 0.29\ 0.28\ 0.28\ 0.28 > \\ < 0.29\ 0.28\ 0.28\ 0.27 > \\ < 0.27\ 0.28\ 0.28\ 0.28 > \\ < 0.28\ 0.29\ 0.29\ 0.29 > \end{bmatrix}$$

(14) Add each entries and find the highest value for AM($\tilde{T}^B$)

$$= \begin{bmatrix} 1.13 \\ 1.12 \\ 1.11 \\ 1.15 \end{bmatrix}$$

(15) Using Definition 5.5, the computation of GM($\tilde{T}^B$) is as below:

$$\text{GM}(\tilde{T}^B) = \begin{bmatrix} < 0.28\ 0.27\ 0.28\ 0.27 > \\ < 0.28\ 0.27\ 0.27\ 0.26 > \\ < 0.26\ 0.27\ 0.27\ 0.27 > \\ < 0.28\ 0.28\ 0.28\ 0.28 > \end{bmatrix}$$

(16) Add each entries and find the highest value for GM($\tilde{T}^B$)

$$= \begin{bmatrix} 1.10 \\ 1.08 \\ 1.07 \\ 1.12 \end{bmatrix}$$

From AM ($\tilde{T}^B$) and GM ($\tilde{T}^B$), it is obvious that m_4 mobile will be preferred.

CONCLUSION

In this document, we investigated some properties of complex intuitionistic fuzzy soft square matrix theory with suitable examples. Further, we constructed complex intuitionistic fuzzy soft square matrices in decision making based on T-norm operators. We hope that our finding will help to enhancing the study on fuzzy soft matrix theory and will open a new direction for applications especially in decision analysis. In future, we extended this concept in complex Pythagorean fuzzy soft matrix theory.

REFERENCES

Atanassov, K. 1986. ”Intuitionisticfuzzy sets , *Fuzzy Sets and Systems*, 20),87-96.

Atanassov K. and Gargov, G. 1989. Interval-valued intuitionistic fuzzy sets, *Fuzzy Sets and Systems*31 (1989), 343–349.

Atanassov, K., Pasi, G. and Yager, R.R. 2005. “Intuitionistic fuzzy interpretationsof multi- criteriamulti-person and multimeasurement tool decision making”,*InternationalJournal of Systems Science,*36 (2005), 859–868.

Bhowmik M, Pal M, 2008. “Generalized intuitionistic fuzzy matrices”, *Far – East J Math Sci,*29 533–554.

Bhowmik. M., Pal. M. 2008. “Some results on intuitionistic fuzzy matrices and intuitionistic circulant fuzzy matrices”, *Int J Math Sci*, 7 (2008) 81–96.

Bustinceand H., Burillo, P.1995. Correlation of intervalvalued intuitionistic fuzzy sets, *Fuzzy Set and Systems,*74, 237–244.

Cagman, N., Enginoglu, S. 2012. ”Fuzzy soft matrix theory and its application in decision-making”, *Iranian Journal of Fuzzy Systems,*9(1), 109-119.

Chinnadurai, V., Thayalanand S., Bobin, A. 2021. “Some Operations of complex interval-valued Pythagorean fuzzy set and its application”.*Communications inMathematics andApplications,* 2021, volume 12,pp. 483–497.

Maji, K., Biswas, R., Roy, A.R., 2001. ”Fuzzy soft sets”, *Journal of Fuzzy Mathematics,*9(3), 589-602.

Molodtsov, D. “Softset theoryfirst result”, *Computer andmathematics with applications*, 37(1999),19-31.

Pal M, Khan S.K., Shyama A.K. 2002. “Intuitionistic Fuzzy Matrices”, *Notes on Intuitionistic Fuzzy Sets*, 8(2) 51-62.

Ramot, D., Milo, R., Friedman, M. and Kandel, A. 2012. “Complex fuzzy sets”, *IEEE Trans Fuzzy Syst.*, volume 10, pp. 171–186.

Szmidt E. and Kacprzyk, J. 2002. Using intuitionistic fuzzy sets in group decision making, *Control and Cybernetics*, 31 1037–1053.

Szmidt E. and Kacprzyk, J. 2010. Correlation of intuitionistic fuzzy sets*, Lectuer Notes in Computer Science,*6178 169–177.

Torra V, Hesitant fuzzy sets.*Int JIntellSyst,*25 ,529–539.

Thomason M.G. 1977. “Convergence of powers of fuzzy matrix”, *J. Mathematical Analysis and Applications,*57 476- 480.

Zadeh. L. A. 1965. ”Fuzzy sets” , *Information and Control*,8 , 330-353.