



AN ITERATIVE BINARY MULTIPLICATION ALGORITHM USING NIKHILAM SUTRA

¹Vedavathi, K. and ²Sitamaha Lakshmi, T.

¹Department of Computer Science, GITAM University, Visakhapatnam, India

²Department of Computer Science and Engineering, GITAM University, Visakhapatnam, India

ARTICLE INFO

Article History:

Received 17th July, 2012
Received in revised form
10th August, 2012
Accepted 30th September, 2012
Published online 30th October, 2012

Key words:

Vedaic Mathematics,
Nikhilam Sutra, Bit Shifting

ABSTRACT

The iterative algorithmic approach for binary multiplication based on ancient Nikhilam Sutra is described. Nikhilam sutra, one of the Multiplication Sutra of Vedic mathematics is efficient in multiplying large decimal numbers as it reduces multiplication of two large decimal numbers to two smaller numbers. The proposed iterative algorithm is taken from Nikhilam Sutra and is further optimized by use of dropping least significant zeros of the binary numbers and performing bit shifting to take the advantage of bit reduction in multiplication.

Copy Right, IJCR, 2012, Academic Journals. All rights reserved.

INTRODUCTION

The name given to ancient mathematics system is Vedic mathematics. Vedic Mathematics is a unique system of calculations based on simple rules and principles with which mathematical problem in arithmetic, algebra, geometry or trigonometry can be solved. The system is based on 16 Vedic sutras. These sutras are the word formulae describing natural ways of solving a whole range of mathematical problems. The rediscovery of Vedic mathematics from the ancient Indian scriptures was done between 1911 and 1918 by Sri Bharati Krishna Tirthaji (1965), a scholar of Sanskrit, mathematics, history and philosophy. He studied these ancient texts and was able to reconstruct a series of mathematical formulae called Sutras. Bharati Krishna Tirthaji, who was also the former Shankaracharya (major religious leader) of Puri, India, delved into the ancient Vedic texts and established the techniques of this system in his pioneering work, Vedic Mathematics, which is considered the starting point on Vedic mathematics. The sutras of Vedic Mathematics are the software for the cosmic computer that runs this universe. Chandra Hari (1999) studied the critical Vedic Mathematics of Sankaracharya Sri Bhakti krsna Tirthakarji Maharaj. Rana Mukherji *et al.* (2011) proposed efficient multiplier architecture based on Urdhva Tiryagbhyam Sutra of ancient Indian Vedic Mathematics using System C. Ritika Jasrotia (2011) explored one particular Vedic mathematics technique for multiplication called Nikhilam Sutra. Kerur *et al.* (2011) designed a Vedic multiplier used for Matrix Multiplication. Saha *et al.* (2011) presented a 32×32 bit high speed low power multiplier design based on the formulas of the ancient Indian Vedic Mathematics which are having wide application in DSP processors. Manoranjan Pradhan *et al.* (2011) presented the concepts behind the

Urdhva Tiryagbhyam Sutra and Nikhilam Sutra multiplication techniques. They have shown the architecture for a 16×16 Vedic multiplier module using Urdhva Tiryagbhyam Sutra and extended the same with Nikhilam Sutra technique. Parth Mehta *et al.* (2009) presented comparison and implementation of normal multiplication and Vedic multiplication using Urdhva Tiryagbhyam Sutra on digital hardware.

Conventional mathematics is an integral part of engineering education since most engineering system designs are based on various mathematical approaches. Major improvements in processor technologies, as well as the search for new algorithms are essential for faster processing. A multiplier is one of the key hardware blocks in most digital systems. With advances in the technology, many researchers have tried to design multipliers which offer high speed, low power consumption. Harpreet Singh Dhillon and Abhijit Mitra (2008) proposed a reduced bit multiplication algorithm for digital arithmetic. Aniruddha Kanhe *et al.* (2012) evaluated the performance of the proposed high speed low power Vedic multiplier. They compared the design with a conventional Array Multiplier and Booth Multiplier. Ganesh Kumar and Charishma (2012) designed a high speed multiplier algorithm based on Ancient Vedic mathematics. Kabiraj Sethi and Rutuparna Panda (2012) proposed the design of a high speed squaring circuit for binary numbers. Saha, *et al.* (2011) presented a 32×32 bit high speed low power multiplier design based on the formulas of the ancient Indian Vedic Mathematics. Prabha S. Kasliwal *et al.* (2011) have given fast algorithm for squaring which when implemented in FPGA given speedup for DSP and encryption. In the present study, an iterative multiplication algorithm for binary numbers using Nikhilam Sutra has been proposed and analyzed. This paper is organized as follows. In section 2, a brief overview of Vedic mathematics and Decimal Multiplication with these rules are

*Corresponding author: vedavathi_k@yahoo.com

provided through illustrations. Section 3 gives the proposed binary multiplication algorithm. Concluding remarks are presented in section 4.

Vedic Mathematics

The word 'Vedic' is derived from the word 'veda' which means the store-house of all knowledge. Vedic mathematics is mainly based on 16 Sutras dealing with various branches of mathematics like arithmetic, algebra, geometry etc. These Sutras are:

- *(Anurupye) Shunyamanyat* – If one is in ratio, the other is zero.
- *Chalana-Kalanabyham* – Differences and Similarities.
- *Ekadhikina Purvena* – By one more than the previous one.
- *Ekanyunena Purvena* – By one less than the previous one.
- *Gunakasmuchyah* – The factors of the sum is equal to the sum of the factors.
- *Gunitasmuchyah* – The product of the sum is equal to the sum of the product.
- *Nikhilam Navatashcaramam Dashatah* – All from 9 and the last from 10.
- *Paraavartya Yojayet* – Transpose and adjust.
- *Puranapuranyam* – By the completion or noncompletion.
- *Sankalana-vyavakalanabhyam* – By addition and by subtraction.
- *Shesanyakena Charamena* – The remainders by the last digit.
- *Shunyam Saamyasamuccaye* – When the sum is the same that sum is zero.
- *Sopaantadvayamantyam* – The ultimate and twice the penultimate.
- *Urdhva-tiryakbyham* – Vertically and crosswise.
- *Vyashtisamstih* – Part and Whole.
- *Yaavadunam* – Whatever the extent of its deficiency and the Up-sutras

The Sub Sutras are

- *Anurupyena*
- *Shishyate Sheshsamjnah*
- *Adyamadye Nantyamantyena*
- *Kevalaih Saptakam Gunyat*
- *Vestanam*
- *Yavadunam Tavadunam*
- *Yavadunam Tavadunikutya Varganka ch Yojayet*
- *Antyayordhshakepi*
- *Antyatoreva*
- *Samucchayagunitah*
- *Lopansthanabhyam*
- *Vilokanam*
- *Gunitasamuchyah Samucchayagunitah*

Decimal Multiplication

In Vedic mathematics there are 3 methods to implement multiplication for decimal numbers. The main purpose of

Vedic Mathematics is to be able to solve complex calculations by simple techniques which can be done mentally. These Vedic formulas require dealing with very small numbers. Hence if the formulas being very short then their practical application becomes very simple. Urdhva triambakam Sutra means "Vertically and Crosswise" where Urdhva means vertically up down and triambakam means left to right or vice versa. To illustrate this multiplication procedure, consider the multiplication of two decimal numbers (327 * 712). The Line diagram for the multiplication using Urdhva triambakam is shown in Fig. 1. The digits on the two ends of the line are multiplied and the result is added with the previous carry. If there are more lines in one step, all the results are added to the previous carry. The obtained least significant digit of the number acts as one of the result digits and the rest act as the carry for the next step. Initially the carry is taken to be zero.

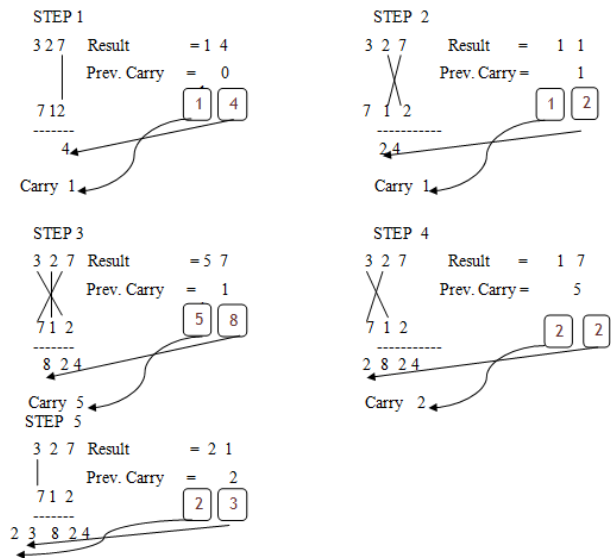


Figure 1 : Urdhvatriambakam

Nikhilam Sutra

Nikhilam Navatashcaram Dashatah means all from 9 and last from 10. This algorithm works for all numbers but it works efficiently for larger numbers. Since it finds out the compliment of the large number from its nearest base to perform the multiplication operation on it, larger is the original number, lesser the complexity of the multiplication. The procedure for Nikhilam Sutra is

1. Take the base of calculation as power of 10 which is nearest to the multiplicands say M and N.
2. Subtract base B from each multiplicand and note two remainders as say M' and N'.
3. Column 1 contains the numbers and Column 2 contain difference from the nearest base.
4. The product will have two parts. Right part (R) is obtained by multiplication of two remainders namely m and n . i.e. $R = m \times n$
5. Left part (L) can be obtained by cross subtracting the second number of Column 2 from the first number of column 1 or vice versa,
6. The answer is obtained by just concatenating left and right part as LR.

The improvement of these methods over conventional multiplication largely depends on closeness of multiplicands to the power of 10.

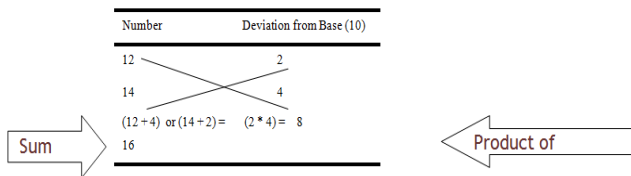
The illustration of this sutra can be given through three cases as :

Case 1: When both the deviations are positive

- Let M & N are 12 and 14.
- These two numbers are nearer to the base 10 with deviation of 2 and 4 respectively
- Add one of the numbers to the deviation of the other.
- Place these numbers side by side : 168

| Number | Deviation from Base (10) |
|--------|--------------------------|
| 12 | (12 - 10) = 2 |
| 14 | (14 - 10) = 4 |
| 8 | |

Product of

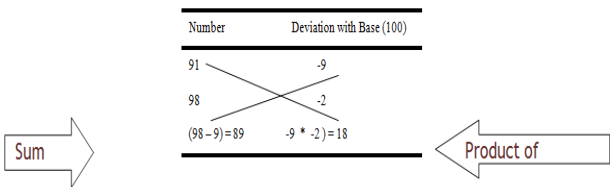


Case 2: When both the deviations are negative.

- Let M & N are 91 and 98. These two numbers are nearer to the base 100 with deviation of -9 and -2.
- Add one of the numbers to the deviation of the other.
- Place these numbers side by side : 8918

| Number | Deviation with Base (100) |
|----------------|---------------------------|
| 91 | (91 - 100) = -9 |
| 98 | (98 - 100) = -2 |
| (-9 * -2) = 18 | |

Product of

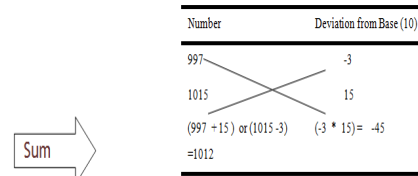


Case 3: When both the deviations have different signs.

- Let m & n are 997 and 1015. These two numbers are nearer to the base 1000 with deviation of -3 and 15.
- Add one of the numbers to the deviation of the other
- The sum and the product have different signs, so they cannot be placed side by side to get the answer. Hence, to get the answer subtract 45 from 101200 after adding two zeros to sum. i.e (101200 - 45 = 1011955)

| Number | Deviation with Base (1000) |
|-----------------|----------------------------|
| 997 | (997 - 1000) = -3 |
| 1015 | (1015 - 1000) = 15 |
| (15 * -3) = -45 | |

Product of



Anurupyena

The reduction in calculation for the multiplication by *Nikhilam* solely depends on closeness of multiplicands, if not both at least one, to the base which has to be power of 10. In case both the multiplicands are not near power of 10, two large remainders are obtained and their multiplication is not a straightforward task. *Anurupyena* means proportionality. In application sense it means while consideration the base and calculation of left part of multiplication, a rational proportionality can be used to reduce the calculations. In other words, while calculating the remainders from multiplicands a base can be chosen as some rational multiple of power of 10 such that multiplication of remainders becomes simple. In turn, while calculating the right part of multiplication same proportionality needs to be considered to calculate correct answer. For example, if a suitable base was found to be k times power of 10, right part has to be multiplied by k times to calculate answer. Suppose we have to multiply 43 by 49, by *Nikhilam* formula, this multiplication translates into multiplication $(10^2 - 43) * (10^2 - 49)$ i.e., 57×51 for the left part and right part also has two bigger numbers to deal with. This happens as both multiplicands are far away from power of 10, here 100. With some observation we can say that base 50 could be effective as it is closer to at least one of the multiplicand, here it is closer to both. With this corollary we make the base suitable by some multiple of power of 10. Here the multiple is $\frac{1}{2}$. While calculating left part this constant has to be multiplied with number obtained by cross adding multiplicand with remainder.

Booth multiplication Algorithm for binary numbers :

The algorithm gives a procedure for multiplying binary integers in signed - 2's complement representation. The algorithm examines the multiplier bits and performs shifting of the partial product bits. Prior to shifting, the multiplicand may be added to the partial product, subtracted from the partial product or left unchanged according to the following rules:

- The multiplicand is subtracted from the partial product upon encountering the first least significant 1 in a string of 1's in the multiplier.
- The multiplicand is added to the partial product upon encountering the first 0 in a string of 0's in the multiplier.
- The partial product does not change when the multiplier bits is identical to the previous multiplier bit.

Iterative Binary Multiplication Algorithm based on Nikhilam Sutra :

1. Read the two multiplicands say X and Y.
2. Let LZ represent the number of least significant zeros in the numbers.
3. Count, set LZ and then discard the least significant zeros in both numbers.
4. While (|X| and |Y| are greater than 1) do begin
 - 4.1 Identify the smaller number.
 - 4.2 If the second most significant bit of the smaller number is 1 then base = b else base = b-1 where b is the number of bits in the smaller number.
 - 4.3 Product = 0
 - 4.4 If X & Y are with different signs, then assign sign ← -1 else assign sign ← 1.
 - 4.5 If the smaller number is positive


```
{
Psum = X + Y - 2base
X = X - 2base
Y = Y - 2base
}
```

 else


```
{
Psum = X + Y + 2base
X = X + 2base
Y = Y + 2base
}
```

 endif
 - 4.6 Product = Product + Psum * 2^{base} * sign
- End // end while //
5. If ((X != 0) & (Y != 0)) Product = Product + (|X| + |Y| - 1)
6. Product = Product * 2^{LZ}
7. Stop

The illustration of this algorithm can be given through the following:

Let X & Y are (997)₁₀ and (499)₁₀

| Iteration | Iteration 0 | Iteration 1 | Iteration 2 | Iteration 3 |
|-----------|---------------------------------|----------------------------------|------------------------------------|------------------------|
| X | 997 = (1111100101) ₂ | (111100101) ₂ | (111110101) ₂ | 111110001 ₂ |
| Y | 499 = (111110011) ₂ | (-1101) ₂ | (11) ₂ | (-01) ₂ |
| Base | 9 | 4 | 2 | 0 |
| Psum | 1111011000 | 111101000 | Psum = 111110110 | - 111110001 |
| Product | 1111011000 * 2 ⁹ | 111100100011000 * 2 ⁴ | 11110011001010100 * 2 ² | 111100101110101111 |

Conclusion

Ancient Indian mathematics, namely Vedic Mathematics can be applied in the Hardware design of processor. The algorithms based on conventional mathematics can be simplified and optimized by the use of Vedic sutras. One such application is the Nikhilam Sutra where the computation of two large numbers is reduced to multiplication of two small numbers. But this method demands for closeness of the multiplicands to nearer to power of 10. The developed iterative algorithm is similar to popular array multiplier where an array of address is required to arrive at the final product.

For multiplication of large numbers the array multiplier suffers with carry propagation delay. This can be solved by the using the developed iterative binary multiplication algorithm

REFERENCES

Aniruddha Kanhe, Shishir Kumar Das and Ankit Kumar Singh. 2012, Design and Implementation of low power Multiplier using Vedic Multiplication technique. International Journal of Computer Science and Communication, 3(1):131-132.

Chandra Hari. 1999. A Critical study of Vedic Mathematics of Sankaracharya Sri Bharati KRSNA Tirthaji Maharaj.

Ganesh Kumar,G. and Charishma, V. 2012. Design of high speed Vedic multiplier using Vedic Mathematics techniques, International Journal of Scientific and Research publications, 2(3):1-5.

Harpreet Singh Dhillon and Abhijit Mitra. 2008. A Reduced bit multiplication Algorithm for Digital Arithmetic, International Journal of Computational and Mathematical Sciences, 2(2) : 64-68.

Jagadguru Swami Sri Bharati Krisna Tirthaji Maharaja, 1965. Vedic Mathematics: Sixteen Simple Mathematical Formulae from the Veda.

Kabiraj Sethi and Rutuparna Panda. 2012. An improved squaring circuit for binary numbers, International Journal of Advanced Computer Science and Applications, 3(2):111-116.

Kerur,S.S. Prakash Narchi. Jayashree, C.N. Harish, M. Kittur. Girish, V.A. 2011. Implementation of Vedic Multiplier for Digital Signal Processing. IJCA Proceedings on International Conference on VLSI , Communication and Instrumentation . 16: 1-5.

Manoranjan Pradhan, Rutuparna Panda and Sushanta Kumar Sahu. 2011. Speed Comparison of 16x16 Vedic Multipliers, International Journal of Computer Applications. 21(6):16-19.

Parth Mehta and Dhanashri Gawali. 2009. Conventional versus Vedic Mathematical Method for Hardware Implementation of a Multiplier, International Conference. on Advances in Computing, Control and Telecommunication, India : 640-642.

Prabha S. Kasliwal, Patil B.P and Gautam, D.K. 2011. Performance evaluation of squaring operation by vedic mathematics, IETE Journal of research. 57(1):39-41.

Rana Mukherji, Amit Kumar Chatterjee, Manishita Das. 2011. Implementation of an efficient multiplier architecture based on Ancient Indian Vedic Mathematics using System C.KIST J ourl of Science and Technology . 1(1): 47-57.

Ritika Jasrotia,Saptrishi Chakrabaroty, Rishabh Sharma and Dushyant Dutta. 2011. Vedic Mathematics: The Future Computing Model, CSI Communications:16 - 17.

Saha,P., Banerjee, A. Dandapat,A. and Bhattacharyya, P. 2011. Vedic Mathematics Based 32-Bit Multiplier Design for High Speed Low Power Processors. International Journal of Smart Sensing and Intelligent System. 4(2): 268 - 284.
