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RESEARCH ARTICLE

CONSTRUCTED AN ALGORITHM FOR FINDING A NON-SPLIT DOMINATING SET OF A CIRCULAR-ARC GRAPH

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ABSTRACT

In graph theory, a connected component of an undirected graph is a sub graph in which any two vertices are connected to each other by paths. For a graph G, if the sub graph of G itself is a connected component then the graph is called connected, else the graph G is called disconnected and each connected component sub graph is called it's components. Circular-arc graphs have variety of applications involving traffic light sequencing, genetics etc. A dominating set D of graph G = (V,E) is a non-split dominating set if the induced sub graph < V-D > is connected. The non-split domination number $\Upsilon_{ns}(G)$ of G is the minimum cardinality of a non-split dominating set .In this paper constructed an algorithm for finding a non-split dominating set of an Circular-Arc graph. Also its relationships with other parameters is investigated.

INTRODUCTION

Consider A = { A_1, A_2, …, A_n } a family of arcs on a circle C. Each endpoint of the arc A_i is assigned a positive integer called a co-ordinate. The endpoints are located at the circumference of C in ascending order of the values of the co-ordinates in the clockwise direction. Suppose that an arc begins at c and ends at point d in the clockwise direction. Then we denote such an arc by [c, d] and the points c and d arc called respectively the head point and tail point of the arc. The arcs are given labels in the increasing order of their head points. If the head point of an arc is less than the tail point of the arc then the arc is called a forward arc. Otherwise it is called a backward arc. A is called a proper arc family if no arc in A contains another arc. A graph G=(V,E) is called a circular-arc graph if there is a one - to - one correspondence between V and A such that two vertices in V are adjacent if and only if their corresponding arcs in A intersect. Let us now denote the arc family by A = {1, 2, …, n}, where arc i=A_i and G is its corresponding Circular-arc graph. We assume that G is a connected graph. Circular-arc graphs have been studied in [1][2]. Let G = (V, E) be a graph. A set D $\subseteq$ V is a dominating set [3] of G if every vertex in < V-D > is adjacent to some vertex in D. The domination number $\Upsilon(G)$ of G is the minimum cardinality of a dominating set. A dominating set D is connected dominating set if the induced sub graph < D > is connected. The connected domination number [4] $\Upsilon_c(G)$ of G is the minimum cardinality of a connected dominating set. A dominating set D of a graph G = (V,E) is a split dominating set if the induced sub graph < V-D > is disconnected. The split domination number [5] $\Upsilon_s(G)$ of G is the minimum cardinality of a split dominating set. The purpose of this paper is to introduce the concept of constructed an algorithm for finding a non- split dominating set of circular-arc graph. A dominating set D of a graph G=(V,E) is a non-split dominating set, if the induced sub graph < V-D > is connected . The non-split domination number $\Upsilon_{ns}(G)$ of G is the minimum cardinality of a non-split dominating set. Kulli .V.R et.all [6] introduced the concept of split and non split domination in graphs and also in Maheswari, *et al.* [7]. The neighbourhood of a vertex v$\in$V is set consisting all vertices adjacent to v (including v). It is denoted by nbd[v]. Let nbd[i] be defined as the set of vertices adjacent to i including i. A subset S of V in G is called a neighbourhood set of G if $G = \bigcup_{v\in S} nbd[v]$,where < nbd [v] > is the vertex induced subgraph of G. The neighbourhood number of G is defined as the minimum cardinality of a neighbourhood set of G. In addition, if the set S is independent then S is called an independent neighbourhood set of G. Let max (i) denotes the largest interval in nbd[i]. Gruprakash. C. D.,Mallikarjuna Swamy.B. P [8], Minimum Matching dominating sets and its apllications in wireless networks define Next(i) = j if and only if $b_i < a_j$ and there does not exist an interval k such that $b_i < a_k < a_j$. If there is no such j, we define Next(i)=null, this means that we should not have consider in terms of next (i). But we should have been considering in neighbourhood sets.

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Verification of an algorithm for finding a non-split dominating set

Input: Circular-arc family A = { A_1, A_2, …, A_n }.
Output: Non-split dominating set and induced sub graph is connected.
Step1: V={v_1,v_2,...,v_n}, T={ j/j∈nbd[1] and forward arc },
P={j/j∈T and j∈nbd[k] ∀ k in T}
Step2: S = The least largest degree interval in P
Step3 : LI = The largest interval in S
Step4: Compute Next (LI). If Next(LI) ∈ nbd[min(S)] then go to step 8
Step5: If Max (Next(LI)) ∈ nbd[r],∀ r∈S
Step6: Max (Next(LI)) = {Max (Next(LI))} –{1} go to step 5
Else go to next step
Step7: S=S ∪ Max (Next(LI)) go to step 3
Step8: V-S = {s_1,s_2,….,$s_{n-ϒ}$}, where ϒ = |S|
Step9: for i = 1 to n-ϒ-1
{
for j = i+1 to n-ϒ
{
If (s_i,s_j) ∈ E(G), then join from s_i to s_j
}
} The induced graph G_1 is obtained
Step10: If w(G_1)=1 (Here w(G_1) indicates number of components)
Therefore the graph G_1 is connected.
Hence S is non-split dominating set.
Else
S is split dominating set
Step11: End.

Main Theorems

Theorem1: Let A={A_1, A_2,….,A_n} be a circular-arc family and G is circular-arc graph corresponding to A. If A_i and A_j are any two arcs in A such that A_j is contained in A_i and A_i∈D and if A_l is any arc in A which is to the left of A_j in clock wise direction such that A_l < A_j and A_l intersect A_j and if there is at least one A_k > A_j such that A_k intersects A_j then non-split domination occurs in G.

Proof: Let A={A_1, A_2,….,A_n } be a circular-arc family and G be circular-arc graph corresponding to A. If A_i and A_j are any two arcs in A such the A_j is contained in A_i and A_i∈D. Suppose there is at least one arc A_k ≠ A_i, A_k > A_j such that A_k intersect A_j then it is obvious that in the induced sub graph < V–D > , A_k is adjacent to A_j. By hypothesis there is at least one A_m < A_j in < V-D > such that A_m intersects A_j. Hence A_j is connected to it's left as well as to it's right. So that there will not be disconnection in the induced sub graph < V–D > .This proves directly. Further find the non-split dominating set apart from an algorithm by using neighbourhood system of a circular-arc graph G corresponding to circular-arc family. The procedure as follows with an illustration.

Illustration

We can draw a circular-arc graph from a circular-arc family .

nbd[1]={1,2,3,10,11},
nbd[2]={1, 2, 3, 4},
nbd[3] ={1,2,3,4,5},
nbd[4]={2,3,4,5,6},
nbd[5]={3,4,5,6,7},
nbd[6]={4,5,6,7},
nbd[7]={5,6,7,8,9},
nbd[8]={7,8,9,10,11},
nbd[9]={7,8,9,10},
nbd[10]={1,8,9,10,11},
nbd[11]={1,8,10,11}

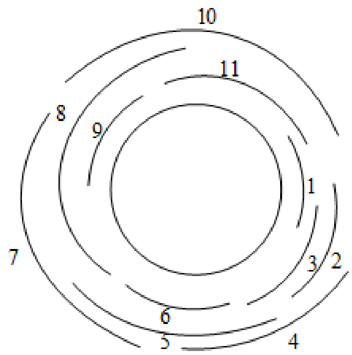

Figure 1.Circular-arc family

Max(1)=11, Max(2)=4, Max(3)=5,Max(4)=6, Max(5)=7,Max(6)=7 Max(7)=9,Max(8)=11 Max(9)=10, Max(10)=11,Max(11)=11.
Next(1)=4, Next(2)=5, Next(3)=6,Next(4)=7, Next(5) =8, Next(6)=8, Next(7)=10,Next(8)=null Next(9)= null, Next(10)= null, Next(11)= null

Procedure

Input**:** Circular-arc family A={1,2,3,4,5,6,7,8,9,10,11}
Step1: V = {1,2,….,11}, T = {1,2,3}, P = {1,2,3}
Step2: S = 1
Step3: LI = 1
Step4: Next (1) = 4
Step5 : Max(4) = 6
Step6: S={1}∪{6}={1,6} go to Step3
Step7: LI = 6
Step8: Next(6)=8
Step9= Max (8)=11-1=10-1=9 go to step3
Step10:S={1,6,9}
Step11:V-S={2,3,4,5,7,8,10,11}
Step12: for i=1 , j = 2 ,(s_1,s_2) = (2,3) ∈ E(G) , join from 2 to 3
j = 3 ,(s_1,s_3) = (2,4) ∈ E(G) , join from 2 to 4
for i=2, j = 3, (s_2,s_3) = (3,4) ∈ E(G) , join from 3 to 4
j = 4, (s_2,s_4) = (3,5) ∈ E(G) , join from 3 to 5
for i=3, j = 4, (s_3,s_4) = (4,5) ∈ E(G) , join from 4 to 5
for i=4, j = 5, (s_4,s_5) = (5,7) ∈ E(G) , join from 5 to 7
for i=5, j = 6, (s_5,s_6) = (7,8) ∈ E(G) , join from 7 to 8
for i=6, j = 7, (s_6,s_7) = (8,10) ∈ E(G) , join from 8 to 10
j = 8, (s_6,s_8) = (8,11) ∈ E(G) , join from 8 to 11
for i=7, j = 8, (s_7,s_8) = (10,11) ∈ E(G) , join from 10 to 11

The induced subgraph G_1 is obtained
Step13: w(G_1)=1
Therefore the graph G_1 is connected.
Hence S={1,6,9} is a non-split dominating set.
Step 14: End
Output: S={1,6,9} is a non-split dominating set and G_1 is connected.

Theorem2: Let A be a circular-arc family and G be a circular-arc graph corresponding to A. If $A_j = 2$ is the arc contained in $A_i = 1$ and $A_i \in D$, where D is a dominating set and there is at least one arc that intersects A_j it's right other than A_i then non-split domination occurs in G.

Proof: Let A be circular-arc family and G is a circular-arc graph corresponding to A. If $A_j = 2$ is the arc contained in $A_i=1$ and $A_i \in D$, where D is a dominating set. Suppose A_k is any arc $A_k \neq A_i$, $A_k > A_j$ such that A_k intersects A_j. Then in the induced sub graph $< V - D >$, A_j is connected to A_k, so that there is no disconnection for A_j to it's right in clockwise direction. After that we prove that the above procedure towards as algorithm and we will verify as the following.

Illustration

We can draw a circular-arc graph from a circular-arc family
nbd[1]={1,2,3,4,10},nbd[2]={1,2,3},nbd[3]={1,2,3,4},
nbd[4]={1,3,4,5},nbd[5]={4,5,6,7}nbd[6]={5,6,7,8},
nbd[7]={5,6,7,8,9},nbd[8]={6,7,8,9,10},
nbd[9]={7,8,9,10},nbd[10]={1,8,9,10}
Max(1)=10,Max(2)=3,Max(3)=4,Max(4)=5,Max(5)=7,Max(6)=8,Max(7)=9,Max(8)=10,Max(9)=10,Max(10)=10Next(1)=5,
Next(2)=4,Next(3)=5,Next(4)=6,Next(5)=8,
Next(6)=9, Next(7)=10,Next(8)=null,Next(9)=null,
Next(10)= null

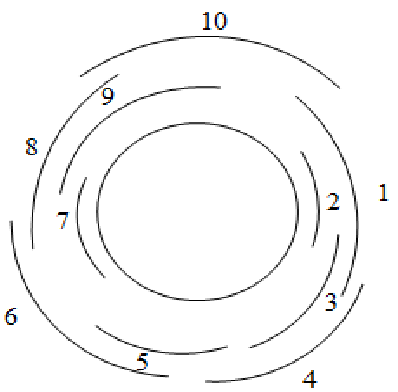

Figure 2. Circular-arc family

Procedure

Input: Circular-arc family A={1,2,3,4,5,6,7,8,9,10}
Step1: V = {1,2,….,10}, T = {1,2,3,4}, P = {1,3}
Step2: S = 1
Step3: LI = 1
Step4: Next (1) = 5
Step5 : Max(5) = 7
Step6: S=1∪7={1,7} go to Step3
Step7: LI = 7
Step8: Next(7)=10
Step9= Max(10)=10-1=9-1=8-1=7
Step10:S={1,7}
Step11:V-S={2,3,4,5,6,8,9,10}
Step12: for i=1 , j = 2, (s_1,s_2) = (2,3) ∈ E(G) , join from 2 to 3
for i=2, j = 3, (s_2,s_3) = (3,4) ∈ E(G) , join from 3 to 4
for i=3, j = 4, (s_3,s_4) = (4,5) ∈ E(G) , join from 4 to 5
for i=4, j = 5, (s_4,s_5) = (5,6) ∈ E(G) , join from 5 to 6
for i=5, j = 6, (s_5,s_6) = (6,8) ∈ E(G) , join from 6 to 8
for i=6, j = 7, (s_6,s_7) = (8,9) ∈ E(G) , join from 8 to 9
j = 8, (s_6,s_8) = (8,10) ∈ E(G) , join from 8 to 10
for i=7, j = 8, (s_7,s_8) = (9,10) ∈ E(G) , join from 9 to 10
The induced subgraph G_1 is obtained
Step13: w(G_1)=1
Therefore the graph G_1 is connected.
Hence S={1,7} is a non-split dominating set.
Step14:End
Output: S={1,7} is a non-split dominating set and G_1 is connected.

Theorem3: Let A be circular-arc family and G be a circular-arc graph corresponding to A. If A_i, A_j, A_k are three consecutive forward arcs in A where A is a proper arc family such that $A_i < A_j < A_k$, $A_j \neq 1$and A_i and A_j intersect, A_j and A_k intersect but A_i and A_k do not intersect and $A_j \in D$ and there is atleast one backward arc in < V-D > then non-split domination occurs in G.

Proof: Let A be circular-arc family and G be a circular-arc graph corresponding to A. If A_i, A_j, A_k are three consecutive forward arcs in A where A is a proper arc family such that $A_i < A_j < A_k$, $A_j \neq 1$ and A_i and A_j intersect, A_j and A_k intersect but A_i and A_k do not intersect and $A_j \in D$ and there is atleast one backward arc in < V-D >. Since $A_j \in D$, A_i and A_k are disconnected in clockwise direction in < V-D >. However if there is a backward arc in < V – D > then there is a path between A_k and A_i in clockwise direction namely (A_k, A_i)-path, so that A_i and A_k are connected in < V-D >. Hence there is no disconnection in < V-D >. The above procedure can verify using a method of algorithms.

Illustration

We can draw a circular-arc graph from a circular-arc family
nbd[1]={1,2,3,10},nbd[2]={1,2,3,10},
nbd[3]={1,2,3,4}, nbd[4]={3,4,5,6}, nbd[5]={4,5,6,7},
nbd[6]={4,5,6,7,8},nbd[7]={5,6,7,8,9},nbd[8]={6,7,8,9,10},nbd [9]={7,8,9,10} nbd[10]={1,2,8,9,10}
Max(1) =10, Max(2)=10, Max(3)=4, Max(4)=6, Max(5)=7, Max(6)=8, Max(7)=9,Max(8)=10, Max(9)=10Max(10)=10
Next(2)=4, Next(3)=5,Next(4)=7, Next(5)=8, Next(1)=4,
Next(6)=9,Next(7)=10, Next(8)=null, Next(9)= null,
Next(10)= null

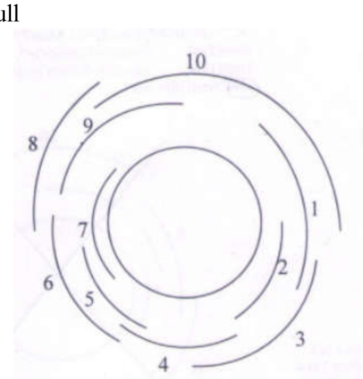

Figure 3. Circular-arc family

Procedure

Input: Circular-arc family A={1,2,3,4,5,6,7,8,9,10}
Step1: V = {1,2,….,10}, T = {1,2,3}, P = {1,2,3}
Step2: S = 1
Step3: LI = 1
Step4: Next(1) = 4

Step5 : Max(4) = 6
Step6: S={1}∪ {6} = {1,6} go to Step3
Step7: LI = 6
Step8: Next (6)=9
Step9:Max (9)=10-1=9
Step10:S = {1,6,9} go to step 3
Step11:V-S = {2,3,4,5,7,8,10}

Step12: for i=1 , j = 2 , (s_1,s_2) = (2,3) $\in$ E(G) , join from 2 to 3
j = 7, (s_1,s_7) = (2,10) $\in$ E(G) , join from 2 to 10
for i=2, j = 3, (s_2,s_3) = (3,4) $\in$ E(G) , join from 3 to 4
for i=3, j = 4, (s_3,s_4) = (4,5) $\in$ E(G) , join from 4 to 5
for i=4, j = 5, (s_4,s_5) = (5,7) $\in$ E(G) , join from 5 to 7
for i=5, j = 6, (s_5,s_6) = (7,8) $\in$ E(G) , join from 7 to 8
for i=6, j = 7, (s_6,s_7) = (8,10) $\in$ E(G) , join from 8 to 10
j = 8, (s_6,s_8) = (8,10) $\in$ E(G) , join from 8 to 10
for i=7, j = 8, (s_7,s_8) = (9,10) $\in$ E(G) , join from 9 to 10

The induced subgraph G_1 is obtained
Step13: w(G_1)=1
Therefore the graph G_1 is connected.
Hence S={1,6,9} is a non-split dominating set.
Step14: End
Output: S={1,6,9} is a non-split dominating set and G_1 is connected

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