## RESEARCH ARTICLE

# MINIMUM RISKS THREE STAGE CHAIN SAMPLING PLANS CHSP $(0,1,2)$ WITH REPETITIVE DEFERRED SAMPLING PLAN INDEXED BY ACCEPTABLE QUALITY LEVEL AND LIMITING QUALITY LEVEL 

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#### Abstract

In this paper, a table and procedure are given for finding the three stage Chain Sampling plan ChSP $(0,1,2)$ with repetitive deferred sampling plan involving minimum sum of producer's and consumer's risks for specified Acceptable Quality Level and Limiting Quality Level.


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## INTRODUCTION

Quality control emerges as an important topic during the industrial economy era. Acceptance sampling is one of its technique. And It began to take root on during in the early nineteenth century and flourished during and after the Second World War. A lot of methodologies were developed in the second half of the last century; in particular, various sampling plans were formulated to cater for various testing situations and quality requirements. Single sampling plans and double sampling plans are the most basic and widely applied testing plans when simple testing is needed. Multiple sampling plans and sequential sampling plans provide marginally better disposition decision at the expense of more complicated operating procedures. Other plans such as the continuous sampling plan, the bulk sampling plan, and the tighten-normal-tighten plan, skip lot sampling plan etc., are well developed and frequently used in their respective working conditions. Among these plans, chain sampling plans have received great attention because of their unique strength in dealing with destructive or costly inspection, where the sample size is kept as low as possible to minimize the total inspection cost without compromising the protection to suppliers and consumers. The objective of this paper is to minimize the consumer risk. In this paper, the starting point is the chain sampling plan (ChSP-1), first introduced by Dodge2. Its original intention was to overcome the problem of the lack of discrimination of a single sampling plan when the acceptance number $c=0$. Today, this plan and its extensions have become the most frequently used plans in destructive or costly inspection.

[^0]Theoretical calculations of the Three stage ChSP $(0,1,2)$ plan are made on the assumptions that:

Step 1: At the outset, select a random sample of $n$ units from the lot and from each succeeding lot.
Step2: Record the number of defectives d, in each sample and sum the number of defectives, $D$, in all samples from the first up to and including in the current sample.
Step 3: Accept the lot associated with each new sample during the cumulaion as long as $\mathrm{D}_{\mathrm{i}} \leq \mathrm{c}_{1 ;}, 1 \leq \mathrm{i} \leq \mathrm{k}_{1}$.
Step4: When $k_{1}$ consecutive samples have all resulted in acceptance continue to sum the defectives in the $\mathrm{k}_{1}$ samples plus additional samples upto not more than $\mathrm{k}_{2}$ samples.
Step 5: Accept the lot associated with each new sample during cumulation as long as $\mathrm{D}_{\mathrm{i}} \leq \mathrm{c}_{2} ; \mathrm{k}_{1} \leq \mathrm{i} \leq \mathrm{k}_{2}$.
Step 6: When $k_{2}$ consecutive samples have all resulted in acceptance continue to sum the defectives in the $\mathrm{k}_{2}$ samples plus additional samples upto not more than $\mathrm{k}_{3}$ samples.
Step 7: Accept the lot associated with each new sample during cumulation as long as $\mathrm{D}_{\mathrm{i}} \leq \mathrm{c}_{3} ; \mathrm{k}_{2} \leq \mathrm{i} \leq \mathrm{k}_{3}$.
Step 8: When the third stage of the restart period has been successfully completed (i.e., $\mathrm{k}_{3}$ consecutive samples have been resulted in acceptance),start cumulation of defectives as moving total over $\mathrm{k}_{3}$ samples by adding the current sample result while dropping from the sum, the sample result of the $\mathrm{k}_{3}$ th preceding sample. Continue this procedure as long as $\mathrm{D}_{\mathrm{i}} \leq \mathrm{c}_{3}$ and in each instance accept the lot.
Step 9: If for any sample at any stage of the above procedure, $D_{i}$ is greater than the corresponding c , reject the lot.
Step 10: When a lot is rejected return to step-1 and fresh restart of the cumulation procedure.

When $\mathrm{k}_{1}=1, \mathrm{k}=2$ and $\mathrm{k}_{3}=3$ and $c_{2}=c_{1}+1, c_{3}=c_{2}+1$, the three stage chain sampling plan becomes a multiple sampling plan It is well-known that in the case of repetitive deferred sampling plan the acceptance or rejection of a lot in deferred state in dependent on the inspection results of the preceding or succeeding lots under Repetitive Group Sampling (RGS) inspection. So, RGS is the particular case of RDS plan. The operation of a repetitive deferred sampling plan as reference plan with four parameters is as follows.

1. Draw a random sample of size n from the lot and determine the number of defectives (d) found there in.
2. Accept the lot if $\mathrm{d} \leq \mathrm{c}_{1}$, Reject the lot if $\mathrm{d}>\mathrm{c}_{2}$.
3. If $\mathrm{c}_{1}<\mathrm{d}<\mathrm{c}_{2}$, accept the lot provided ' i ' preceding or succeeding lots are accepted under RGS inspection plan, otherwise reject the lot. Here $c_{1}$ and $c_{2}$ are acceptance numbers such that $c_{1}<c_{2}$ when $i=1$ this plan reduces to RGS plan. The operating characteristic function $\mathrm{P}_{\mathrm{a}}$ (p) for RDS plan is derived by Shankar and Mahopatra ( 1991) using Poisson model as

$$
\begin{gather*}
p_{a}(p)=\frac{p_{a}\left(1-p_{c}\right)^{i}+p_{c} p_{a}^{i}}{\left(1-p_{c}\right)^{i}} \ldots \ldots \ldots \ldots \ldots  \tag{1}\\
p_{a}=\sum_{r=0}^{c_{1}} \frac{e^{-x} x^{r}}{r!} \\
p_{c}=\sum_{r=0}^{c_{2}} \frac{e^{-x} x^{r}}{r!}-\sum_{r=0}^{c_{1}} \frac{e^{-x} x^{r}}{r!}
\end{gather*}
$$

Where $x=n p$
Thus the RDS plan is characterized with parameters namely $\mathrm{n}, \mathrm{c}_{1}, \mathrm{c}_{2}, \mathrm{r}$ and the acceptance criterion i .

## Selection of sampling Plan

## Minimum Risk three stages Chain Sampling Plan with repetitive deferred sampling plan

Table 1 is used to select a three stage Chain Sampling Plan with repetitive deferred sampling plan [6] for given $p_{1}$ and $p_{2}$ involving the minimum sum of risks. For the plans of Table 1, the producer's and consumer's risk will be atmost $10 \%$ each. Against fixed value of the operating ratio $p_{2} / p_{1}$, Table 1 gives the parameters $\mathrm{i}, \mathrm{c}_{1}, \mathrm{c}_{2}, \mathrm{k}_{1}, \mathrm{k}_{2}$, and $\mathrm{k}_{3}$, and the associated producer's and consumer's risks in the body of the Table against the product of sample size and the Acceptable Quality Level $\left(\mathrm{np}_{1}\right)$. The following procedure is used for selecting the plans for given $\mathrm{p}_{1}, \mathrm{p}_{2}, \alpha$ and $\beta$.

1. Compute the operating ratio $p_{2} / p_{1}$.
2. With the computed value of $p_{2} / p_{1}$, enter Table 1 in the row headed by $p_{2} / p_{1}$ which is equal to or just smaller than the computed ratio. 3. For determining the parameters i and k , one proceeds form left to right in the row identified in Step 2 such that the tabulated producer's and consumer's risks are equal to or just less than the desired values.
3. The sample size $n_{2}$ is obtained as $n_{2}=n=n p_{1} / p_{1}$, where $n p_{1}$ values are given in the column heading corresponding to the parameter i $, \mathrm{c}_{1}, \mathrm{c}_{2}, \mathrm{k}_{1}, \mathrm{k}_{2}$, and k 3 identified in step 3. The sample size $\mathrm{n}_{1}$ is found as $\mathrm{n}_{1}=\mathrm{k}_{1} \mathrm{k}_{2} \mathrm{k}_{3} \mathrm{n}_{2}(\mathrm{k}>1)$.

For example, for given $p_{1}=0.005, p_{2}=0.1, \alpha=0.05$ and $\beta=0.05$, one obtain a three stage chain sampling plan with RDS by the following steps from Table 1.

Table 1. Parametric values for three stage Chain sampling plan with repetitive deferred sampling plan using Minimum Sum of risks


Table 2. Parametric values for three stage Chain sampling plan with repetitive deferred sampling plan using Minimum Sum of risks

|  | 0.55 | 0.60 | 0.65 | 0.70 | 0.75 | 0.80 | 0.85 | 0.90 | 0.95 | 0.70 | 0.75 | 0.80 | 0.85 | 0.90 | 0.95 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 31 | $\begin{aligned} & 9,10,11, \\ & 1,2,3, \\ & 0.5,17 \end{aligned}$ | $\begin{aligned} & 9,10,11, \\ & 1,2,3, \\ & 0,19 \end{aligned}$ | $\begin{aligned} & 9,10,11, \\ & 1,2,3, \\ & 0.5,20 \end{aligned}$ | $\begin{aligned} & 9,10,11, \\ & 1,2,3, \\ & 0,22 \end{aligned}$ | $\begin{aligned} & 9,10,11, \\ & 1,2,3, \\ & 0.5,23 \end{aligned}$ | $\begin{aligned} & 9,10,11, \\ & 1,2,3, \\ & 0,25 \end{aligned}$ | $\begin{aligned} & 9,10,11, \\ & 1,2,3, \\ & 0.5,26 \end{aligned}$ | $\begin{aligned} & 9,10,11, \\ & 1,2,3, \\ & 0,28 \end{aligned}$ | $\begin{aligned} & 9,10,11, \\ & 1,2,3, \\ & 0.5,29 \end{aligned}$ |  |  |  |  |  |  |
| 30 | $\begin{aligned} & 9,10,20 \\ & 1,2,3 \\ & 0.95,16 \\ & 10,11,12 \end{aligned}$ | $\begin{aligned} & 9,10,20 \\ & 1,2,3 \\ & 0.4,18 \\ & 10,11,12 \end{aligned}$ | $\begin{aligned} & 9,10,20 \\ & 1,2,3 \\ & 0.85,19 \end{aligned}$ | $\begin{aligned} & 9,10,20 \\ & 1,2,3 \\ & 0.3,21 \end{aligned}$ | $\begin{aligned} & 9,10,20 \\ & 1,2,3 \\ & 0.75,22 \\ & 10,11,12 \end{aligned}$ | $\begin{aligned} & 9,10,20 \\ & 1,2,3 \\ & 0.2,24 \\ & 10.11 .12 \end{aligned}$ | $\begin{aligned} & 9,10,20, \\ & 1,2,3, \\ & 0.65,25 \\ & 10,11,12 \end{aligned}$ | $\begin{aligned} & 9,10,20 \\ & 1,2,3 \\ & 0.1,27 \end{aligned}$ | $\begin{aligned} & 9,10,20, \\ & 1,2,3, \\ & 0.55,28 \\ & 10,11,12 \end{aligned}$ | $\begin{aligned} & 9,10,20, \\ & 1,2,3, \\ & 0 \\ & 10,11,12 \end{aligned}$ | $\begin{aligned} & 9,10,20, \\ & 1,2,3, \\ & 0 \end{aligned}$ | $\begin{aligned} & 9,10,20, \\ & 1,2,3, \\ & 0 \end{aligned}$ | $\begin{aligned} & 9,10,20, \\ & 1,2,3 \\ & 0 \\ & 10,11.12 \end{aligned}$ | $\begin{aligned} & 9,10,20, \\ & 1,2,3, \\ & 0, \\ & 10,11,12, \end{aligned}$ | $\begin{aligned} & 9,10,20, \\ & 1,2,3, \\ & 0 \end{aligned}$ $10,11,12$ |
| 29 | $\begin{aligned} & 1,2,3, \\ & 0.4,16 \end{aligned}$ | $\begin{aligned} & 1,2,3 \\ & 0.8,17 \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0.2,19 \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0.6,20 \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0,22 \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0.4,23 \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0.8,24 \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0.2,26 \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0.6,27 \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0 \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0 \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0 \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0 \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0 \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0 \end{aligned}$ |
| 28 | $\begin{aligned} & 11,12,13, \\ & 1,2,3, \\ & 0.85,15 \\ & 11,12,19 \end{aligned}$ | $\begin{aligned} & 11,12,13, \\ & 1,2,3 \\ & 0.2,17 \\ & 11,12,19 \end{aligned}$ | $\begin{aligned} & 11,12,13, \\ & 1,2,3, \\ & 0.55,18 \\ & 11,12,19 \end{aligned}$ | $\begin{aligned} & 11,12,13, \\ & 1,2,3 \\ & 0.9,19 \\ & 11,12,19 \end{aligned}$ | $\begin{aligned} & 11,12,13, \\ & 1,2,3, \\ & 0.25,21 \\ & 11,12,19 \end{aligned}$ | $\begin{aligned} & 11,12,13, \\ & 1,2,3, \\ & 0.6,22 \\ & 11,12,19 \end{aligned}$ | $\begin{aligned} & 11,12,13, \\ & 1,2,3, \\ & 0.95,23 \\ & 11,12,19 \end{aligned}$ | $\begin{aligned} & 11,12,13, \\ & 1,2,3, \\ & 0.3,25 \\ & 11,12,19 \end{aligned}$ | $\begin{aligned} & 11,12,13, \\ & 1,2,3, \\ & 0.65,26 \\ & 11,12,19 \end{aligned}$ | $\begin{aligned} & 11,12,13, \\ & 1,2,3, \\ & 0, \\ & 11,12,19 \end{aligned}$ | $\begin{aligned} & 11,12,13, \\ & 1,2,3, \\ & 0, \\ & 11,12,19 \end{aligned}$ | $\begin{aligned} & 11,12,13, \\ & 1,2,3, \\ & 0, \\ & 11,12,19 \end{aligned}$ | $\begin{aligned} & 11,12,13, \\ & 1,2,3, \\ & 0, \\ & 11,12,19 \end{aligned}$ | $\begin{aligned} & 11,12,13, \\ & 1,2,3, \\ & 0, \\ & 11,12,19 \end{aligned}$ | $\begin{aligned} & 11,12,13, \\ & 1,2,3, \\ & 0, \\ & 11,12,19 \end{aligned}$ |
| 27 | $\begin{aligned} & 1,2,3, \\ & 0.3,15 \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0.6,16 \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0.9,17 \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0.2,19 \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0.5,20 \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0.8,21 \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0.1,23 \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0.4,24 \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0.7,25 \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0, \end{aligned}$ |
| 26 | $\begin{aligned} & 11,17,20, \\ & 1,2,3 \\ & 0.75,14 \end{aligned}$ | $\begin{aligned} & 11,17,20, \\ & 1,2,3 \\ & 0,16 \end{aligned}$ | $\begin{aligned} & 11,17,20, \\ & 1,2,3 \\ & 0.25,17 \end{aligned}$ | $\begin{aligned} & 11,17,20, \\ & 1,2,3 \\ & 0.5,18 \end{aligned}$ | $\begin{aligned} & 11,17,20, \\ & 1,2,3 \\ & 0.75,19 \end{aligned}$ | $\begin{aligned} & 11,17,20, \\ & 1,2,3, \\ & 0,21 \end{aligned}$ | $\begin{aligned} & 11,17,20, \\ & 1,2,3, \\ & 0.25,22 \end{aligned}$ | $\begin{aligned} & 11,17,20, \\ & 1,2,3, \\ & 0.5,23 \end{aligned}$ | $\begin{aligned} & 11,17,20, \\ & 1,2,3 \\ & 0.75,24 \end{aligned}$ | $\begin{aligned} & 11,17,20, \\ & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 11,17,20, \\ & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 11,17,20, \\ & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 11,17,20, \\ & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 11,17,20, \\ & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 11,17,20, \\ & 1,2,3, \\ & 0, \end{aligned}$ |
| 25 | $\begin{aligned} & 12,13,14, \\ & 1,2,3 \\ & 0.2,14 \end{aligned}$ | $\begin{aligned} & 12,13,14, \\ & 1,2,3, \\ & 0.4,15 \end{aligned}$ | $\begin{aligned} & 12,13,14, \\ & 1,2,3 \\ & 0.6,16 \end{aligned}$ | $\begin{aligned} & 12,13,14, \\ & 1,2,3 \\ & 0.8,17 \end{aligned}$ | $\begin{aligned} & 12,13,14, \\ & 1,2,3 \\ & 0,19 \end{aligned}$ | $\begin{aligned} & 12,13,14, \\ & 1,2,3 \\ & 0.2,20 \end{aligned}$ | $\begin{aligned} & 12,13,14, \\ & 1,2,3 \\ & 0.4,21 \end{aligned}$ | $\begin{aligned} & 12,13,14, \\ & 1,2,3 \\ & 0.6,22 \end{aligned}$ | $\begin{aligned} & 12,13,14, \\ & 1,2,3, \\ & 0.8,23 \end{aligned}$ | $\begin{aligned} & 12,13,14, \\ & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 12,13,14, \\ & 1,2,3 \\ & 0, \end{aligned}$ | $\begin{aligned} & 12,13,14, \\ & 1,2,3 \\ & 0, \end{aligned}$ | $\begin{aligned} & 12,13,14, \\ & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 12,13,14, \\ & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 12,13,14, \\ & 1,2,3 \\ & 0, \end{aligned}$ |
| 24 | $\begin{aligned} & 13,14,15, \\ & 1,2,3, \\ & 0.65,13 \end{aligned}$ | $\begin{aligned} & 13,14,15, \\ & 1,2,3 \\ & 0.8,14 \end{aligned}$ | $\begin{aligned} & 13,14,15, \\ & 1,2,3, \\ & 0.95,15 \end{aligned}$ | $\begin{aligned} & 13,14,15, \\ & 1,2,3 \\ & 0.1,17 \end{aligned}$ | $\begin{aligned} & 13,14,15, \\ & 1,2,3, \\ & 0.25,18 \end{aligned}$ | $\begin{aligned} & 13,14,15, \\ & 1,2,3, \\ & 0,4,19 \end{aligned}$ | $\begin{aligned} & 13,14,15, \\ & 1,2,3, \\ & 0.55,20 \end{aligned}$ | $\begin{aligned} & 13,14,15, \\ & 1,2,3, \\ & 0.7,21 \end{aligned}$ | $\begin{aligned} & 13,14,15, \\ & 1,2,3, \\ & 0.85,22 \end{aligned}$ | $\begin{aligned} & 13,14,15, \\ & 1,2,3 \\ & 0, \end{aligned}$ | $\begin{aligned} & 13,14,15, \\ & 1,2,3 \\ & 0, \end{aligned}$ | $\begin{aligned} & 13,14,15, \\ & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 13,14,15, \\ & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 13,14,15, \\ & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 13,14,15, \\ & 1,2,3, \\ & 0, \end{aligned}$ |
| 23 | $\begin{aligned} & 13,14,19, \\ & 1,2,3, \\ & 0.1,13 \end{aligned}$ | $\begin{aligned} & 13,14,19, \\ & 1,2,3 \\ & 0.2,14 \\ & 13,14,22, \end{aligned}$ | $\begin{aligned} & 13,14,19, \\ & 1,2,3, \\ & 0.3,15 \end{aligned}$ | $\begin{aligned} & 13,14,19, \\ & 1,2,3, \\ & 0.4,16 \end{aligned}$ | $\begin{aligned} & 13,14,19, \\ & 1,2,3 \\ & 0.5,17 \end{aligned}$ | $\begin{aligned} & 13,14,19, \\ & 1,2,3 \\ & 0.6,18 \end{aligned}$ | $\begin{aligned} & 13,14,19, \\ & 1,2,3, \\ & 0.7,19 \end{aligned}$ | $\begin{aligned} & 13,14,19, \\ & 1,2,3 \\ & 0.8,20 \end{aligned}$ | $\begin{aligned} & 13,14,19, \\ & 1,2,3, \\ & 0.9,21 \end{aligned}$ | $\begin{aligned} & 13,14,19, \\ & 1,2,3 \\ & 0 \\ & 13,14,22, \end{aligned}$ | $\begin{aligned} & 13,14,19, \\ & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 13,14,19, \\ & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 13,14,19, \\ & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 13,14,19, \\ & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 13,14,19, \\ & 1,2,3, \\ & 0, \end{aligned}$ |
| 22 | $\begin{aligned} & 1,2,3 \\ & 0.55,12 \end{aligned}$ | $\begin{aligned} & 1,2,3 \\ & 0.6,13 \end{aligned}$ | $\begin{aligned} & 13,14,22 \\ & 1,2,3, \\ & 0.65,14 \end{aligned}$ | $\begin{aligned} & 1,2,3 \\ & 0.7,15 \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0.75,16 \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0.8,17 \end{aligned}$ | $\begin{aligned} & 1,2,3 \\ & 0.85,18 \end{aligned}$ | $\begin{aligned} & 1,2,3 \\ & 0.9,19 \end{aligned}$ | $\begin{aligned} & 1,2,3 \\ & 0.95,20 \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 1,2,3, \\ & 0 \end{aligned}$ |
| 21 | $\begin{aligned} & 13,17,22, \\ & 1,2,3, \\ & 0,12 \end{aligned}$ | $\begin{aligned} & 13,17,22, \\ & 1,2,3 \\ & 0,13 \end{aligned}$ | $\begin{aligned} & 13,17,22, \\ & 1,2,3 \\ & 0,14 \end{aligned}$ | $\begin{aligned} & 13,17,22, \\ & 1,2,3 \\ & 0,15 \end{aligned}$ | $\begin{aligned} & 13,17,22, \\ & 1,2,3, \\ & 0,16 \end{aligned}$ | $\begin{aligned} & 13,17,22, \\ & 1,2,3, \\ & 0,17 \end{aligned}$ | $\begin{aligned} & 13,17,22, \\ & 1,2,3, \\ & 0,18 \end{aligned}$ | $\begin{aligned} & 13,17,22, \\ & 1,2,3 \\ & 0,19 \end{aligned}$ | $\begin{aligned} & 13,17,22, \\ & 1,2,3, \\ & 0,20 \end{aligned}$ | $\begin{aligned} & 13,17,22, \\ & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 13,17,22, \\ & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 13,17,22, \\ & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 13,17,22, \\ & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 13,17,22, \\ & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 13,17,22, \\ & 1,2,3, \\ & 0, \end{aligned}$ |
| 20 | $\begin{aligned} & 13,20,22, \\ & 1,2,3, \\ & 0,10 \end{aligned}$ | $\begin{aligned} & 13,20,22, \\ & 1,2,3, \\ & 0,9 \end{aligned}$ | $\begin{aligned} & 13,20,22, \\ & 1,2,3, \\ & 0,8 \end{aligned}$ | $\begin{aligned} & 13,20,22, \\ & 1,2,3, \\ & 0,7 \end{aligned}$ | $\begin{aligned} & 13,20,22, \\ & 1,2,3, \\ & 0,6 \end{aligned}$ | $\begin{aligned} & 13,20,22, \\ & 1,2,3, \\ & 0,5 \end{aligned}$ | $\begin{aligned} & 13,20,22, \\ & 1,2,3 \\ & 0,4 \end{aligned}$ | $\begin{aligned} & 13,20,22, \\ & 1,2,3, \\ & 0,3 \end{aligned}$ | $\begin{aligned} & 13,20,22, \\ & 1,2,3, \\ & 0,2 \end{aligned}$ | $\begin{aligned} & 13,20,22, \\ & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 13,20,22, \\ & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 13,20,22, \\ & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 13,20,22, \\ & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 13,20,22, \\ & 1,2,3, \\ & 0, \end{aligned}$ | $\begin{aligned} & 13,20,22, \\ & 1,2,3, \\ & 0, \end{aligned}$ |
| 19 | $\begin{aligned} & 14,15,16, \\ & 1,2,3, \\ & 0.55,9 \end{aligned}$ | $\begin{aligned} & 14,15,16, \\ & 1,2,3, \\ & 0.6,8 \end{aligned}$ | $\begin{aligned} & 14,15,16, \\ & 1,2,3, \\ & 0.65,7 \\ & \hline \end{aligned}$ | $\begin{aligned} & 14,15,16, \\ & 1,2,3, \\ & 0.7,6 \end{aligned}$ | $\begin{aligned} & 14,15,16, \\ & 1,2,3, \\ & 0.75,5 \end{aligned}$ | $\begin{aligned} & 14,15,16, \\ & 1,2,3, \\ & 0.8,4 \end{aligned}$ | $\begin{aligned} & 14,15,16, \\ & 1,2,3, \\ & 0.85,3 \end{aligned}$ | $\begin{aligned} & 14,15,16, \\ & 1,2,3, \\ & 0.9,2 \end{aligned}$ | $\begin{aligned} & 14,15,16, \\ & 1,2,3, \\ & 0.95,1 \\ & \hline \end{aligned}$ | $\begin{aligned} & 14,15,16, \\ & 1,2,3, \\ & 0 \end{aligned}$ | $\begin{aligned} & 14,15,16, \\ & 1,2,3, \\ & 0 \end{aligned}$ | $\begin{aligned} & 14,15,16, \\ & 1,2,3, \\ & 0 \end{aligned}$ | $\begin{aligned} & 14,15,16, \\ & 1,2,3, \\ & 0 \end{aligned}$ | $\begin{aligned} & 14,15,16, \\ & 1,2,3, \\ & 0 \end{aligned}$ | $\begin{aligned} & 14,15,16, \\ & 1,2,3, \\ & 0 \end{aligned}$ |


| $\mathrm{k}_{1}$ | $\mathrm{k}_{2}$ | $\mathrm{k}_{3}$ |  |
| :--- | :--- | :--- | :---: |
| i | $\mathrm{c}_{1}$ | $\mathrm{c}_{2}$ |  |
| $\alpha$ | $\beta$ |  |  |

1. $\mathrm{p}_{2} / \mathrm{p}_{1}=0.01 / 0.005=20.0$,
2. Tabulated $\mathrm{p}_{2} / \mathrm{p}_{1}=20.0$,
3. Corresponding to $\mathrm{i}=1$ and $\mathrm{k}_{1}=1, \mathrm{k}_{2}=4, \mathrm{k}_{3}=5$ given in the body of the Table 1, one obtain $\alpha=5$ and $\beta=0.1$ against the desired $\alpha=0.05$ and $\beta=0.05$.
4. $\mathrm{n}_{2}=\mathrm{n}=\mathrm{np}_{1} / \mathrm{p}_{1}=0.01 / 0.005=20$
$\mathrm{n}_{1}=\mathrm{k}_{1} \mathrm{k}_{2} \mathrm{k}_{3} * \mathrm{n}_{2}=20(20)=400$
Table 1 given here does not assume any fixed values of $\alpha$ and $\beta$, and gives three stage chin sampling plan with repetitive deferred sampling plan for rounded values of the operating ratio. The Table presented here directly assume $p_{2} / p_{1}$ values and give the parameters with corresponding producer's and consumer's risks such that their sum is minimum. For example, if one fixes $p_{1}=0.004, p_{2}=0.07$ with $\alpha=$ 0.05 and $\beta=0.10$, one gets the following plan using the Table 2 of Subramani [8], for which the tabulated operating ratio is 17.16 with $\mathrm{np}_{1}=0.0610: \mathrm{n}_{1}=33, \mathrm{n}_{2}=15, \mathrm{i}=6$ and $\mathrm{k}=2.2$

For the same conditions, one obtains the following two stage plan of Bagchi from Table 1 of this paper as $n_{1}=40, n_{2}=8, i=2$ and $k=5$.
This plan has $\alpha=0.02$ and $\beta=0.07$, giving $\alpha+\beta=0.09$.
Table 1 can also be used to select a three stage chain sampling plan with repetitive deferred sample plan when the sample size is fixed. For example, if one fixes $\mathrm{n}_{2}=\mathrm{n}=15, \mathrm{p}_{1}=0.004, \mathrm{p}_{2}=0.04$, one gets $\mathrm{np}_{1}=0.06$ and $\mathrm{p}_{2} / \mathrm{p}_{1}=10$. With the computed value of $\mathrm{np}_{1}$ and $\mathrm{p}_{2} /$ $\mathrm{p}_{1}$, one obtains from Table 1 the following parameters corresponding to the minimum sum of producer's and consumer's risks [9]. $\mathrm{n}_{2}=15$, $\mathrm{n}_{1}=75, \mathrm{i}=2$ and $\mathrm{k}=5(\alpha=0.05 ; \beta=0.05)$
The OC function for three stage chain sampling plan with RDS obtained bySuresh and Anamiya is

$$
\begin{aligned}
& P_{a}(P)= \\
& p_{0}+p_{1} p_{0}^{k_{2}-1}+\left(k_{3}-k_{2}-1\right) p_{1}^{2} p_{0}^{k_{3}-2}+\frac{p_{a}\left(1-p_{c}\right)^{i}+p_{c} p_{a}^{i}}{\left(1-p_{c}\right)^{i}} p_{0}^{k_{3}-1}+p_{1} p_{0}^{k_{1}}\left[\frac{1-p_{0}^{k_{2}-k_{1}-1}}{1-p_{0}}\right]+\left(k_{2}-k_{1}\right) p_{1}^{2} p_{0}^{k_{2}-1}+ \\
& \frac{p_{1}^{2} p_{0}^{k_{2}}\left[\frac{1-\left(k_{3}-k_{2}-1\right) p_{0}^{k_{3}-k-2_{2}}}{1-p_{0}}+\frac{p_{0}\left(1-p_{0}^{k_{3}-k-2_{2}}\right)}{\left(1-p_{0}\right)^{2}}\right]+\frac{p_{a}\left(1-p_{c}\right)^{i}+p_{c} p_{a}^{i}}{\left(1-p_{c}\right)^{i}} p_{0}^{k_{2}}\left[\frac{1-p_{0}^{k-k_{2}-1_{3}}}{1-p_{0}}\right]}{1+p_{1} p_{0}^{k_{i}}\left[\frac{1-p_{0}^{k_{2}-k_{1}-1}}{1-p_{0}}\right]+\left(k_{2}-k_{1}\right) p_{1}^{2} p_{0}^{k_{2}-1}+} . \\
& \quad p_{1}^{2} p_{0}^{k_{2}}\left[\frac{1-\left(k_{3}-k_{2}-1\right) p_{0}^{k_{3}-k-2_{2}}}{1-p_{0}}+\frac{p_{0( }\left(1-p_{0}^{k_{3}-k-2_{2}}\right)}{\left(1-p_{0}\right)^{2}}\right]+\frac{p_{a}\left(1-p_{c}\right)^{i}+p_{c} p_{a}^{i}}{\left(1-p_{c}\right)^{i}} p_{0}^{k_{2}}\left[\frac{1-p_{0}^{k-k_{2}-1_{3}}}{1-p_{0}}\right]
\end{aligned}
$$

The expression for the sum of producer's and consumer's risks is given by

$$
\begin{equation*}
\alpha+\beta=1-\mathrm{Pa}\left(\mathrm{p}_{1}\right)+\mathrm{Pa}\left(\mathrm{p}_{2}\right) . \tag{4}
\end{equation*}
$$

If the operating ratio $\mathrm{p}_{2} / \mathrm{p}_{1}$ are known, then $\mathrm{np}_{2}$ can be written as
$\mathrm{np}_{2}=\left(\mathrm{p}_{2} / \mathrm{p}_{1}\right)\left(\mathrm{np}_{1}\right)$
Under the Poisson assumptions, one has the expression for the sum of producer's and consumer's risks is given by For fixed $\mathrm{np}_{1}$ the value of $\mathrm{np}_{2}$ is calculated from equation (5) and used in equation (4). The parameters i and k corresponding to the minimum $1-\mathrm{Pa}\left(\mathrm{p}_{1}\right)+\mathrm{Pa}\left(\mathrm{p}_{2}\right)$ are obtained searching for $\mathrm{i}=1(1) 10$ and $\mathrm{k}=1(0.1) 5$ for fixed value of $\mathrm{np}_{1}$ and $\mathrm{p}_{2} / \mathrm{p}_{1}$ using a computer program. The producer's and consumer's risks are then obtained corresponding to the i and k values for which the sum of risks is minimum.

## Conclusion

Acceptance sampling is the techniques which deals with the procedure in which decision either accept or reject lots or processes which are based on the examination of samples. This paper relates to the new procedure for the selection of three stage chain sampling plan with repetitive deferred sampling plan as reference plan using Minimum sum of risks.

In acceptance sampling the producer and consumer plays a dominant role and hence one allows certain level of risk for producer and consumer, namely $\alpha=0.05, \beta=0.10$. In practice it is desirable to design any sampling plan with the associated quality levels which concern to producer and consumer. The result presented in this paper are mainly related with new procedure and necessary tables for selection of sampling system through minimum sum of risks involving producer's and onsumer's quality levels. The emphasis in the present work is that the selection of sampling system with this procedure is more advantages to the producer and consumer. Tables are provided here which are tailor made, handy and ready- made uses to the industrial shop-floor condition. These tables are useful for both producer and consumer for obtaining good quality products with less cost for inspection.

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