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RESEARCH ARTICLE

GENERALIZED FRACTIONAL INTEGRAL OPERATORS ASSOCIATED WITH ALEPH-FUNCTION

Biniyam Shimelis, Nigussie Abeye and G. Venkat Reddy

Department of Mathematics, Wollo University, Dessie Campus P.O. Box: 1145, South Wollo, Amhara Region, (Ethiopia)

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ABSTRACT

The aim of this paper to evaluate generalized fractional integral operator involving the product of the general class of polynomials associated with the Aleph-function. Some interesting special cases of our main results are also considered.

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1. INTRODUCTION

Let $\alpha, \alpha', \beta, \beta', \eta \in \mathbb{C}$ and $x > 0$, then the generalized fractional calculus operators involving Appell function F_3 are defined by Saigo and Maeda [9] by means of the following equations:

$$(I_{+}^{\alpha,\alpha',\beta,\beta',\eta} f)(x) = \frac{x^{-\alpha}}{\Gamma(\eta)} \int_0^x (x-t)^{\eta-1} t^{-\alpha'} F_3(\alpha, \alpha', \beta, \beta'; \eta; 1-t/x, 1-x/t) f(t) dt, \quad \Re(\eta)>0, \qquad \text{...........(1.1)}$$

$$(I_{-}^{\alpha,\alpha',\beta,\beta',\eta} f)(x) = \frac{x^{-\alpha'}}{\Gamma(\eta)} \int_x^\infty (t-x)^{\eta-1} t^{-\alpha} F_3(\alpha, \alpha', \beta, \beta'; \eta; 1-x/t, 1-t/x) f(t) dt, \quad \Re(\eta)>0, \qquad \text{...........(1.2)}$$

The general class of polynomials is defined by Srivastava [16, p.1, Eq. (1)] in the following manner:

$$S_w^u [x] = \sum_{s=0}^{[w/u]} \frac{(-w)_{us}}{s!} A_{w,s} \, x^s, \qquad w=0,1,2,\ldots \qquad \text{.............(1.3)}$$

where u is an arbitrary positive integer and the coefficients $A_{w,s}$ (w, s $\geq$ 0) are arbitrary constants, real or complex.

The series representation of $\aleph$-function is introduced by Chaurasia et al. [2] as follow:

$$\aleph_{p_i,q_i,\tau_i;r}^{m,n}\left[z \left| \begin{matrix} (a_j,\alpha_j)_{1,n}, \ldots, [\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i;r} \\ (b_j,\beta_j)_{1,m}, \ldots, [\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i;r} \end{matrix} \right. \right] = \sum_{k=0}^{\infty} \sum_{h=1}^{m} \frac{(-1)^k \, \Omega_{p_i,q_i,\tau_i;r}^{m,n}(\zeta)}{\beta_h k!} (z)^{-\zeta}, \qquad \text{...........(1.4)}$$

Corresponding author: **G. Venkat Reddy,**
Department of Mathematics, Wollo University, Dessie Campus P.O. Box: 1145, South Wollo, Amhara Region, (Ethiopia)

with $\zeta = \dfrac{b_h + k}{\beta_h}, p_i < q_i, |z| < 1$ and

$$\Omega^{m,n}_{p_i,q_i,\tau_i;r}(\zeta) = \frac{\Pi^{m}_{j=1}\Gamma(b_j + \beta_j\zeta)\,\Pi^{n}_{j=1}\Gamma(1 - a_j - \alpha_j\zeta)}{\sum_{i=1}^{r}\tau_i\Pi^{q_i}_{j=m+1}\Gamma(1 - b_{ji} - \beta_{ji}\zeta)\,\Pi^{p_i}_{j=n+1}\Gamma(a_{ji} + \alpha_{ji}\zeta)}, \qquad \text{(1.5)}$$

The existence of the $\aleph$-function defined on (1.4) depends on the following conditions.

$$\varphi_l > 0,\ |\arg(z)| < \frac{\pi}{2}\varphi_l,\ l = 1,\dots,r, \qquad \text{(1.6)}$$

and

$$\varphi_l \geq 0,\ |\arg(z)| < \frac{\pi}{2}\varphi_l,\ l = 1,\dots,r\ \Re(\zeta) + 1 < 0, \qquad \text{(1.7)}$$

where

$$\varphi_l = \sum_{j=1}^{n}\alpha_j + \sum_{j=1}^{m}\beta_j - \tau_l\left(\sum_{j=n+1}^{p_l}\alpha_{jl} + \sum_{j=m+1}^{q_l}\beta_{jl}\right), \qquad \text{(1.8)}$$

and

$$\zeta = \sum_{j=1}^{m}b_j - \sum_{j=1}^{n}a_j + \tau_l\left(\sum_{j=m+1}^{q_l}b_{jl} - \sum_{j=n+1}^{p_l}a_{jl}\right) + \frac{1}{2}(p_l - q_l),\ l = 1,\dots,r. \qquad \text{(1.9)}$$

For the convergence conditions and other details of Aleph-function, (see: S ü dland et al. [18], [19]) and is defined in terms of the Mellin- Barnes type integrals as following manner (see, e.g., [12], [13]).

Remark 1.1 On setting $\tau_i = 1 (i = 1,\dots,r)$ in (1.4), yields the I-function due to Saxena [11], defined in following manner:

$$I^{m,n}_{p_i,q_i;r}[z] = \aleph^{m,n}_{p_i,q_i,1;r}[z] = \aleph^{m,n}_{p_i,q_i,1;r}\left[z\left|\begin{matrix}(a_j,\alpha_j)_{1,n},\dots,[(a_{ji},\alpha_{ji})]_{n+1,p_i}\\(b_j,\beta_j)_{1,m},\dots,[(b_{ji},\beta_{ji})]_{m+1,q_i}\end{matrix}\right.\right]$$
$$= \frac{1}{2\pi i}\int_L \Omega^{m,n}_{p_i,q_i,1;r}(\zeta) z^{-\zeta}\, d\zeta. \qquad \text{(1.10)}$$

Remark 1.2 If we set $\tau_i = 1 (i = 1,\dots,r)$ and $r = 1$, then (1.4) reduces to the familiar Fox H-function [3]:

$$H^{m,n}_{p,q}[z] = \aleph^{m,n}_{p_i,q_i,1;1}[z] = \aleph^{m,n}_{p_i,q_i,1;1}\left[z\left|\begin{matrix}(a_j,\alpha_j)\\(b_j,\beta_j)\end{matrix}\right.\right] = \frac{1}{2\pi i}\int_L \Omega^{m,n}_{p_i,q_i,1;1}(\zeta) z^{-\zeta}\, d\zeta. \qquad \text{(1.11)}$$

are the kernel $\Omega^{m,n}_{p_i,q_i,1;1}(\zeta)$ can be obtained from (1.5).

A thorough and wide-ranging account of the H-function is obtainable from the monographs written by Kilbas and Saigo [4], Mathai et al. [6], Prudnikov et al. [7] and Srivastava et al. [17].

Now, we recall the generaliged hypergeometric series defined by (see: [5, 8]):

$${}_pF_q\left[\begin{matrix}\alpha_1,\dots,\alpha_p;\\ \beta_1,\dots,\beta_q;\end{matrix}\ z\right] = \sum_{n=0}^{\infty}\frac{(\alpha_1)_n\cdots(\alpha_p)_n z^n}{(\beta_1)_n\cdots(\beta_q)_n n!} = {}_pF_q(\alpha_1,\dots,\alpha_p;\beta_1,\dots,\beta_q;z), \qquad \text{(1.12)}$$

where $(\lambda)_n$ is the Pochhammer symbol defined (for $\lambda \in \mathbb{C}$) by

$$(\lambda)_n = \begin{cases}1 & (n = 0)\\ \lambda(\lambda+1)\dots(\lambda+n-1) & (n \in N = \{1,2,3,\dots\})\end{cases} = \frac{\Gamma(\lambda+n)}{\Gamma(\lambda)}, \qquad \text{(1.13)}$$

We now establish:

Lemma 1: If $\Re(\gamma) > 0$, $\delta > 0$, $\varepsilon = 1, 2, 3, \dots$, c is a positive number and ρ is a complex number, then there holds the relation

$$\left(I^{\alpha,\alpha',\beta,\beta',\eta}_{0+}[x^{\delta}(x^{\varepsilon}+c^{\varepsilon})^{-\rho}]\right)(x) = x^{\delta-\alpha-\alpha'+\eta}c^{-\varepsilon\rho}\frac{\Gamma(\delta+1)\Gamma(\delta-\alpha'+\beta'+1)\Gamma(\delta-\alpha-\alpha'-\beta+\eta+1)}{\Gamma(\delta+\beta'+1)\Gamma(\delta-\alpha-\alpha'+\eta+1)\Gamma(\delta-\alpha'-\beta+\eta+1)}$$

$$\times {}_{3\varepsilon+1}F_{3\varepsilon}\left[\begin{matrix}\rho,\ \Delta(\varepsilon,\delta-\alpha-\alpha'-\beta+\eta+1),\ \Delta(\varepsilon,\lambda+1),\ \Delta(\varepsilon,\delta+\beta'-\alpha'+1);\\ \Delta(\varepsilon,\delta-\alpha-\alpha'+\eta+1),\ \Delta(\varepsilon,\delta-\alpha'-\beta+\eta+1),\ \Delta(\varepsilon,\lambda+\beta'+1);\end{matrix}\middle| -\left(\frac{x}{c}\right)^{\varepsilon}\right] \qquad \ldots\ldots(1.14)$$

where $\Re(\eta)>0,\ \Re(\delta-\alpha')+\min\{-\Re(\alpha'),-\Re(\beta'),\Re(\eta-\alpha-\beta)\}$, $\Delta(\varepsilon,\alpha)$ represent the sequence of parameters $\frac{\alpha}{\varepsilon}, \frac{\alpha+1}{\varepsilon}, \ldots, \frac{\alpha+\varepsilon-1}{\varepsilon}$, and ${}_{3\varepsilon+1}F_{3\varepsilon}(\cdot)$ is the generalized hypergeometric function, defined in [4].

Proof: We first operate the fractional integral operator (1.1) with $f(t)=t^{\delta}(t^{\varepsilon}+c^{\varepsilon})^{-\rho}$ and express Appell Function F_3 and $(t^{\varepsilon}+c^{\varepsilon})^{-\rho}$ in terms of their equivalent series by means of the formula

$$(t^{\varepsilon}+c^{\varepsilon})^{-\rho}=c^{-\varepsilon\rho}\sum_{q=0}^{\infty}\frac{(\rho)_q}{q!}\left(-\frac{t^{\varepsilon}}{c^{\varepsilon}}\right)^{q} \qquad \ldots\ldots(1.15)$$

On interchange the order of integration and summation, which is permissible due to the absolute convergence, and evaluate the inner integral by means of the formula given by Saxena, Ram and Kalla [14, p. 100, eq .(1.20)]

$$\int_0^x t^{s-1}(x-t)^{\eta-1}F_3(\alpha,\alpha',\beta,\beta';\eta;1-t/x,1-x/t)\,dt$$

$$=\Gamma(\eta)\,x^{\eta+s-1}\,\Gamma\left[\begin{matrix}s+\alpha',\ s+\beta',\ s+\eta-\alpha-\beta\\ s+\alpha'+\beta',\ s+\gamma-\alpha,\ s+\eta-\beta\end{matrix}\right] \qquad \ldots\ldots(1.16)$$

where $\Re(\eta)>0$, $\Re(s)>\max\{-\Re(\alpha'),-\Re(\beta'),\Re(\alpha+\beta-\eta)\}$, the result (1.14) follows. When $\alpha'=0$, (1.8) reduces to the result given in [1, p. 334, Eq. (1.6)].

Lemma 2: If $\Re(\eta)>0,\ \delta>0,\ \varepsilon=1,2,3,\ldots$, c is a positive number and ρ is a complex number, then we have

$$\left(I_{-}^{\alpha,\alpha',\beta,\beta',\eta}[x^{\delta}(x^{\varepsilon}+c^{\varepsilon})^{-\rho}]\right)(x)=x^{\delta-\alpha-\alpha'+\eta}\,c^{-k\rho}\,\frac{\Gamma(\alpha+\alpha'-\eta-\delta)\,\Gamma(\alpha+\beta'-\eta-\delta)\,\Gamma(-\beta-\delta)}{\Gamma(\alpha+\alpha'+\beta'-\eta-\delta)\,\Gamma(-\delta)\,\Gamma(\alpha-\beta-\delta)}$$

$$\times {}_{3\varepsilon+1}F_{3\varepsilon}\left[\begin{matrix}\rho,\ \Delta(k,\lambda-\alpha-\alpha'-\beta'+\gamma+1),\ \Delta(k,\lambda+1),\ \Delta(k,\lambda-\alpha-\beta+1)\\ \Delta(k,\lambda-\alpha-\alpha'+\gamma+1),\ \Delta(k,\lambda-\alpha-\beta'+\gamma+1),\ \Delta(k,\lambda+\beta+1)\end{matrix}\middle| -\left(\frac{x}{c}\right)^{\varepsilon}\right] \qquad \ldots\ldots(1.17)$$

where $\Re(\eta)>0,\quad \Re(\eta+\delta-\varepsilon\rho)+\min\{-\Re(\alpha'),\ -\Re(\beta'),\ \Re(\eta-\alpha-\beta)\}+1>0$

Proof: To establish lemma 2, we take $f(t)=t^{\delta}(t^{\varepsilon}+c^{\varepsilon})^{-\rho}$ in equation (1.2) and write series expansions for the Appell function and $(t^{\varepsilon}+c^{\varepsilon})^{-\rho}$, then interchanging the order of integration and summation, which is permissible due to the absolute convergence and evaluating the inner integral by means of the formula given by Saxena, Ram and Kalla [14, p. 100, eq. (1.21)]

$$\int_x^{\infty} t^{s-1}(t-x)^{\eta-1}F_3(\alpha,\alpha',\beta,\beta';\eta;1-x/t,1-t/x)\,dt$$

$$=\Gamma(\eta)\ x^{\eta+s-1}\,\Gamma\left[\begin{matrix}1+\alpha'-\eta-s,\ 1+\beta'-\eta-s,\ 1-\alpha-\beta-s\\ 1-s+\alpha'+\beta'-\eta,\quad 1-s-\alpha,\quad 1-s-\beta\end{matrix}\right] \qquad \ldots\ldots(1.18)$$

where $\Re(\eta)>0$, $\Re(\rho)<1+\min\{\Re(\alpha'-\eta),\ \Re(\beta'-\eta),\ -\Re(\alpha+\beta)\}$ and using the well known relation $(\alpha)_{-n}=\frac{(-1)^n}{(1-\alpha)_n}$, the desired result (1.17) is obtained. For $\alpha'=0$, (1.17) yields the result given in [1, p. 335, Eq. (1.10)].

2. Generalized Fractional Integral Formulas

If $$f(t)=t^{\delta}(t^{\varepsilon}+c^{\varepsilon})^{-\rho}\,S_w^u\,[yt^m(t^{\varepsilon}+c^{\varepsilon})^{-n}]\,\aleph_{p_i,q_i,\tau_i;r}^{m,n}\left[zt^h(t^{\varepsilon}+c^{\varepsilon})^{-\mu}\middle|\begin{matrix}(s_1)\\(s_2)\end{matrix}\right] \qquad \ldots\ldots(2.1)$$

where

$s_1 = (a_j, \alpha_j)_{1,n}, ..., (\tau_i(a_{ji}, \alpha_{ji}))_{n+1, p_i; r}$ and $s_2 = (b_j, \beta_j)_{1,m}, .., (\tau_i(b_{ji}, \beta_{ji}))_{m+1, q_i; r}$

then, we have following relation:

(I) $$\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\eta} f\right)(x) = \sum_{s=0}^{[w/u]} \sum_{h=1}^{M} \sum_{\upsilon=0}^{\infty} \frac{(-1)^{\upsilon} (-w)_{us} A_{w,s} \ x(\xi) \ z^{\xi} y^{s}}{s! \ \upsilon! \ B_h \ c^R} x^{\Im-\alpha-\alpha'+\eta}$$

$$\times \frac{\Gamma(\Im+1)\,\Gamma(\Im-\alpha'-\beta'+1)\,\Gamma(\Im-\alpha-\alpha'-\beta+\eta+1)}{\Gamma(\Im+\beta'+1)\,\Gamma(\Im-\alpha-\alpha'+\eta+1)\,\Gamma(\Im-\alpha'-\beta+\eta+1)}$$

$$\times {}_{3\varepsilon+1}F_{3\varepsilon}\left[\begin{matrix} \rho+ns+r\xi,\ \Delta(\varepsilon, \Im+1),\ \Delta(\varepsilon, \Im+\beta'+1-\alpha'),\ \Delta(\varepsilon, \Im+\eta+1-\alpha-\alpha'-\beta) \\ \Delta(\varepsilon, \Im+\beta'+1),\ \Delta(\varepsilon, \Im+\eta+1-\alpha-\alpha'),\ \Delta(\varepsilon, \Im+\eta+1-\alpha'-\beta) \end{matrix} \middle| -\left(\frac{x}{c}\right)^{\varepsilon}\right] \quad \text{.......................(2.2)}$$

where

$\Im = \delta + ms + h\xi$ and R = $\varepsilon\rho + \varepsilon n s + \varepsilon r \xi$; $\Im' = \sum_{j=1}^{N} A_j - \sum_{j=N+1}^{P} A_j + \sum_{j=1}^{M} B_j - \sum_{j=M+1}^{Q} B_j > 0$

The result (2.2) is valid for $\Re(\eta) > 0$, $1 + \Re(\delta - \alpha' + ms + h\frac{b_j}{B_j}) + \min_{1 \le j \le M} \{-\Re(\alpha'), -\Re(\beta'), -\Re(\eta - \alpha - \beta)\} > 0$,

$|\arg z| < \Im' \pi/2$, $\Im' > 0$, c is a positive number and ρ, m, n, h, r are complex numbers, k = 1, 2, 3, … , u is an arbitrary positive integer and the coefficients $A_{w,s}$

(w, s $\ge$ 0) are arbitrary constants, real or complex.

(II) $$\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\eta} f\right)(x) = \sum_{s=0}^{[w/u]} \sum_{h=1}^{M} \sum_{\upsilon=0}^{\infty} \frac{(-1)^{\upsilon} (-w)_{us} A_{w,s} \ x(\xi) \ z^{\xi} y^{s}}{s! \ \upsilon! \ B_h \ c^R} x^{\Im-\alpha-\alpha'+\eta}$$

$$\times \frac{\Gamma(\alpha+\alpha'-\eta-\Im)\,\Gamma(\alpha+\beta'-\eta-\Im)\,\Gamma(-\beta-\Im)}{\Gamma(-\Im)\,\Gamma(\alpha+\alpha'+\beta'-\eta-\Im)\,\Gamma(\alpha-\beta-\Im)}$$

$$\times {}_{3\varepsilon+1}F_{3\varepsilon}\left[\begin{matrix} \rho+ns+r\xi,\ \Delta(\varepsilon, \Im-\alpha-\alpha'-\beta'+\eta+1),\ \Delta(\varepsilon, \Im+1),\ \Delta(\varepsilon, \Im-\alpha+\beta+1) \\ \Delta(\varepsilon, \Im-\alpha-\alpha'+\eta+1),\ \Delta(\varepsilon, \Im-\alpha-\beta'+\eta+1),\ \Delta(\varepsilon, \Im+\beta+1) \end{matrix} \middle| -\left(\frac{x}{c}\right)^{\varepsilon}\right]$$

…………………(2.3)

where c is a positive number and ρ, m, n, h, r are complex numbers, k = 1, 2, 3, …; $\Re(\eta) > 0$,

$\Re(\eta+\delta-\alpha-\varepsilon\rho) + \min\{-\Re(\alpha'), -\Re(\beta'), \Re(\eta-\alpha-\beta) + ms + (h-\varepsilon r)\max_{1 \le j \le N}\left\{\frac{a_j - 1}{A_j}\right\} < 0 \ |\arg z| < \Im' \pi/2$,

$\Im' > 0$. u is an arbitrary positive integer and the coefficients $A_{w,s}$ (w, s $\ge$0) are arbitrary constants, real or complex. The proof of the results (2.2) and (2.3) can be developed on similar lines to that followed for the results (1.12) and (1.15).

3. Interesting Special Cases

(I) If we set h = r = 0, (2.2) and (2.3) yield

$$\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\eta}\left[x^{\delta}\,(x^{\varepsilon}+c^{\varepsilon})^{-\rho}\right] S_w^u\,[y x^m\,(x^{\varepsilon}+c^{\varepsilon})^{-n}]\right)(x)$$

$$= \frac{x^{\delta-\alpha-\alpha'+\eta}}{c^{\varepsilon\rho}} \sum_{s=0}^{[w/u]} \frac{(-w)_{us} A_{w,s} \ y^s}{s!} \left(\frac{x^m}{c^{\varepsilon n}}\right)^s$$

$$\times \frac{\Gamma(\delta+ms+1)\quad \Gamma(\delta+ms-\alpha'+\beta'+1)\quad \Gamma(\delta+ms-\alpha-\alpha'-\beta+\eta+1)}{\Gamma(\delta+ms+\beta'+1)\quad \Gamma(\delta+ms-\alpha-\alpha'+\eta+1)\quad \Gamma(\delta+ms-\alpha'-\beta+\eta+1)}$$

$$\times\ {}_{3\varepsilon+1}F_{3\varepsilon}\left[\begin{matrix}\rho+ns,\ \Delta(\varepsilon,\ \delta+ms+1),\Delta(\varepsilon,\delta+ms+\beta'-\alpha'),\Delta(\varepsilon,\delta+ms+\eta+1-\alpha-\alpha'-\beta)\\ \Delta(\varepsilon,\delta+ms+\beta'+1),\Delta(\varepsilon,\delta+ms+\eta+1-\alpha-\alpha'),\Delta(\varepsilon,\delta+ms+\eta+1-\alpha'-\beta)\end{matrix}\middle|-\left(\frac{x}{c}\right)^{\varepsilon}\right] \quad \text{.........(3.1)}$$

and

$$\left(I_{-}^{\alpha,\alpha',\beta,\beta',\eta}\left[x^{\delta}\ (x^{\varepsilon}+c^{\varepsilon})^{-\rho}\right] S_w^u\ [y x^m\ (x^{\varepsilon}+c^{\varepsilon})^{-n}]\right)(x)$$

$$=\frac{x^{\delta-\alpha-\alpha'+\eta}}{c^{\varepsilon\rho}}\sum_{s=0}^{[w/u]}\frac{(-w)_{us}\ A_{w,s}\ y^s}{s!}\left(\frac{x^m}{c^{\varepsilon n}}\right)^s$$

$$\times\frac{\Gamma(\alpha+\alpha'-\eta-\delta-ms)\ \ \Gamma(\alpha+\beta'-\eta-\delta-ms)\ \ \Gamma(-\beta-\delta-ms)}{\Gamma(\alpha+\alpha'+\beta'-\eta-\delta-ms)\ \ \Gamma(-\eta-ms)\ \ \Gamma(\alpha-\beta-\delta-ms)}$$

$$\times\ {}_{3\varepsilon+1}F_{3\varepsilon}\left[\begin{matrix}\rho+ns,\Delta(\varepsilon,\ \delta+ms+1),\Delta(\varepsilon,\ \lambda+ms-\alpha-\alpha'-\beta'+\eta+1),\ \Delta(\varepsilon,\delta+ms-\alpha+\beta+1)\\ \Delta(\varepsilon,\delta+ms-\alpha-\alpha'+\eta+1),\ \Delta(\varepsilon,\delta+ms-\alpha-\beta'+\eta+1),\Delta(\varepsilon,\delta+ms+\beta+1)\end{matrix}\middle|-\left(\frac{x}{c}\right)^{\varepsilon}\right] \quad \text{.................(3.2)}$$

(II) If we put α' = 0 in (2.2), where the right hand sides represents the Saigo operators, we get

$$\left(I_{0+}^{\alpha,\beta,\gamma}\ f\right)(x)=\sum_{s=0}^{[w/u]}\sum_{h=1}^{M}\sum_{\upsilon=0}^{\infty}\frac{(-1)^{\upsilon}\ (-w)_{us}\ A_{w,s}\ \ x(\xi)\ z^{\xi}\ y^s}{s!\ \upsilon!\ B_h\ c^R}x^{\Im-\beta}$$

$$\times\ \frac{\Gamma(\Im+1)\ \Gamma(\Im+\gamma-\beta+1)}{\Gamma(\Im-\beta+1)\ \Gamma(\Im+\alpha+\gamma+1)}\ {}_{2\varepsilon+1}F_{2\varepsilon}\left[\begin{matrix}\rho+ns+r\xi,\ \ \Delta(\varepsilon,\Im+1),\ \ \Delta(\varepsilon,\Im+\gamma+1-\beta)\\ \Delta(\varepsilon,\Im+1-\beta),\ \ \Delta(\varepsilon,\Im+\alpha+\gamma+1)\end{matrix}\middle|-\frac{x^k}{c^k}\right] \quad \text{.................(3.3)}$$

which holds under the same conditions as given with (2.2) for α' = 0. Next if we put α' = 0 in (2.3) and use the identity

we arrive at

$$\left(I_{-}^{\alpha,\beta,\gamma}\ f\right)(x)=\sum_{s=0}^{[w/u]}\sum_{h=1}^{M}\sum_{\upsilon=0}^{\infty}\frac{(-1)^{\upsilon}\ (-w)_{us}\ A_{w,s}\ \ x(\xi)\ x^{\Im-\beta}\ y^s\ \Gamma(\beta-\Im)\ \ \Gamma(\eta-\Im)}{s!\ \upsilon!\ B_h\ c^R\qquad\qquad \Gamma(-\Im)\ \ \Gamma(\alpha+\beta+\eta-\Im)}$$

$$\times\ {}_{2\varepsilon+1}F_{2\varepsilon}\left[\begin{matrix}\rho+ns+r\xi,\ \ \Delta(\varepsilon,\ \Im+1),\Delta(\varepsilon,\Im-\alpha-\beta-\gamma+1)\\ \Delta(\varepsilon,\Im-\gamma+1),\ \ \Delta(\varepsilon,\Im-\beta+1)\end{matrix}\middle|-\frac{x^k}{c^k}\right] \quad \text{..............(3.4)}$$

Which holds under the same conditions as given with (2.3) for α' = 0, (3.3) and (3.4) are recently given by Suthar et al. [22]. When $\rho=\alpha'=0$, Lemma 1 and 2 are reducing to the results given by Saigo and Raina [10].

4. Conclusion

We have established two new integral relations involving the product of the Srivastava's polynomials and the ℵ-function. We can also derived analogous result in the form of Riemann-Liouville and Erdélyi-Kober fractional integral operators, which have been depicted in corollaries. In another direction, using remark (1.1) and (1.2), we can also find the numerous result in the form of I-function and H-function. Therefore, the results presented in this article are easily converted in terms of a similar type (1, 10, 14, 15, 20, 21) of new interesting integrals with different arguments after some suitable parametric replacements.

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REFERENCES

Chaurasia, V.B.L. and N. Gupta, 1999. General fractional integral operators, General class of polynomials and Fox's H-function. Sochow J. Math., 25, 333-339

Chaurasia, V.B.L. and Y. Singh, 2014. New generalization of integral equations of fredholm type using Aleph function, Int. J. of Modern Math. Sci., 9(3), 208–220.

Fox: C. 1961. The G and H-functions as symmetrical Fourier kernels, Trans. Amer. Math. Soc., 98 (1961), 395-429.

Kilbas, A.A. and Saigo, M. 2004. H-Transforms: Theory and Application, Chapman & Hall/CRC Press, Boca Raton, London, New York.

Mathai, A.M. and Saxena, R. K. 1978. The H- functions with Application in Statistics and Other Disciplines, Halsted press, John Wiley and Sons, New York, London and Sydney.

Mathai, A.M., Saxena, H.J. Haubold, 2010. The H-function: Theory and Applications, Springer, New York.

N.S$\ddot{u}$dland, B. Baumann and T.F. Nannenmacher, Fractional driftless Fokker-Planck equation with power law diffusion coefficients, in V.G. Gangha, E.W. Mayr, W.G. Vorozhtsov (Eds.), Computer Algebra in Scientific Computing (CASC Konstanz 2001), Springer, Berlin, (2001), 513–525.

N.S$\ddot{u}$dland, B. Baumann and T.F. Nannenmacher, Open problem: Who knows about the $\aleph$-function?. Appl. Anal., 1(4), (1998), 401- 402.

Prudnikov, A.P. Yu. A. Brychkov and O.I. Marichev, 1990. Integrals and Series, More Special Functions, Vol. 3, Gordon and Breach Science Publishers, New York,

Purohit, S.D., Suthar, D.L. and S.L. Kalla, 2010. Some results on fractional calculus operators associated with the M-function. Hadronic J., 33(3), 225-235.

Saigo, M. and Maeda N. 1996. More generalization of fractional calculus, Transform Methods and Special Function, Verna Bulgaria, 386-400.

Saigo, M. and R. K. Raina, 1988. Fractional calculus operators associated with a general class of polynomials, Fukuoka Univ. Sci. Reports, 18(1), 15-22.

Saxena, R.K. and T.K. Pogány, 2010. Mathieu-type series for the $\aleph$-function occurring in Fokker-Planck equation. Eur. J. Pure Appl. Math., 3(6), 980-988.

Saxena, R.K. and T.K. Pogány, On fractional integration formulae for Aleph functions. Appl. Math. Comput., 218, (2011), 985-990.

Saxena, R.K., J. Ram and D.L. Suthar, Integral formulas for the H-function generalized fractional calculus II. South East Asian J. Math. & Math. Sc., 5(2), (2007), 23-31.

Saxena, R.K., J. Ram and S.L. Kalla, Unified fractional integral formulas for the generalized H-function, Rev. Acad. Canar. Cienc. XIV, (2002), 97- 109.

Saxena, V.P. 19821. Formal solution of certain new pair of dual integral equations involving H-functions. Proc. Nat. Acad. Sci. India Sect., A 52, 366–375.

Srivastava, H.M. A contour integral involving Fox's H-function, Indian J. Math., 14, (1972), 1-6.

Srivastava, H.M., K.C. Gupta and S.P. Goyal, The H-Functions of One and Two Variables with Applications, South Asian Publishers, New Delhi, (1982).

Suthar, D.L. G.V. Reddy and B. Shimelis, On certain unified fractional integrals pertaining to Product of srivastava's polynomials and Aleph-function,International Journal of Mathematics Trends and Technology, 47(1), (2017), 66-73.

Suthar, D.L. H. Habenom and H. Tadesse, Certain integrals involving Aleph function and Wright's generalized hypergeometric function, Acta Univ. Apulensis Math. Inform., 52, (2017), 01-10.

Suthar, D.L. and B. Debalkie, A new class of integral relation Involving Aleph-functions, Surv. Math. Appl. , 12 , (2017), 193-201.

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