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RESEARCH ARTICLE

DISPERSION OF MHD COUPLE STRESS FLUID FLOW ON MULTI-STENOS IS ARTERIES IN BLOOD ATTRIBUTES WITH PRESENCE OF HALL CURRENT

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ABSTRACT

In this paper, we have examine the analysis of pulsatile flow in rectangular plate with multi-stenosis artery. The impact of Hall current in external magnetic field and electrically conducting blood fluid. Dispersion of blood vessels from couple stress fluid using method of solution is (i) Taylor dispersion and (ii) Generalized dispersion model. The velocity field, dispersion coefficient and mean concentration are solved analytically. Variation of different dimensionless parameters are graphically through the plots.

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INTRODUCTION

Multi stenosis is cardiovascular diseases which causes blood flow is abnormal in cardiac arrest arteries. The cardiac arrest (heart disease) it's heart conditions particular diseased blood vessels, formation of the problems and blood thicken. The blood flow have investigated in many researchers analyzed dispersion of soluable matter in solvent flowing slowly through a tube by Taylor (1953). Exact analysis of unsteady convective diffusion investigated by Gill and Sankarasubramanian (1970)(1971) and (1973). Patel and Sirs (1983) examined study the dispersion of solutes during blood flow through curved tubes. Developed Shivakumar et.al.,(1987) closed form solution for unsteady diffusion in a fluid saturated sparsely packed porous medium. Ikbal *et al.,* (2009) studying unsteady response of non newtonian blood flow through a stenosed artery in magnetic field. Obtained mathematical anlysis of unsteady sloute dispersion with chemical reaction through a stenosed artery Nurul Aini Jaafar *et al.,* (2016) and (2021). Sankar(2016) and Nirmala Ratchagar and Vijayakumar (2019) studied generalized dispersion method is analyticallysolving blood flow have an casson fluid. Effect of externally applied transverse magnetic field on unsteady flow of blood in tapered stenosed artery investigated by Veena (2019). Mukesh Roy *et al.,* (2017) showed modelling of blood flow in stenosed arteries. Nagarani (2017) investigated effect of flow unsteadiness on dispersion in non newtonian fluid in an annulus. Studied dispersion of waves and transmission reflection in blood vessels with structured stents by Frecentese. Meenapriya (2011) and Vijayakumar (2015) studied dispersion of analytical solution of the results. Analyzed pulsatile through a generalized blood flow in non linear equation solving and solution part in numerical values using finite difference method investigated by sathyasaran changdar and Soumen De (2016). In our model is steady convective diffusion, impact of Hall current on externally magnetic field with mass shift by make use of Taylordispersion model. The blood fluid modeled is couple stress fluid, rectangular channel with pulsatile blood flow on a porous medium, in a multi-stenosed artery. Considered the numerical values for several parameters are plotted and discussed.

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MATHEMATICAL FORMULATION

Consider the viscous incompressible, laminar, pulsatile and fully developed unidirectional flow in rectangular channel. The geometry of pipe flow decribes the multi-stenosis artery. To external magnetic field apply to impact of Hall current.The geometry of the multiple stenosed artery $R(x)$ is radius artery,R_0 is normal artery, l_i and δ_i ($i = 1, 2, 3$) are length and maximum thickness of three stenosis. d_i is location of the stenosis ($i = 1, 2, 3$) , are illustrated in figure1. The three multiple-stenosis flow of blood at boundary of the wall is
$(y* = 0\,,\ y* = R\,)$.

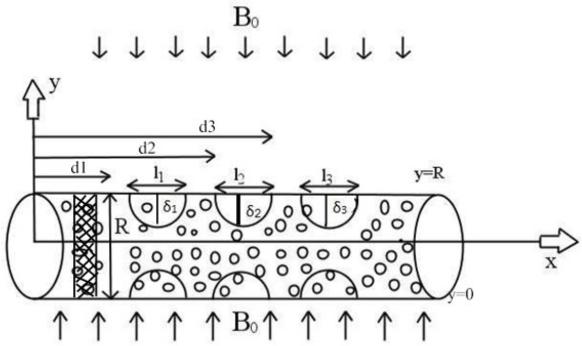

Figure: 1 Physical model.

The governing blood fluid film region can be written as follows:

Conservation of continuity:

$$\frac{\partial u^*}{\partial x^*} = 0 \tag{1}$$

Conservation of momentum:

$$0 = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \vartheta\frac{\partial^2 u^*}{\partial x^*} - \frac{\lambda}{\rho}\frac{\partial^4 u^*}{\partial y^{*4}} - \frac{\vartheta}{k}u^* - \frac{\sigma B_0^2}{\rho(1+m^2)}u^* \tag{2}$$

$$0 = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} \tag{3}$$

Conservation of concentration:

$$\frac{\partial c^*}{\partial t^*} + u^*\frac{\partial c^*}{\partial x^*} = D\frac{\partial^2 c^*}{\partial y^{*2}} \tag{4}$$

with boundary conditions,

$$\left.\begin{aligned} u^* = 0, \frac{\partial^2 u^*}{\partial y^{*2}} = 0 \ \ at\ \ y^* = R^* \\ u^* = 0, \frac{\partial^2 u^*}{\partial y^{*2}} = 0 \ \ at\ \ y^* = 0 \end{aligned}\right\} \tag{5}$$

$$\left.\begin{aligned} c^* = 0,\ \ at\ \ y^* = R^* \\ \frac{\partial c^*}{\partial y^*} = 0 \ \ at\ \ y^* = 0 \end{aligned}\right\} \tag{6}$$

where,

$u*$ is the axial field in x-way, g gravitation force, ρ is blood fluid of the density, p^* is the pressure gradient, k is permeability, λ denoted as slip parameter, v denoted as kinematic viscosity, μ is dynamic viscosity, $\beta_0$is viscoelastic coefficient, m is Hall current effect, B_0 is external force on magnetic field, C is concentration, R_e Reynolds number, D is diffusive flux. The blood liquid is handle to closed the pulsating move to the heart resulting in an unsteady pulsating pressure gradient approaching, Ogulu(1993)

$$-\frac{\partial p^*}{\partial x^*} = P_s + \in\ P_0\ Cos(\omega\ t) > 0 \tag{7}$$

Where, $P_s + \in P_0$ the amplitude of the pulsating component causing blood vessel or artery pressure and heart pressure $w = 2\pi f$ with f, the heart burst frequency.

The blood arterial of the multi-stenosis dimension mathematical form as,

$$(x^*) = \begin{cases} R_0 & 0 \le x^* \le {d_1}^* \\ R_0 - \frac{{\delta_1}^*}{2}\left(1 + Cos\frac{2\pi}{{l_1}^*}\left(x^* - {d_1}^* - \frac{{l_1}^*}{2}\right)\right) & {d_1}^* \le x^* \le {d_1}^* + {l_1}^* \\ R_0 & {d_1}^* + {l_1}^* \le x^* \le {d_2}^* \\ R_0 - \frac{{\delta_2}^*}{2}\left(1 + Cos\frac{2\pi}{{l_2}^*}\left(x^* - {d_2}^* - \frac{{l_2}^*}{2}\right)\right) & {d_2}^* \le x^* \le {d_2}^* + {l_2}^* \\ R_0 & {d_2}^* + {l_2}^* \le x^* \le {d_3}^* \\ R_0 - \frac{{\delta_3}^*}{2}\left(1 + Cos\frac{2\pi}{{l_3}^*}\left(x^* - {d_3}^* - \frac{{l_3}^*}{2}\right)\right) & {d_3}^* \le x^* \le {d_3}^* + {l_3}^* \\ R_0 & {d_3}^* + {l_3}^* \le x^* \le l^* \end{cases}$$

Introducing the following non dimensional quantities,

$$u = \frac{u^*}{u_0}, x = \frac{x^*}{R_0}, y = \frac{y^*}{R_0}, u = \frac{u^*}{U_0}, R_e = \frac{U_0\, R_0}{\vartheta}, p = \frac{R_0 p^*}{\rho\, \vartheta u_0}, M^2 = \frac{\sigma B_0^2\, R_0^2}{\mu}$$

$$l^2 = \frac{\lambda}{\mu}, l = \sqrt{\frac{\lambda}{\mu}}, l = \frac{l^*}{R_0}, d = \frac{d^*}{R_0}, \delta = \frac{\delta^*}{R_0} R = \frac{R^*}{R_0}, C = \frac{c^*}{c_0},\ t^* = \frac{t}{t'} \tag{8}$$

By using above non dimensional quantities, the equations (2)and (3) becomes,

$$-Re = \frac{1}{a^2}\frac{\partial^4 u}{\partial y^4} - \frac{\partial^2 u}{\partial y^2} + \left(\frac{M^2}{1+m^2} + \frac{1}{\lambda}\right) u \tag{9}$$

$$0 = \frac{\partial p}{\partial y} \tag{10}$$

Solving equation (9) using the boundary condition (5) we get,

$$u = c_1 e^{m_3 y} + c_2\, e^{-m_4\, y} + c_3 e^{m_5 y} + c_4 e^{-m_6 y} - \frac{Re\ P}{a^{2\left(\frac{M^2}{1+m^2}+\frac{1}{\lambda}\right)}} \tag{11}$$

u is average velocity specified by,

$$\overline{u} = \frac{1}{R}\int_R^0 u\, dy \tag{12}$$

$$\overline{u} = \frac{1}{R}\left[\frac{c_1 e^{m_3 R}}{m_3} - \frac{c_2 e^{-m_4 R}}{m_4} + \frac{c_3 e^{m_5 R}}{m_5} - \frac{c_4 e^{-m_6 R}}{m_6} - \frac{Re\ P\ R}{a^2\left(\frac{M^2}{1+m^2}+\frac{1}{\lambda}\right)}\right] \tag{13}$$

$$V = u - \overline{u}$$

$$= c_1 e^{m_3 y} + c_2 e^{-m_4 y} + c_3 e^{m_5 y} + c_4 e^{-m_6 y} - \frac{Re\ P}{a^2\left(\frac{M^2}{1+m^2}+\frac{1}{\lambda}\right)} - \frac{1}{R}\left[\frac{c_1 e^{m_3 R}}{m_3} - \frac{c_2 e^{-m_4 R}}{m_4} + \frac{c_3 e^{m_5 R}}{m_5} - \frac{c_4 e^{-m_6 R}}{m_6} - \frac{Re\ P\ R}{a^2\left(\frac{M^2}{1+m^2}+\frac{1}{\lambda}\right)}\right] \tag{14}$$

Taylor Dispersion:

equation (4) and (8) becomes,

$$\frac{1}{t'}\frac{\partial c}{\partial t} + \frac{u-\overline{u}}{L}\frac{\partial c}{\partial \zeta} = \frac{D}{R_0^2}\frac{\partial^2 c}{\partial y^2} \tag{15}$$

$$\frac{V R_0^2}{D\ L}\frac{\partial c}{\partial \xi} = \frac{\partial^2 c}{\partial y^2}$$

where , $\frac{R_0^2}{D}\frac{\partial c}{L\,\partial\xi} = Q$

$$QV = \frac{\partial^2 c}{\partial y^2} \tag{16}$$

dimensionless in multi-stenosis artery is below,

$$(x) = \begin{cases} 1 & 0 \le x \le d_1 \\ 1 - \frac{\delta_1}{2}\left(1 + Cos\frac{2\pi}{l_1}\left(x - d_1 - \frac{l_1}{2}\right)\right) & d_1 \le x \le d_1 + l_1 \\ 1 & d_1 + l_1 \le x \le d_2 \\ 1 - \frac{\delta_2}{2}\left(1 + Cos\frac{2\pi}{l_2}\left(x - d_2 - \frac{l_2}{2}\right)\right) & d_2 \le x \le d_2 + l_2 \\ 1 & d_2 + l_2 \le x \le d_3 \\ 1 - \frac{\delta_3}{2}\left(1 + Cos\frac{2\pi}{l_3}\left(x - d_3 - \frac{l_3}{2}\right)\right) & d_3 \le x \le d_3 + l_3 \\ 1 & d_3 + l_3 \le x \le l \end{cases}$$

$$c = Q(\left(\frac{c_1 e^{m_3 y}}{m_3^2} + \frac{c_2 e^{-m_4 y}}{m_4^2} + \frac{c_3 e^{m_5 y}}{m_5^2} + \frac{c_4 e^{-m_6 y}}{m_6^2} - \frac{Re\ P\ y^2}{2a^2\left(\frac{M^2}{1+m^2} + \frac{1}{\lambda}\right)}\right)$$

$$-\frac{c_1\, e^{m_3 R}\, y^2}{2m_3 R} + \frac{c_2\, e^{-m_4 R}\, y^2}{2m_4 R} - \frac{c_3\, e^{m_5 R}\, y^2}{2m_5 R} + \frac{c_4\, e^{-m_6 R}\, y^2}{2m_6 R} + \frac{Re\ P\ y^2 R}{2a^2\left(\frac{M^2}{1+m^2} + \frac{1}{\lambda}\right)}) + Ay + B \tag{17}$$

where,

$$A = Q\left[-\frac{c_1 m_3}{m_3^2} + \frac{c_2 m_4}{m_4^2} - \frac{c_3 m_5}{m_5^2} + \frac{c_4 m_6}{m_6^2}\right]$$

$$B = Q(-\frac{c_1 e^{m_3 R}}{m_3^2} - \frac{c_2 e^{-m_4 R}}{m_4^2} - \frac{c_3 e^{m_5 R}}{m_5^2} - \frac{c_4 e^{-m_6 R}}{m_6^2} + \frac{Re\ P\ R^2}{2a^2(\frac{M^2}{1+m^2} + \frac{1}{\lambda})}$$

$$\frac{c_1 e^{m_3 R} R^2}{2m_3 R} - \frac{c_2 e^{-m_4 R} R^2}{2m_4 R} + \frac{c_3 e^{m_5 R} R^2}{2m_5 R} - \frac{c_4 e^{-m_6 R} R^2}{2m_6 R} - \frac{Re\ P\ R\ R^2}{2a^2 R\left(\frac{M^2}{1+m^2} + \frac{1}{\lambda}\right)} +$$

$$\frac{c_1 m_3\, R}{m_3^2} - \frac{c_2 m_4\, R}{m_4^2} + \frac{c_3 m_5\, R}{m_5^2} - \frac{c_3 m_6\, R}{m_6^2})$$

Following Taylor (1953, 1954), we consider C variation and ξ longitudinal $\partial\xi$ direction, C_m is mean concentration over a section ∂C distingishable from $\partial c_m\ \partial\xi$ *so that equation (17) can be written as,*

$$M = \frac{R^3}{D\,L} G \frac{\partial c_m}{\partial\xi} \tag{18}$$

This shows that c_m is disperised relative to a plates which moves with velocity exactly through a difffusive by porous region. Which is molecular diffusion itis

(18) becomes,

$$\frac{\partial M}{\partial\xi} = -\frac{2}{L}\frac{\partial c_m}{\partial\xi} \tag{19}$$

equation (18) and (19) we get

$$-\frac{2}{L}\frac{\partial c_m}{\partial t} = \frac{R^3}{D\,L} G\, \frac{\partial^2 c_m}{\partial\xi^2}$$

$$\frac{\partial c_m}{\partial t} = -\frac{R^3}{2D} G \frac{\partial^2 c_m}{\partial\xi^2}$$

$$D^* = -\frac{R^3}{2D}G \tag{20}$$

Generalized Dispersion

To obtained generalized dispersion an evaluated Gill and Sankarasubramanian(1970), The solution of equation (4) becomes, with boundary conditions,

$$\left.\begin{array}{c} c^*(0,x^*,y^*)=c_0 \; for \; |x|\le x_S, \\ c^*(0,x^*,y^*)=c_0 \; for \; |x|>x_S, \\ \frac{\partial c^*}{\partial y^*}(t^*,x^*,0)=0 \\ \frac{\partial c^*}{\partial y^*}(t^*,x^*,R^*(x))=0 \\ c^*(t^*,\infty,R^*)=\frac{\partial c^*}{\partial x^*}(t^*,\infty,y^*)=0 \end{array}\right\} \tag{21}$$

where c_0 is concentration of the initial slug insert of length and equation (21)represents the initial concentration we obtain equation (4) and (21) dimensionless using

$$\theta = \frac{c^*}{c_0}, u^* = \frac{u}{\bar{u}}, Y = \frac{y}{R}, X = \frac{D_x}{R^2\bar{u}}, p_e = \frac{\bar{u}R}{D}, \tau = \frac{tD}{R^2} \tag{22}$$

using the above equation (22) becomes,

$$\frac{\partial\theta}{\partial\tau} + u^*\frac{\partial\theta}{\partial X} = \frac{1}{p_e^2}\frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial Y^2} \tag{23}$$

Where $u^* = \frac{u}{\bar{u}}$, dimensionless velocity of the fluid. We define axial coordinate moving with the average velocity of flow as $x_1 = x - \bar{u}t$ which dimensionless form is $X_1 = X - \tau$

where $X_1 = \frac{x_1 D}{R^2\bar{u}}$ using equation (23) becomes,

$$\frac{\partial\theta}{\partial\tau} + U\frac{\partial\theta}{\partial X_1} = \frac{1}{p_e^2}\frac{\partial^2\theta}{\partial X_1^2} + \frac{\partial^2\theta}{\partial Y^2}$$

with $U = \frac{u-\bar{u}}{u}$

The dimensionless along with initial and boundary conditions above equation (21) are given by,

$$\left.\begin{array}{c} \theta(0,X_1,Y) = R \; for \; |x| \le X, \\ \theta(0,X_1,Y) = R \; for \; |x| > X, \\ \frac{\partial\theta}{\partial Y}(\tau,X_1,0) = 0 \\ \frac{\partial\theta}{\partial Y}(\tau,X_1,R(x)) = 0 \\ 0(\tau,\infty,Y) = \frac{\partial\theta}{\partial X_1}(\tau,\infty,Y) = 0 \end{array}\right\} \tag{25}$$

solution of equation (24) and boundary conditions (25) can be written as,

$$\theta(\tau,X_1,Y) = \theta_m(\tau,X_1) + f_1(\tau,Y)\frac{\partial\theta_m}{\partial X_1}(\tau,Y) + f_2(\tau,Y)\frac{\partial^2\theta_m}{\partial X_1^2}(\tau,X_1) + \cdots$$

$$\theta = \theta_m + \sum_{k=1}^{\infty} f_k(\tau,Y)\frac{\partial^k\theta_m}{\partial X_1^k} \tag{26}$$

where average concentration is denoted as θm (cross section) and we get,

$$\theta_m(\tau,X_1) = \int_0^R \theta(\tau,X_1,Y)dY \tag{27}$$

Integrating equation (24) limits in [0, R] using this equation (27) we get,

(28)

$$\frac{\partial\theta_m}{\partial\tau} = \frac{1}{p_e^2}\frac{\partial^2\theta}{\partial X_1^2} + \int_0^R \frac{\partial^2\theta}{\partial Y^2} dY - \frac{\partial}{\partial X_1}\int_0^R U\,\theta\,dY$$

using the equation (26) in (27) we get,

$$\frac{\partial\theta_m}{\partial\tau} = \frac{1}{p_e^2}\frac{\partial^2\theta_m}{\partial X_1^2} - \frac{\partial}{\partial X_1}\int_0^R U(\theta_m(\tau,X_1) + f_1(\tau,Y)\frac{\partial\theta_m}{\partial X_1}(\tau,X_1) +$$

$$f_2(\tau,Y)\frac{\partial^2\theta_m}{\partial X_1^2}(\tau,X_1) + \cdots)d\tau \qquad (29)$$

Gill and Sankarasubramanian developed generalized dispersion representationwith time dependent coefficient of equation follows,

$$\frac{\partial\theta_m}{\partial\tau} = K_1\frac{\partial\theta_m}{\partial X_1} + K_2\frac{\partial^2\theta_m}{\partial X_1^2} + K_3\frac{\partial^3\theta_m}{\partial X_1^3} + \cdots \qquad (30)$$

equation (30) and (29) we get,

$$K_1\frac{\partial\theta_m}{\partial X_1} + K_2\frac{\partial^2\theta_m}{\partial X_1^2} + K_3\frac{\partial^3\theta_m}{\partial X_1^3} + \cdots =$$

$$\frac{1}{p_e^2}\frac{\partial^2\theta_m}{\partial X_1^2} - \frac{\partial}{\partial X_1}\int_0^R U(\theta_m(\tau,X_1) + f_1(\tau,Y)\frac{\partial\theta_m}{\partial X_1}(\tau,X_1) +$$

$$f_2(\tau,Y)\frac{\partial^2\theta_m}{\partial X_1^2}(\tau,X_1) + \cdots)d\tau \qquad (31)$$

Equating the coefficient $\frac{\partial\theta_m}{\partial X_1}, \frac{\partial^2\theta_m}{\partial X_1^2}, \ldots$ we get

$$K_1 = -\int_0^R U\,dY$$

$$K_2 = \frac{1}{p_e^2} - \int_0^R U\,f_1(\tau,Y)dY$$

$$K_3 = -\int_0^R U\,f_2(\tau,Y)dY$$

$$K_i(\tau) = \frac{\delta_{ij}}{p_e^2} - \int_0^R f_{i-1}(\tau,Y)dY \; (i = 1,2,\ldots \;\; j = 2) \qquad (32)$$

where δ_{ij} identified is kroneaker delta is,

$$\delta_{ij} = \begin{cases} 1, & if \quad i = 1 \\ 0, & if \quad i \neq j \end{cases}$$

equation (26) and (24) we get,

$$\frac{\partial}{\partial\tau}(\theta_m(\tau,X_1) + f_1(\tau,Y)\frac{\partial\theta_m}{\partial X_1}(\tau,X_1) + f_2(\tau,Y)\frac{\partial^2\theta_m}{\partial X_1^2}(\tau,X_1) + \cdots)$$

$$+U\frac{\partial}{\partial X_1}\left(\theta_m(\tau,X_1) + f_1(\tau,Y)\frac{\partial\theta_m}{\partial X_1}(\tau,X_1) + f_2(\tau,Y)\frac{\partial^2\theta_m}{\partial X_1^2}(\tau,X_1) + \cdots\right)$$

$$= \frac{1}{p_e^2}\frac{\partial^2}{\partial X_1^2}\left(\theta_m(\tau,X_1) + f_1(\tau,Y)\frac{\partial\theta_m}{\partial X_1}(\tau,X_1) + f_2(\tau,Y)\frac{\partial^2\theta_m}{\partial X_1^2}(\tau,X_1) + \cdots\right)$$

$$+\frac{\partial^2}{\partial Y^2}\left(\theta_m(\tau,X_1)+f_1(\tau,Y)\frac{\partial\theta_m}{\partial X_1}(\tau,X_1)+f_2(\tau,Y)\frac{\partial^2\theta_m}{\partial X_1^2}(\tau,X_1)+\cdots\right) \tag{33}$$

Following Gill and Sankarasubramanian (1970) using the equation (30) and(33)

$$\frac{\partial^{K+1}\theta_m}{\partial\tau\,\partial X_1^K}=\sum_{i=1}^{K}k_i(\tau)\frac{\partial^{K+1}\theta_m}{\partial X_1^{K+1}} \tag{34}$$

1

we have,

$$\left[\frac{\partial f_1}{\partial\tau}-\frac{\partial^2 f_1}{\partial Y^2}+U+K_1(\tau)\right]\frac{\partial\theta_m}{\partial X_1}$$

$$+[\frac{\partial f_2}{\partial\tau}-\frac{\partial^2 f_2}{\partial Y^2}+Uf_1+K_1(\tau)+K_2(\tau)-\frac{1}{p_e^2}]\frac{\partial^2\theta_m}{\partial X_1^2}$$

$$+\sum_{i=1}^{\infty}(\left[\frac{\partial f_{k+2}}{\partial\tau}-\frac{\partial^2 f_{k+2}}{\partial Y^2}+Uf_{k+1}+K_1(\tau)f_{k+1}+K_1(\tau)f_{k+1}+\left(K_2(\tau)-\frac{1}{p_e^2}\right)f_k\right]$$

$$+\sum_{i=1}^{k+2}K_i\,f_{k+2-i})\frac{\partial^{k+2}\theta_m}{\partial X_1^{k+2}}=0 \tag{35}$$

with f_0= 1 equating the coefficient of $\frac{\partial^k\theta_m}{\partial X_1^k}$ (35) to zero, we obtain following partial differential equation

$$\frac{\partial f_1}{\partial\tau}=\frac{\partial^2 f_1}{\partial Y^2}-U-K_1(\tau) \tag{36}$$

$$\frac{\partial f_2}{\partial\tau}=\frac{\partial^2 f_2}{\partial Y^2}-U\,f_1-K_1(\tau)f_1-K_2(\tau)+\frac{1}{p_e^2} \tag{37}$$

$$\frac{\partial f_{k+2}}{\partial\tau}=\frac{\partial^2 f_k}{\partial Y}-U\,f_{k+1}-K_1(\tau)f_{k+1}-\left(K_2(\tau)-\frac{1}{p_e^2}\right)f_k-\sum_{i=1}^{k+2}K_i\,f_{k+1-i} \tag{38}$$

using the boundary conditions,

$$\left.\begin{aligned} f_k(0,Y)&=0\\ \frac{\partial f_k}{\partial Y}(\tau,0)&=0\\ \frac{\partial f_k}{\partial Y}(\tau,R)&=0\\ \int_0^R f_k\,(\tau,Y)dY&=0\end{aligned}\right\} \tag{39}$$

for k=1,2,3,... from equation (32) for $i=1$ using $f_0=1$

$$K_1(\tau)=0 \tag{40}$$

From equation (32) for $i=2$,

$$K_2(\tau)=\frac{1}{p_e^2}-\int_0^R U\,f_1\,dY \tag{41}$$

To evaluate $K_2(\tau)$

Put

$$f_1=f_{10}(Y)+f_{11}(\tau,Y) \tag{42}$$

where $f_{10}(Y)$ is corresponds to an infinitely wide slug which is independentof τ and f_{11} is τ dependent satisfied with boundary conditions

$$\frac{df_{10}}{dY} = 0 \ \ at \ \ Y = 0 \tag{43}$$

$$f_{10} = 0 \quad at \quad Y = R \tag{44}$$

$$\int_0^R f_{10}\, dY = 0 \tag{45}$$

using the (42) in (12) implies that

$$\frac{d^2 f_{10}}{dY^2} = \frac{1}{\frac{c_1 e^{m_3 R}}{m_3 R} - \frac{c_2 e^{-m_4 R}}{m_4 R} + \frac{c_3 e^{m_5 R}}{m_5 R} - \frac{c_4 e^{-m_6 R}}{m_6 R} - \frac{Re\ P\ R}{Ra^2\left(\frac{M^2}{1+m^2} + \frac{1}{\lambda}\right)}}$$

$$(c_1 e^{m_3 Y} + c_2 e^{-m_4 Y} + c_3 e^{m_5 Y} + c_4 e^{-m_6 Y} - \frac{Re\ P}{a^2\left(\frac{M^2}{1+m^2} + \frac{1}{\lambda}\right)}$$

$$-\frac{c_1 e^{m_3 R}}{m_3 R} + \frac{c_2 e^{-m_4 R}}{m_4 R} - \frac{c_3 e^{m_5 R}}{m_5 R} + \frac{c_4 e^{-m_6 R}}{m_6 R} + \frac{Re\ P\ R}{Ra^2\left(\frac{M^2}{1+m^2} + \frac{1}{\lambda}\right)}$$

$$f_{10} = \frac{1}{\bar{u}}\left(\frac{c_1 e^{m_3 Y}}{m_3^2} + \frac{c_2 e^{-m_4 Y}}{m_4^2} + \frac{c_3 e^{m_5 Y}}{m_5^2} + \frac{c_4 e^{-m_6 Y}}{m_6^2} - \frac{Re\ P\ Y^2}{2a^2\left(\frac{M^2}{1+m^2} + \frac{1}{\lambda}\right)}\right.$$

$$-\frac{c_1 e^{m_3 R} Y^2}{2m_3 R} + \frac{c_2 e^{-m_4 R} Y^2}{2m_4 R} - \frac{c_3 e^{m_5 R} Y^2}{2m_5 R} + \frac{c_4 e^{-m_6 R} Y^2}{2m_6 R} + \frac{Re\ P\ Y^2 R}{2Ra^2\left(\frac{M^2}{1+m^2} + \frac{1}{\lambda}\right)} + A_1 Y + B_1$$

$$\frac{\partial f_{11}}{\partial \tau} = \frac{\partial^2 f_{11}}{\partial Y^2} \tag{46}$$

The above equation is heat conduction equation is solving separation ofvariables steady state condition time increases,

$$f_{11}(Y, \tau) = e^{-\lambda^2(\tau)}(A\, cos[\lambda\, Y]) + B\, sin[\lambda\, Y] \tag{47}$$

$$f_{11} = \sum_{n=1}^{\infty} A_n\, e^{-\lambda_n^2}\, cos[\lambda_n\, Y] \qquad 2$$

where,

$$A_n = -\int_0^R f_{10}\,(Y)\, cos[\lambda_n Y] dY \tag{48}$$

$$A_n = \left(-\frac{c_1 e^{m_3 R}}{m_3^3} + \frac{c_2 e^{-m_4 R}}{m_4^3} - \frac{c_3 e^{m_5 R}}{m_5^3} + \frac{c_4 e^{-m_6 R}}{m_6^3} + \frac{Re\ P\ R^3}{6a^2\left(\frac{M^2}{1+m^2} + \frac{1}{\lambda}\right)}\right.$$

$$+\frac{c_1 e^{m_3 R} R^3}{6m_3 R} - \frac{c_2 e^{-m_4 R} R^3}{6m_4 R} + \frac{c_3 e^{m_5 R} R^3}{6m_5 R} - \frac{c_4 e^{m_6 R} R^3}{6m_6 R} - \frac{Re\ P\ R^3}{6a^2\left(\frac{M^2}{1+m^2} + \frac{1}{\lambda}\right)}$$

$$+\frac{c_1 m_3 R^2}{2m_3^2} - \frac{c_2 m_4 R^2}{2m_4^2} + \frac{c_3 m_5 R^2}{2m_5^2} - \frac{c_4 m_6 R^2}{2m_6^2}$$

$$\left(+\frac{c_1 e^{m_3 R} R}{m_3^2} + \frac{c_2 e^{-m_4 R} R}{m_4^2} + \frac{c_3 e^{m_5 R} R}{m_5^2} + \frac{c_4 e^{-m_6 R} R}{m_6^2} - \frac{Re\ P\ R^2 R}{2a^2\left(\frac{M^2}{1+m^2} + \frac{1}{\lambda}\right)}\right.$$

$$-\frac{c_1 e^{m_3 R} R^2 R}{2m_3 R} - \frac{c_2 e^{-m_4 R} R^2 R}{2m_4 R} + \frac{c_3 e^{m_5 R} R^2 R}{2m_5 R} - \frac{c_4 e^{m_6 R} R^2 R}{2m_6 R} - \frac{Re\ P\ R^2 RR}{2a^2\left(\frac{M^2}{1+m^2} + \frac{1}{\lambda}\right)}$$

$$-\frac{c_1 m_3 RR}{2m_3^2}+\frac{c_2 m_4 RR}{2m_4^2}-\frac{c_3 m_5 RR}{2m_5^2}+\frac{c_4 m_6 RR}{2m_6^2}\Bigg) sin\lambda_n R$$

substituting (46) and (48) in equation (42) we get,

$$f_1=\frac{1}{\bar{u}}\Bigg(\frac{c_1 e^{m_3 Y}}{m_3^2}+\frac{c_2 e^{-m_4 Y}}{m_4^2}+\frac{c_3 e^{m_5 Y}}{m_5^2}+\frac{c_4 e^{-m_6 Y}}{m_6^2}-\frac{Re\,P\,Y^2}{2a^2\left(\frac{M^2}{1+m^2}+\frac{1}{\lambda}\right)}$$

$$-\frac{c_1 e^{m_3 R} Y^2}{2m_3 R}+\frac{c_2 e^{-m_4 R} Y^2}{2m_4 R}-\frac{c_3 e^{m_5 R} Y^2}{2m_5 R}+\frac{c_4 e^{-m_6 R} Y^2}{2m_6 R}+\frac{Re\,P\,Y^2 R}{2Ra^2\left(\frac{M^2}{1+m^2}+\frac{1}{\lambda}\right)}+A_1 Y+B_1\Bigg)$$

$$+\sum_{n=1}^{3} e^{\lambda_n^2\tau} cos[Y\lambda_n]\Bigg(-\frac{c_1}{2m_3^2(m_3-i\lambda_n)}+\frac{c_1 e^{m_3 R} cos[R\lambda_n]}{2m_3^2(m_3-i\lambda_n)}-\frac{ic_1 e^{m_3 R} sin[R\lambda_n]}{2m_3^2(m_3-i\lambda_n)}$$

$$-\frac{c_1}{2m_4^2(m_4-i\lambda_n)}+\frac{c_2 e^{m_4 R} cos[R\lambda_n]}{2m_4^2(m_4-i\lambda_n)}-\frac{ic_2 e^{m_4 R} sin[R\lambda_n]}{2m_4^2(m_4-i\lambda_n)}-\frac{c_3}{2m_5^2(m_5-i\lambda_n)}$$

$$+\frac{c_3 e^{m_5 R} cos[R\lambda_n]}{2m_5^2(m_5-i\lambda_n)}-\frac{ic_3 e^{m_5 R} sin[R\lambda_n]}{2m_5^2(m_5-i\lambda_n)}-\frac{c_4}{2m_6^2(m_6-i\lambda_n)}+\frac{c_4 e^{m_6 R} cos[R\lambda_n]}{2m_6^2(m_6-i\lambda_n)}$$

$$-\frac{ic_4 e^{m_6 R} sin[R\lambda_n]}{2m_6^2(m_6-i\lambda_n)}-\frac{c_1}{2m_3^2(m_3+i\lambda_n)}+\frac{c_1 e^{m_3 R} cos[R\lambda_n]}{2m_3^2(m_3+i\lambda_n)}-\frac{ic_1 e^{m_3 R} sin[R\lambda_n]}{2m_3^2(m_3+i\lambda_n)}$$

$$-\frac{c_2}{2m_4^2(m_4+i\lambda_n)}+\frac{c_2 e^{m_4 R} cos[R\lambda_n]}{2m_4^2(m_4+i\lambda_n)}+\frac{ic_2 e^{m_4 R} sin[R\lambda_n]}{2m_4^2(m_4+i\lambda_n)}-\frac{c_3}{2m_5^2(m_5+i\lambda_n)}$$

$$+\frac{c_3 e^{m_5 R} cos[R\lambda_n]}{2m_5^2(m_5+i\lambda_n)}-\frac{ic_3 e^{m_5 R} sin[R\lambda_n]}{2m_5^2(m_5+i\lambda_n)}-\frac{c_4}{2m_6^2(m_6+i\lambda_n)}+\frac{c_4 e^{m_6 R} cos[R\lambda_n]}{2m_6^2(m_6+i\lambda_n)}$$

$$\frac{ic_4 e^{m_6 R} sin[R\lambda_n]}{2m_6^2(m_6+i\lambda_n)}+\frac{c_1 e^{m_3 R} sin[R\lambda_n]}{m_3 R\lambda_n^3}-\frac{c_2 e^{-m_4 R} sin[R\lambda_n]}{m_4 R\lambda_n^3}+\frac{c_3 e^{m_5 R} sin[R\lambda_n]}{m_5 R\lambda_n^3}$$

$$-\frac{c_4 e^{-m_6 R} sin[R\lambda_n]}{m_6 R\lambda_n^3}-\frac{c_1 e^{m_3 R} cos[R\lambda_n]}{m_3\lambda_n^3}+\frac{c_2 e^{-m_4 R} cos[R\lambda_n]}{m_4\lambda_n^2}-\frac{c_3 e^{-m_5 R} cos[R\lambda_n]}{m_5\lambda_n^2}$$

$$\frac{c_4 e^{-m_6 R} cos[R\lambda_n]}{m_6\lambda_n^2}+\frac{A\,Y\,sin[R\lambda_n]}{\lambda_n}-\frac{c_1 e^{m_3 R} sin[R\lambda_n]}{m_3^2\lambda_n}-\frac{c_1\,sin[R\lambda_n]}{m_4^2\lambda_n}$$

$$\frac{c_2\,sin[R\lambda_n]}{m_4\lambda_n}-\frac{c_3 e^{m_5 R} sin[R\lambda_n]}{m_5^2\lambda_n}-\frac{c_3\,sin[R\lambda_n]}{m_5^2\lambda_n}-\frac{c_4\,sin[R\lambda_n]}{m_6\lambda_n}\Bigg)$$

substituting f_1 into equation (41) and integrating, results in dispersion coefficient solution using more help to mathematica software and A_1 and B_1 are constant values are Appendix. Similarly $K_3(\tau)$, $K_4(\tau)$ and so on are obtained and we found that $K_i(\tau)$, $i > 2$ are negulating terms. Then small comparing value $K_2(\tau)$. The dispersion model from (41) now we obtain,

$$\frac{\partial\theta_m}{\partial\tau}=K_2\frac{\partial^2\theta_m}{\partial\xi^2} \quad (49)$$

$\lambda_n = n\pi$

The analytical solution of(49) satisfying the condition (25) and examinedFourier Transform (Sankara (1995)) given by,

$$\theta_m(\tau,\xi)=\frac{1}{2}\left[erf\left(\frac{\frac{\xi_s}{2}+\xi}{2\sqrt{T}}\right)+erf\left(\frac{\frac{\xi_s}{2}-\xi}{2\sqrt{T}}\right)\right]$$

$$where\ T = \int_0^T K_2(Y)\,dY \ \ and\ \ erf(x) = \frac{2}{\sqrt{T}}\int_0^x e^{-z^2}\,dz$$

RESULTS AND DISCUSSION

The results of the present study is development dispersion analysis for several various parameter values Hartmann number(M), Reynolds number (Re), Couple stress parameter (a) and Hall parameter (m). It's observed in physical problem velocity field, mean concentration, and dispersion diffusive are dispute by apply numerical values of varied in $l_1 = l_2 = l_3 = 0.2$, $w = 1$, $t = 1$,height of the three stenosis is equal to distance of the stenosis when the blockage ofthe vessels are 10 , 30 , and 20 (δ_1 =0.1, δ_2 =0.3 and δ_3 =0.2) R_e =0.2,0.6,0.8, $M = 0, 1, 2, 3$, $m = 0, 1$, $a = 5, 10, 15$, $\lambda = 0.2, 0.4, 0.6$ The above values areconsider same in the entire investigation exclude for different values as showed in Figure 2 to 13.

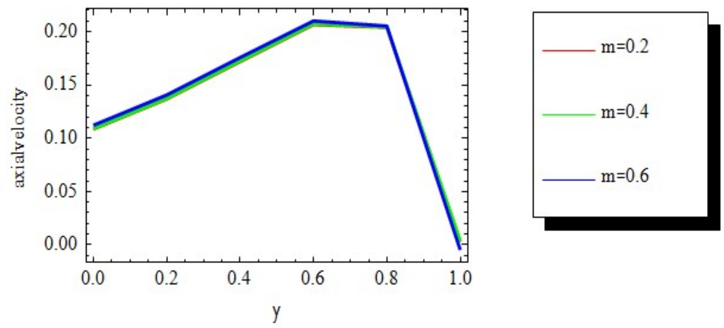

Figure: 2 Plots of velocity of axial field for several values "m".

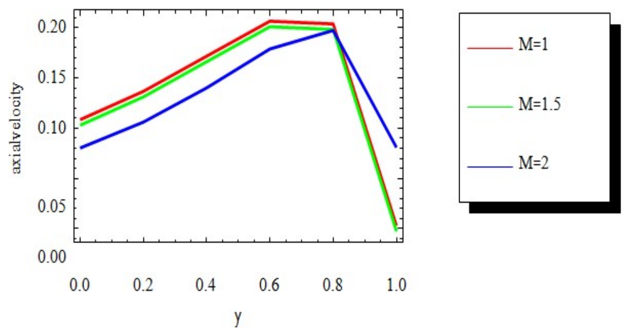

Figure: 3 Plots of velocity of axial field for several values "M".

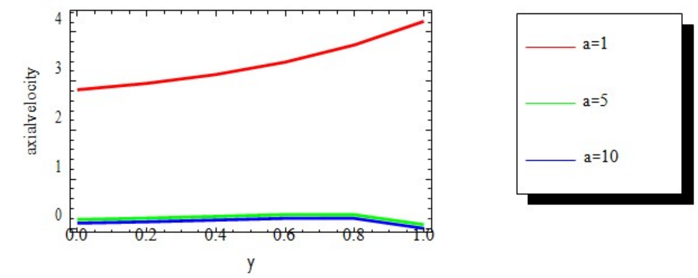

Figure: 5 Plots of velocity of axial field for several values "a".

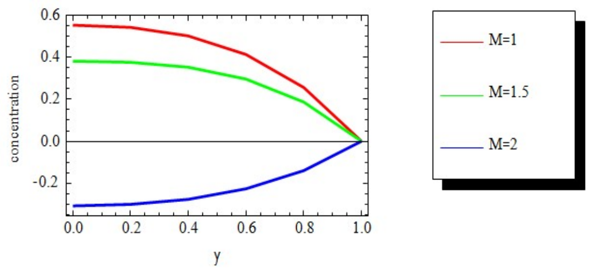

Figure: 6 Plots of concentration of axial field for several values "M"

Figure 2 show velocity field decreases with increasing Hall parameter. Figure 3 depits that the velocity field decreases with magnetic parameter is increases.figure 4 that velocity field of the increasing in blood with decreasing the Reynolds number. The couple stress parameter increasing with increasing Figure 5. The variation of the species of the several values of couple stress parameter, Reynolds number, Hall parameter and magnetic parameter. Shows that the figure 6 increases with increasing magnetic parameter.

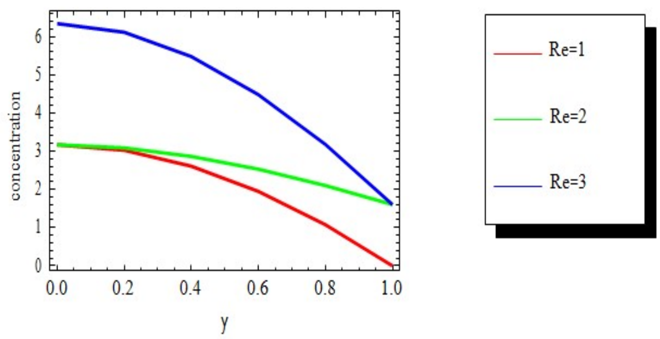

Figure: 7 Plots of concentration of axial field for several values "Re"

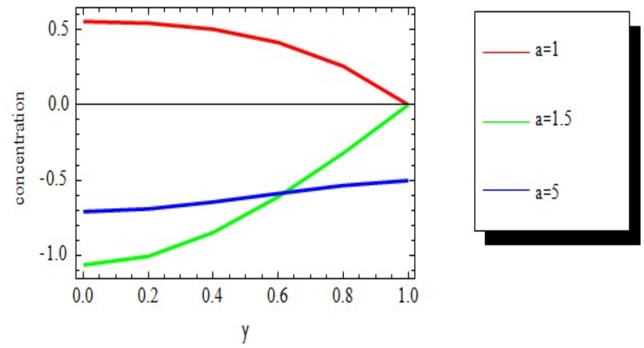

Figure: 8 Plots of concentration of axial field for several values "a"

Figure 7 and9 is concentration is decreases with Hall parameter and Reynolds numberis increasing. Figure 8 display in couple stress parameter increases withconcentration increasing. The mean concentration θ_m with ξ for various values of a , Re , M , m . Figure 10 and Figure 11 θ_m mean concentration increases with increasingRe , m . Shows that figure 12 and 13 variation of mean concentration θ_m increases with decreases in a , M . Figure 14, 15 dispersion of increased while decreases in Re and m .

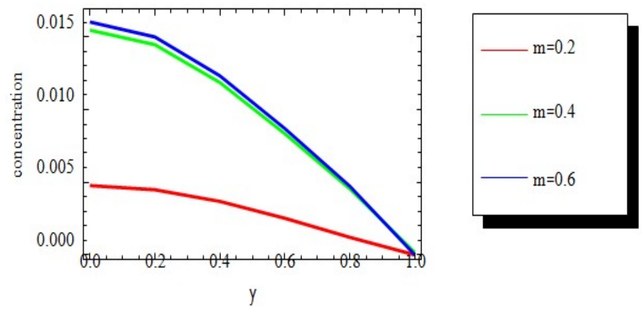

Figure: 9 Plots of concentration of axial field for several values "m"

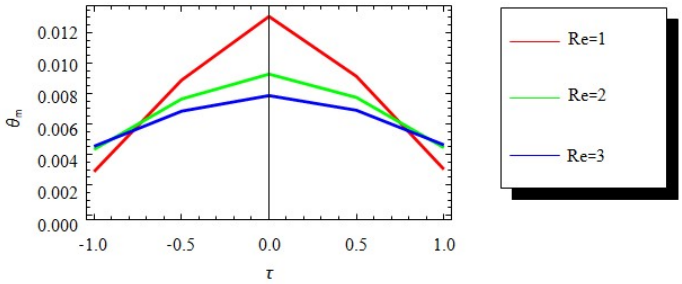

Figure: 10 Plots of mean concentration of axial field for several values "Re"

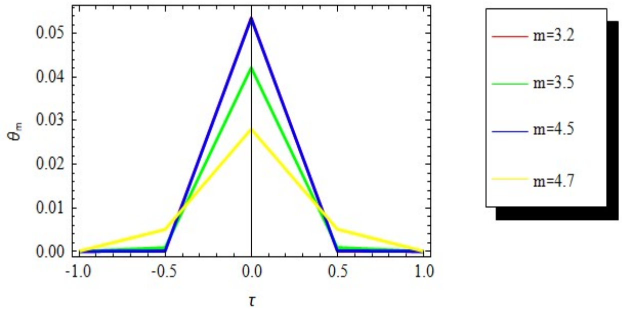

Figure 11 Plots of mean concentration of axial field for several values "m"

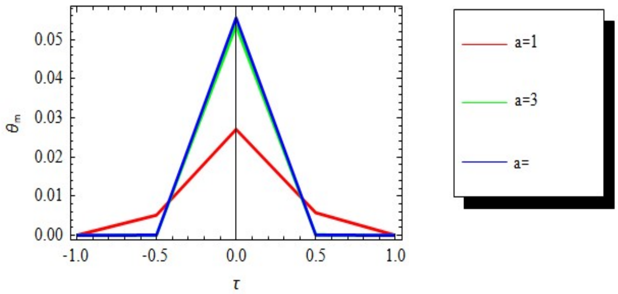

Figure 12 Plots of mean concentration of axial field for several values "a"

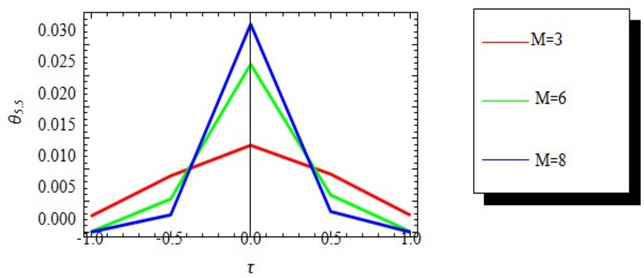

Figure 13 Plots of mean concentration of axial field for several values "M"

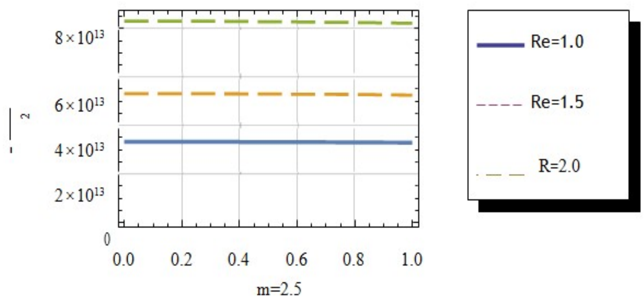

Figure 14 Plots of dispersion of Reynolds number for several values "Re"

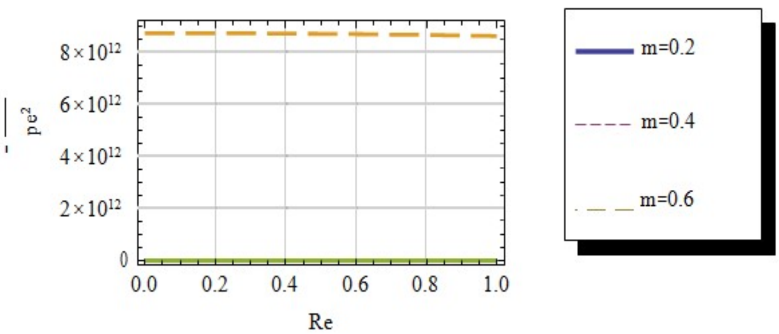

Figure 15. Plots of dispersion of Reynolds number for several values "m"

Conclusion

Dispersion process in flow of blood an couple stress fluid in the porous channel, impact of Hall current and external magnetic field is developed using the generalized dispersion model. Unsteady in flow of blood is nature, it's solute of dispersion in multi-stenosed artery.

In this study currently limited to the exact solution. The results are more useful to medical industrial and high pressure control. Flow of blood two method in different works.

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Appendix:

$$c_1 = -((1+m^2)(-m_4^2m_5^2 + e^{(m_6R)}m_4^2m_5^2 + e^{(m_4R+m_5R)}m_4^2m_5^2 - e^{(m_4R+m_5R+m_6R)}m_4^2m_5^2 + e^{(m_4R)}m_4^2m_6^2 - e^{(m_6R)}m_4^2m_6^2 - e^{(m_4R+m_5R)}m_4^2m_6^2 + e^{(m_5R+m_6R)}m_4^2m_6^2 + m_5^2m_6^2 - e^{(m_4R)}m_5^2m_6^2 - e^{(m_5R+m_6R)}m_5^2m_6^2 + e^{(m_4R+m_5R+m_6R)}\, m_5^2m_6^2)pq\lambda/(a^2(m_3^2m_4^2 - e^{(m_3R+m_4R)}m_3^2m_4^2 - e^{(m_5R+m_6R)}m_3^2m_4^2 + e^{(m_3R+m_4R+m_5R+m_6R)}m_3^2m_4^2 + e^{(m_3R+m_4R)}m_3^2m_5^2 - e^{(m_4R+m_5R)}m_3^2m_5^2 - e^{(m_3R+m_6R)}m_3^2m_5^2 + e^{(m_5R+m_6R)}m_3^2m_5^2 -$$

$$m_4^2m_5^2 + e^{(m_4R+m_5R)}m_4^2m_5^2 + e^{(m_3R+m_6R)}m_4^2m_5^2 - e^{(m_3R+m_4R+m_5R+m_6R)}m_4^2m_5^2 - m_3^2m_6^2 + e^{(m_4R+m_5R)}m_3^2m_6^2 + e^{(m_3R+m_6R)m_3^2m_6^2 - e^{(m_3R+m_6R+m_5R+m_6R)}}m_3^2m_6^2 + e^{(m_3R+m_4R)}m_4^2m_6^2 - e^{(m_4R+m_5R)}m_4^2m_6^2 - e^{(m_3R+m_6R)}m_4^2m_6^2 + e^{(m_5R+m_6R)}m_4^2m_6^2 + m_5^2m_6^2 - e^{(m_3R+m_4R)}m_5^2m_6^2 - e^{(m_5R+m_6R)}m_5^2m_6^2 + e^{(m_3R+m_4R+m_6R)}m_5^2m_6^2)(1+m^2+M^2\lambda));$$

$$c_2 = (e^{(m_4R)}(1+m^2)(e^{(m_3R)}m_3^2m_5^2 - e^{(m_5R)}m_3^2m_5^2 - e^{(m_3R+m_6R)}m_3^2m_5^2 + e^{(m_5R+m_6R)} m_3^2m_5^2 - m_3^2m_6^2 + e^{(m_5R)}m_3^2m_6^2 + e^{(m_3R+m_6R)}m_3^2m_6^2 - e^{(m_3R+m_5R+m_6R)}m_3^2m_6^2 + m_5^2m_6^2 - e^{(m_3R)}m_5^2m_6^2 - e^{(m_5R+m_6R)}m_5^2m_6^2 + e^{(m_3R+m_4R+m_6\,R)}m_5^2m_6^2)pq\lambda)/(a^2(-m_3^2m_4^2 + e^{(m_3R+m_4R)}m_3^2m_4^2 + e^{(m_5R+m_6R)}m_3^2m_4^2 - e^{(m_3R+m_4R+m_5R+m_6R)}m_3^2m_4^2 - e^{(m_3R+m_4R)} m_3^2m_5^2 + e^{(m_4R+m_5R)}m_3^2m_5^2 + e^{(m_3R+m_6R)}m_3^2m_5^2 - e^{(m_5R+m_6R)}m_3^2m_5^2 + m_4^2m_5^2 - -e^{(m_4R+m_5R)}m_4^2m_5^2 - e^{(m_3R+m_6R)}m_4^2m_5^2 + e^{(m_3R+m_4R+m_5R+m_6R)}m_3^2m_6^2 - e^{(m_3R+m_4R)}m_4^2m_6^2 + e^{(m_4R+m_5R)}m_4^2m_6^2 + e^{(m_3R+m_6R)}m_4^2m_6^2 - e^{(m_5R+m_6R)}m_4^2m_6^2 -$$

$$m_5^2 m_6^2 + e^{(m_3R+m_4R)} m_5^2 m_6^2 + e^{(m_5R+m_6R)} m_5^2 m_6^2 - e^{(m_3R+m_4R+m_5R+m_6R)} m_5^2 m_6^2) \\ 1 + m^2 + M^2\lambda));$$

$$c_3 = ((1+m^2)(m_3^2 m_4^2 - e^{(m_6R)} m_3^2 m_4^2 - e^{(m_3R+m_4R)} m_3^2 m_4^2 + e^{(m_3R+m_4R+m_6R)} m_3^2 m_4^2 \\ -m_3^2 m_6^2 + e^{(m_4R)} m_3^2 m_6^2 + e^{(m_3R+m_6R)} m_3^2 m_6^2 - e^{(m_3R+m_4R+m_6R)} m_3^2 m_6^2 - e^{(m_4R)} m_4^2 m_6^2 + e^{(m_3R+m_4R)} m_4^2 m_6^2 \\ - e^{(m_3R+m_6R)} m_4^2 m_6^2) pq\lambda / (a^2(-m_3^2 m_4^2 + e^{(m_3R+m_4R)} m_3^2 m_4^2 + e^{(m_5R+m_6R)} m_3^2 m_4^2 \\ - e^{(m_3R+m_4R+m_5R+m_6R)} m_3^2 m_4^2 \\ -e^{(m_3R+m_4R)} m_3^2 m_5^2 + e^{(m_4R+m_5R)} m_3^2 m_5^2 + e^{(m_3R+m_6R)} m_3^2 m_5^2 - e^{(m_5R+m_6R)} m_3^2 m_5^2 + \\ m_4^2 m_5^2 - e^{(m_4R+m_5R)} m_4^2 m_5^2 - e^{(m_3R+m_6R)} m_4^2 m_5^2 + e^{(m_3R+m_4R+m_5R+m_6R)} m_4^2 m_5^2 + \\ m_3^2 m_6^2 - e^{(m_4R+m_5R)} m_3^2 m_6^2 - e^{(m_3R+m_6R)} m_3^2 m_6^2 + e^{(m_3R+m_4R+m_5R+m_6R)} m_3^2 m_6^2 - \\ e^{(m_3R+m_4R)} m_4^2 m_6^2 + e^{(m_4R+m_5R)} m_4^2 m_6^2 + e^{(m_3R+m_6R)} m_4^2 m_6^2 - e^{(m_5R+m_6R)} m_4^2 m_6^2 - \\ m_5^2 m_6^2 + e^{(m_3R+m_4R)} m_5^2 m_6^2 + e^{(m_5R+m_6R)} m_5^2 m_6^2 - e^{(m_3R+m_4R+m_5R+m_6R)} m_5^2 m_6^2) \\ (1 + m^2 + M^2\lambda));$$

$$c_4 = (e^{(m_6R)}(1+m^2)(-m_3^2 m_4^2 + e^{(m_5R)} m_3^2 m_4^2 + e^{(m_3R+m_4R)} m_3^2 m_4^2 - e^{(m_3R+m_4R+m_5R)} m_3^2 m_4^2 + e^{(m_3R)} m_3^2 m_5^2 - e^{(m_5R)} m_3^2 m_5^2 \\ - \\ e^{(m_3R+m_4R)} m_3^2 m_5^2 + e^{(m_4R+m_5R)} m_3^2 m_5^2 + m_4^2 m_5^2 - e^{(m_3R)} m_4^2 m_5^2 - e^{(m_4R+m_5R)} m_4^2 m_5^2 + \\ e^{(m_3R+m_4R+m_5R)} m_4^2 m_5^2) pq\lambda) / (a^2(m_3^2 m_4^2 - e^{(m_3R+m_4R)} m_3^2 m_4^2 - e^{(m_3R+m_6R)} m_3^2 m_4^2 + \\ e^{(m_3R+m_4R+m_5R+m_6R)} m_3^2 m_4^2 + e^{(m_3R+m_4R)} m_3^2 m_5^2 - e^{(m_4R+m_5R)} m_3^2 m_5^2 - e^{(m_3R+m_6R)} \\ m_3^2 m_5^2 + e^{(m_5R+m_6R)} m_3^2 m_5^2 - m_4^2 m_5^2 + e^{(m_4R+m_5R)} m_4^2 m_5^2 + e^{(m_3R+m_6R)} m_4^2 m_5^2 - e^{(m_3R+m_6R)} m_4^2 m_5^2 \\ - e^{(m_3R+m_4R+m_5R+m_6R)} m_4^2 m_5^2 - m_3^2 m_6^2 \\ + e^{(m_3R+m_4R)} m_4^2 m_6^2 - e^{(m_4R+m_5R)} m_4^2 m_6^2 - e^{(m_3R+m_6R)} m_4^2 m_6^2 + e^{(m_5R+m_6R)} m_4^2 m_6^2 + \\ m_5^2 m_6^2 - e^{(m_3R+m_4R)} m_5^2 m_6^2 - e^{(m_5R+m_6R)} m_5^2 m_6^2 + e^{(m_3R+m_4R+m_5R+m_6R)} m_5^2 m_6^2) \\ (1 + m^2 + M^2\lambda));$$

$$m_3 = +\frac{\sqrt{a^2 + \sqrt{a^4 - 4a^2\left(\frac{M^2}{1+m^2} + \frac{1}{\lambda}\right)}}}{2};$$

$$m_4 = -\frac{\sqrt{a^2 + \sqrt{a^4 - 4a^2\left(\frac{M^2}{1+m^2} + \frac{1}{\lambda}\right)}}}{2};$$

$$m_5 = +\frac{\sqrt{a^2 - \sqrt{a^4 - 4a^2\left(\frac{M^2}{1+m^2} + \frac{1}{\lambda}\right)}}}{2};$$

$$m_6 = -\frac{\sqrt{a^2 - \sqrt{a^4 - 4a^2\left(\frac{M^2}{1+m^2} + \frac{1}{\lambda}\right)}}}{2};$$

$$A = -\frac{c_1 m_3}{m_3^2} + \frac{c_2 m_4}{m_4^2} - \frac{c_3 m_5}{m_5^2} + \frac{c_4 m_6}{m_6^2};$$

$$t_1 = \frac{c_1 e^{(m_3R)}}{m_3} - \frac{c_2 e^{(-m_4R)}}{m_4} + \frac{c_3 e^{(m_5R)}}{m_5} - \frac{c_4 e^{(-m_6R)}}{m_6} + \frac{Re\ P\ R}{a^2\left(\frac{M^2}{1+m^2} + \frac{1}{\lambda}\right)};$$

$$B = \left(-\frac{c_1 e^{(m_3R)}}{m_3^2} + \frac{c_2 e^{(-m_4R)}}{m_4^2} + \frac{c_3 e^{(m_5R)}}{m_5^2} + \frac{c_4 e^{(-m_6R)}}{m_6^2} + \frac{Re\ P\ R^2}{a^2\left(\frac{M^2}{1+m^2} + \frac{1}{\lambda}\right)} - \frac{R^2}{2R} t_1\right)$$

$$+\frac{c_1 m_3 R}{m_3^2} - \frac{c_2 m_4 R}{m_4^2} + \frac{c_3 m_5 R}{m_5^2} - \frac{c_4 m_6 R}{m_6^2};$$

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