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RESEARCH ARTICLE

NEAR PRODUCT CORDIAL ON BOOK GRAPHS

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ABSTRACT

In this paper we discuss about Near product cordial labeling graphs on Book graphs. If the labeling in the graph satisfies the condition of Near product cordial then it is called Near product cordial graphs. In this paper we have proved that the above mentioned graphs are Near product cordial graphs Except $B+K_1$.It is not Near product cordial for both even and odd rectangular pages but for Triangular Book and Pentagonal Book it is not Near product cordial when t is odd.

Key Words: Cordial labeling, Product cordial, Near Product cordial labeling.

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INTRODUCTION

The concept of cordial labeling was introduced by Cahit. The concept of product cordial labeling is introduced by Sundaram, Ponraj and Somasundaram. Motivated by the above definitions, Near Product cordial is defined.

THEOREM 1:

A Book with t triangular pages is Near product cordial graph if and only if t is even, $t \geq 3$.

Proof:

Suppose t is even

Let B be a book with triangular pages and t be the number of pages.

Let v_1,v_n be the vertices of the common edge and $v_2,v_3, \ldots,v_{n-1}$ be the vertices of the other ends of the triangular pages. There are 2n+1 edges in B.

Suppose one of the vertices v_1 or v_n is even then clearly $e_f(0)-e_f(1) \geq 3$ for $n \geq 3$

Then v_1 and v_n should be labeled with odd numbers.

As t is even, then n = t+2 is even and

n = 2k say.

Out of 2k numbers in f(B) there are k+1 odd numbers and k-1 even numbers.

So $v_1,v_2, \ldots,v_{n-1}$ are labelled by k-1 odd numbers and k-1 even numbers.

Then $e_f(0) = 2\,(k-1)$ and $e_f(1) = 2\,(k-1) +1$

Hence, $|\,e_f(0) - e_f(1)\,| = 1$

If t = n-2 is even, then B is Near cordial graph.

Conversely, Suppose t is odd, then n = t+2 is odd and n = 2k+1, say

Out of 2k+1 numbers in f(B) there are k odd numbers and k+1 even numbers for any Near product cordial labeling f.

In order to get maximum number of ones, v_1 and v_n are labeled with odd numbers,as deg v_1 and deg v_n are high. Further to obtain maximum number of ones, we should label odd numbers for a maximal connected subgraph of G. So label $v_2, \ldots,v_{n-1}$ by odd numbers and the remaining vertices are labeled by n+2 even numbers

Then, $e_f(0) \geq 2(k+1)$ and $e_f(1) \leq 2(k-2)+1 = 2k-1$

Then for any near product cordial labeling f, $e_f(0) \geq 2k+2$ and $e_f(1) \leq 2(k-2)+1$

Then $e_f(0) - e_f(1) \geq 3$.

Hence, B is not Near product cordial graph.

Theorem 2:

A Book with t rectangular pages is Near product cordial

Theorem 3:

A Book with t pentagonal pages is Near product cordial graph if and only if t is even.

Theorem 4:

Let B be a Book with Triangular pages. Then $B+K_1$ is Near product cordial graph if and only t is odd, $t \geq 3$.

Proof:

Let B be a Book with t Triangular pages and $G = B + K_1$

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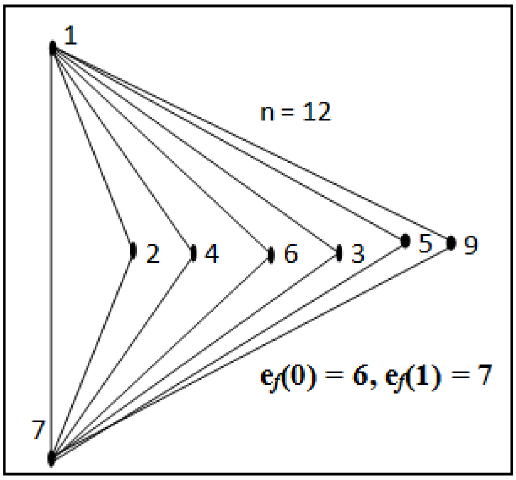


Fig. 1.

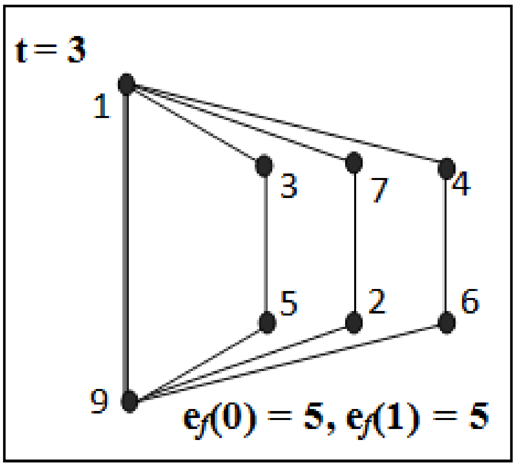


Fig. 2.

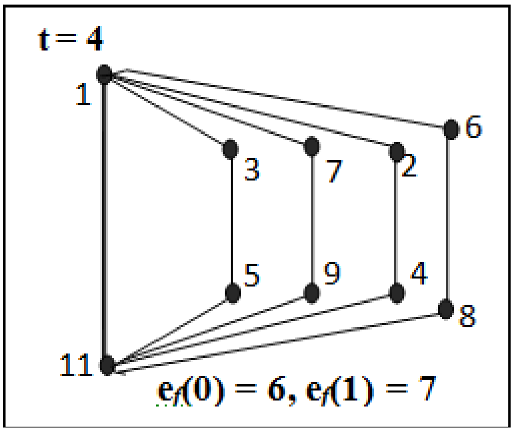


Fig. 3.

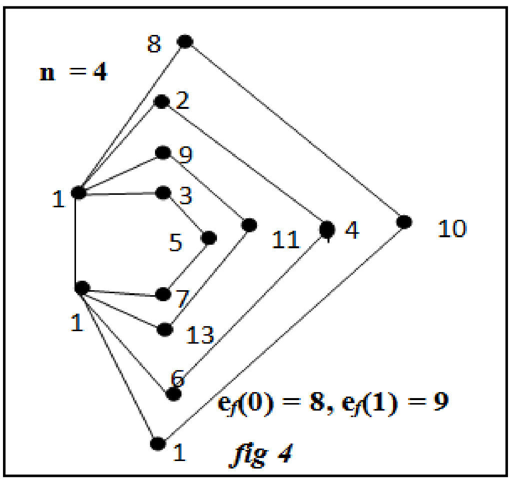


fig 4

Let V(G) = { u,v_1,v_2,w_i : $1\leq i \leq t$} and

E(G) = {v_1v_2} ∪ {uv_1, uv_2} ∪ {uw_i : $1\leq i \leq t$} ∪ {v1wi,v2wi : $1\leq i \leq t$}

There are t+3 vertices and 3t+3 edges. Let v_1v_2 be the common edge.

Suppose t is odd

Define f: V(G) → { 1, 2, 3, . . . , t+2, t+4} as follows:

$f(v_1) = 1$

$f(v_2) = 3$

$$f(w_i) = \begin{cases} 5 + 2(i-1), 1 \leq i \leq \frac{t-1}{2} \\ 2 + 2\left(i - \frac{t+1}{2}\right), \frac{t+1}{2} \leq i \leq t \end{cases}$$

f(u) = t+4

Edge Condition

ef(0) = $\frac{3t+3}{2}$ and ef(1) = $\frac{3t+3}{2}$

Thus, $|e_f(0) - e_f(1)| = 0$

Conversely suppose t is even

Here we have, $\frac{t+2}{2}$ odd numbers and $\frac{t+4}{2}$ even number exactly.

In this case, if u is labeled by even number or one of the vertices of v_1,v_2 is labeled by even number, then $e_f(1) \leq t+2$ and $e_f(0) \geq 2t+1$.

For any near product cordial labeling f

Then $e_f(0) - e_f(1) \geq t-1 \geq 3$ as t is even,$t \geq 4$

Hence, B + K_1 is not Near product cordial graph

Hence, G = B+K_1 is not Near product cordial.

Theorem 5:

Let B be a Book with Pentagonal pages.Then B+K_1 is Near product cordial if and only if t is odd.

Theorem 6:

Let B be a Book with t Rectangular pages.Then B+K_1 is not near product cordial graph.

Theorem 7:

The Book $K_{1,n}\times K_2$ is Near product cordial.

Proof:

Let V(K1,n×K2) = {u,ui,v,vi : $1\leq i \leq n$}
E(K1,n×K2) = {uv}∪{uui : $1\leq i \leq n$}∪{uivi : $1\leq i \leq n$}∪{viv : $1\leq i \leq n$}

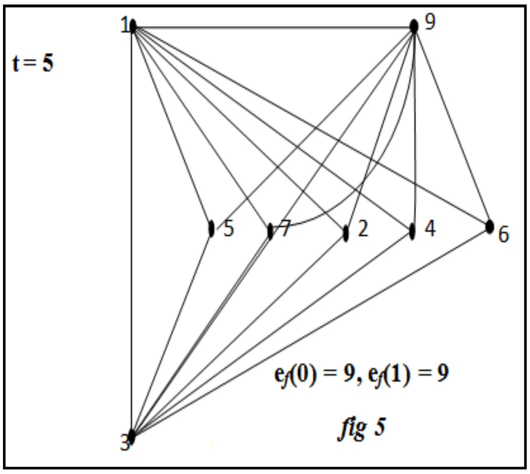
Number of vertices is 2n+2 and number of edges is 3n+1

Define f: V($K_{1,n}\times K_2$) → { 1, 2, 3, ... , 2n, 2n+1, 2n+3} as follows:

Case (i):

When n is odd

f(u) = 1



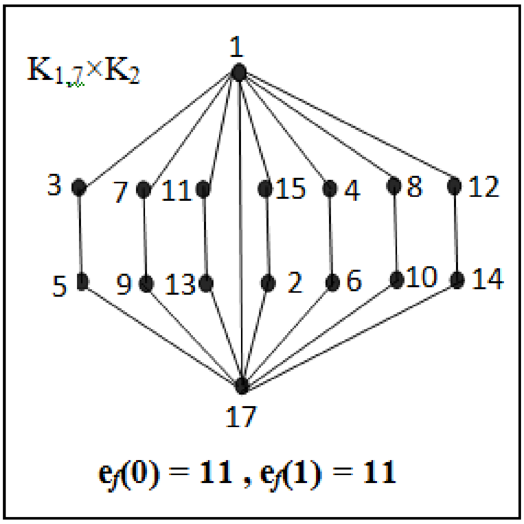


Fig. 6.

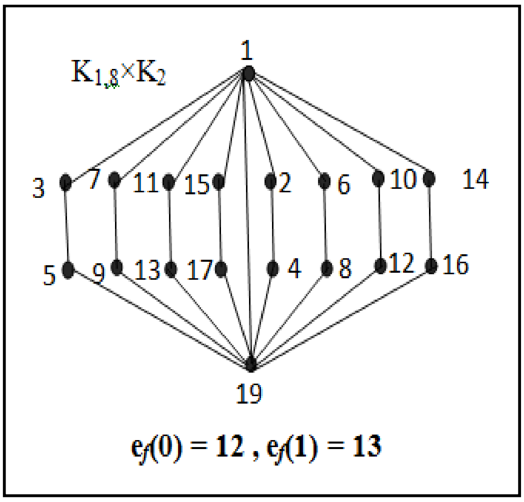


Fig. 7.

$$f(u_i) = \begin{cases} 3+4(i-1), 1 \le i \le \frac{n+1}{2} \\ 4+4\left(i-\frac{n+3}{2}\right), \frac{n+3}{2} \le i \le n \end{cases}$$

$$f(v_i) = \begin{cases} 5+4(i-1), 1 \le i \le \frac{n-1}{2} \\ 2+4\left(i-\frac{n+1}{2}\right), \frac{n+1}{2} \le i \le n \end{cases}$$

f(v) = 2n+3

Edge Condition:

ef(0) = $\frac{3n+1}{2}$ and ef(1) = $\frac{3n+1}{2}$

Thus, $|e_f(0) - e_f(1)| = 0$, When n is odd

Hence, $(K_{1,n}\times K_2)$ is Near product cordial when n is odd

Case (ii)

When n is even

f(u) = 1

$$f(u_i) = \begin{cases} 3+4(i-1), 1 \le i \le \frac{n}{2} \\ 2+4\left(i-\frac{n+2}{2}\right), \frac{n+2}{2} \le i \le n \end{cases}$$

$$f(vi) = \begin{cases} 5+4(i-1), 1 \le i \le \frac{n}{2} \\ 4+4\left(i-\frac{n+2}{2}\right), \frac{n+2}{2} \le i \le n \end{cases}$$

f(v) = 2n+3

Edge Condition

ef(0) = $\frac{3n}{2}$ and ef(1) = $\frac{3n+2}{2}$

Thus, $|e_f(0) - e_f(1)| = 1$, When n is even

Hence, $(K_{1,n}\times K_2)$ is Near product cordial when n is even.

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