ISSN: 0975-833X

Available online at http://www.journalcra.com

International Journal of Current Research
Vol. 9, Issue, 02, pp.46080-46086, February, 2017

INTERNATIONAL JOURNAL OF CURRENT RESEARCH

RESEARCH ARTICLE

KAMAL TRANSFORM INTEGRATED WITH ADOMIAN DECOMPOSITION METHOD FOR SOLVING PREDATOR-PREY SYSTEMS

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ARTICLE INFO

Article History:
Received 06th November, 2016
Received in revised form
29th December, 2016
Accepted 19th January, 2017
Published online 28th February, 2017

Key words:
Kamal Transform,
Adomian Decomposition Method,
Predator-Prey Systems,
Nonlinear Systems.

ABSTRACT

In this paper, we use Kamal transform and Decomposition Method (KTADM), which is integrated of Kamal transform and Decomposition Method for solving the approximate numerical solutions of Predator-Prey systems. We can easily decompose the nonlinear terms by the help of special kind of Adomian polynomials. This technique provides a sequence of functions, which converges fast to the accurate solution of the problems. Finally, numerical examples prepared to illustrate and given to show the effectiveness and applicability of this method in solving these kind of systems.

Citation: Zahra.I. Adam Mahamoud and Abdelilah K. Hassan Sedeeg, 2017. "Kamal transform integrated with Adomian decomposition method for solving predator-prey systems", *International Journal of Current Research*, 9, (02), 46080-46086.

INTRODUCTION

Recently, differential equations have become highly crucial and widely utilized in many aspects in our daily life .In recent years, a lot of scientists and engineers have focused on some numerical methods to solve plenty of nonlinear ordinary differential equations and partial differential equations that are not susceptible to analytical solution in any reasonably appropriate manner. These numerical methods give generally approximate solutions which converge fast to exact solutions easily and correctly. One of these numerical methods is Adomian decomposition method which has been introduced by George Adomian (Adomian, 1988; Adomian, 2013). Adomian decomposition method has been applied to a large class of linear and nonlinear equations that are useful for many research work (Hassan *et al*., 2016; Bulut *et al*., 2004; Asil *et al*., 2005). The essential property of this method is the ability to solve these equations appropriately and accurately. Systems of ordinary differential equations have been appeared frequently in a wide class of scientific applications in physics, engineering and other fields. So, their importance cannot be undervalued. Adomian decomposition method has been also used efficiently for solving systems of differential equations (Biazar, 2004). After the invention of the computer and the successful utilization of it in mathematics, again, mathematicians made their best to drive many transformations that solve differential equations. One of the newly inferred transformation is Kamal transform was introduced by Abdelilah Kamal in 2016, to facilitate the process of solving ordinary equations and partial differential equations in the time domain. This transformation has deeper connection with the Laplace and Sumudu Transform (Abdelilah Kamal, 2016; Abdelilah Kamal, 2017). The Predator-Prey equations a first-order, nonlinear differential equations frequently used to model the dynamics of ecological systems. This model is one of the most interesting and important application of stability theory involves the interactions between two or more biological populations. It shows the situation in which one species (the predator) preys on the other species (the prey), while the prey lives on a different source of food.

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These kind of examples contain rather simple equations, but they characterize a wide class of problems of competing species (Boyce *et al*., 1992; Edwards *et al*., 2008; Necdet Bildik and Sinan Deniz, 2016). The main idea of this paper we use (KTADM) to get the solutions of Predator-Prey systems, and Mathematica program to graph that shows the relations between the number of predators and the preys in time.

Fundamental Facts of Kamal Transform

A new transform called Kamal transform is defined on the set of functions:

$$A = \left\{ f(t) : \exists M, k_1, k_2 > 0, |f(t)| < Me^{\frac{|t|}{k_j}}, if \ \ t \in (-1)^j \times [0, \infty) \right\} \tag{1}$$

by the following formula

$$K[f(t)] = \int_0^\infty f(t) e^{\frac{-t}{v}} dt = G(v). \quad t \geq 0 \quad , \quad k_1 \leq v \leq k_2 \tag{2}$$

Theorem 2.1:

Let $G(v)$ is Kamal transform of $f(t)$ then :

$$K[f^{(n)}(t)] = v^{(-n)} G(v) - \sum_{k=0}^{n-1} v^{k-n+1} f^{(k)}(0) \text{ for } \quad n \geq 0 \tag{3}$$

For $n = 1$, $K[f'(t)] = \frac{1}{v} G(v) - f(0)$

We use this theorem to get the approximate solution of given problems. Additionally, the properties and theorems of Kamal transform and its derivatives we refer to (Abdelilah Kamal, 2016; Abdelilah Kamal, 2017).

Basic Idea of Kamal Transform and Adomian Decomposition Method (KTADM)

We consider the particular form of inhomogeneous nonlinear ordinary differential equation with initial condition given below:

$$Ly(t) + Ry(t) + Ny(t) = f(t) \tag{4}$$

With the following initial conditions

$$y(0) = a$$

Where L is the first order derivative and also is invertible, R is a linear differential operator, Ny represents the nonlinear terms and $f(t)$ is the source term. Applying Kamal transform to the both sides of the Eq.(4), we get

$$K[Ly(t) + Ry(t) + Ny(t)] = K[f(t)] \tag{5}$$

Using the differentiation property of Kamal transform and above initial conditions, we have:

$$K[y(t)] = vK[f(t)] + av - vk[Ry(t) + Ny(t)] \tag{6}$$

Operating with Kamal inverse on both sides of Eq.(6) gives:

$$y(t) = a + K^{-1}[vK[f(t) - Ry(t) - Ny(t)]] \tag{7}$$

So, we may represent the solution as an infinite series:

$$y(t) = \sum_{n=0}^{\infty} y_n(t) \tag{8}$$

And the nonlinear term can be decomposed as:

$$Ny(t) = \sum_{n=0}^{\infty} A_n(y) \tag{9}$$

Where $A_n(y)$are Adomian polynomials of $y_0, y_1, y_2 \cdots$ and we given by:

$$A_n = \frac{1}{n!}\frac{\partial^n}{\partial \lambda^n}[N(\sum_{n=0}^{\infty} \lambda^i y_i)]_{\lambda=0} \ , n = 0,1,2, \dots \tag{10}$$

Substituting Eqs. (8) and (9) in Eq. (7) we get:

$$\sum_{n=0}^{\infty} y_n(t) = F(t) - K^{-1}[vK[R\sum_{n=0}^{\infty} y_n(t) + \sum_{n=0}^{\infty} A_n]] \tag{11}$$

where $F(t)$ is the term arising from the source term and the given initial condition. On comparing both sides of last equation and by using standard ADM we have:

$$y(t) = F(t) \tag{12}$$

$$y_1(t) = -K^{-1}[vK[Ry_0(t) + A_0]] \tag{13}$$

$$y_2(t) = -K^{-1}[vK[Ry_1(t) + A_1]] \tag{14}$$

And the general relation is given by:

$$y_{n+1}(t) = -K^{-1}[vK[Ry_n(t) + A_n]], n \geq 0 \tag{15}$$

Finally, applying Kamal transform of the right hand side of the last equation and then taking inverse Kamal transform, we get $y_0, y_1, y_2 \cdots$ which are the series form of the desired solutions.

Numerical Examples

In this section we discuss some examples to illustrate Kamal transform and Decomposition Method. We select small numbers for initial conditions to make the calculations easier, also consider four terms approximation to the solutions.

Example 4.1.

Let's consider the predator-prey model:

$$\begin{cases} \dot{x} = \frac{dx}{dt} = x(2-y) \\ \dot{y} = \frac{dy}{dt} = y(-3+x) \end{cases} \tag{16}$$

With initial conditions:

$$x(0) = 1\,, \ \ y(0) = 2 \tag{17}$$

First let us take Kamal transform of above system as:

$$\begin{aligned} K[\dot{x}] &= \frac{1}{v}K(x) - x(0) = K[x(2-y)] \\ K[\dot{y}] &= \frac{1}{v}K(y) - y(0) = K[y(-3+x)] \end{aligned} \tag{18}$$

By applying the initial condition, we get

$$\begin{aligned} K(x) &= v + vK[2x - xy] \\ K(y) &= 2v + vK[-3y + xy] \end{aligned} \tag{19}$$

Taking inverse Kamal transform of last equations, then we get:

$$\begin{aligned} x(t) &= 1 + K^{-1}[vK[2x - xy]] \\ y(t) &= 2 + K^{-1}[vK[-3y + xy]] \end{aligned} \tag{20}$$

If we assume an infinite series solution of the unknown functions:

$$\begin{aligned} \sum_{n=0}^{\infty} x_n(t) &= 1 + K^{-1}[vK[2\sum_{n=0}^{\infty} x_n(t) - \sum_{n=0}^{\infty} A_n]] \\ \sum_{n=0}^{\infty} y_n(t) &= 2 + K^{-1}[vK[-3\sum_{n=0}^{\infty} y_n(t) + \sum_{n=0}^{\infty} A_n]] \end{aligned} \tag{21}$$

where A_n are Adomian polynomials that represents nonlinear term and the first three components of the Adomian polynomials are given as follows:

$$A_0 = x_0y_0,\ A_1 = x_1y_0 + x_0y_1\ ,\ A_2 = x_0y_2 + x_1y_1 + x_2y_0 \tag{22}$$

The recursive relation can be written as:

$$x_0(t) = 1$$
$$x_{n+1}(t) = K^{-1}[vK[2x_n(t) - A_n]]$$

And,

$$y_0(t) = 2$$
$$y_{n+1}(t) = K^{-1}[vK[-3y_n(t) + A_n]]$$

Calculating the necessary terms:

$$A_0 = 2\ ,$$

$$x_1(t) = K^{-1}\big[vK[2x_0(t) - A_0]\big] = 0$$
$$y_1(t) = K^{-1}\big[vK[-3y_0(t) + A_0]\big] = -4t$$
$$A_1 = -4t\ ,$$
$$x_2(t) = K^{-1}\big[vK[2x_1(t) - A_1]\big] = 2t^2$$
$$y_2(t) = K^{-1}\big[vK[-3y_1(t) + A_1]\big] = 4t^2$$

Proceeding in a similar manner, we have:

$$A_3 = 8t^2\ ,$$
$$x_3(t) = -\frac{4t^3}{3}$$
$$y_3(t) = -\frac{4t^3}{3}$$
$$\vdots$$

Therefore, the solutions are given by:

$$x(t) = 1 + 2t^2 - \frac{4t^3}{3} + \cdots \tag{23}$$
$$y(t) = 2 - 4t + 4t^2 - \frac{4t^3}{3} + \cdots \tag{24}$$

For the initial values $x(0) = 1.0$ and $y(0) = 2.0$ in Eqs. (23), (24) by using Kamal transform method, the sample values are tabulated (Table 1).

Table 1. (Predator- Prey)

t	$x(t)$	$y(t)$
0	1.0000	2.0000
0.1	1.0187	1.6387
0.2	1.0693	1.3493
0.3	1.1440	1.1240
0.4	1.2347	0.9547
0.5	1.3333	0.8333
0.6	1.4320	0.7520
0.7	1.5227	0.7027
0.8	1.5973	0.6773
0.9	1.6480	0.6680
1	1.6667	0.6667

We sketch the following graph that shows the relations between the number of predators and the preys in time. As the graph states that the number of predators increases, as the number of preys decrease. Predators will reach

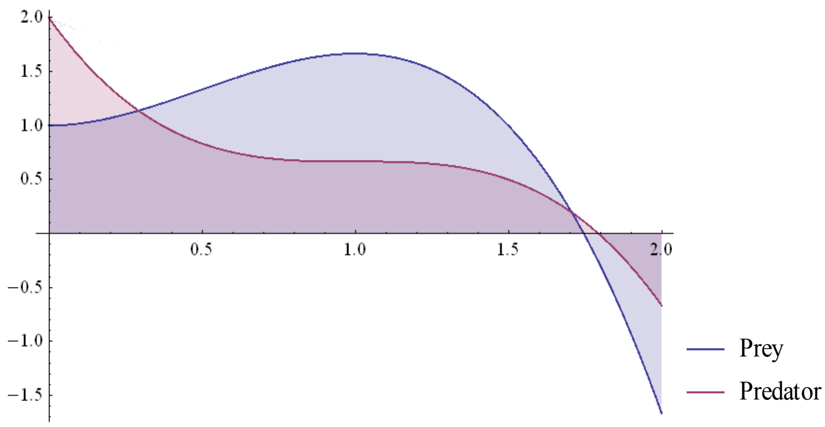

Fig. 1. Predator- Prey

Their maximum as the preys reach their minimum. Since the number of preys has decreased, there is not enough food for predators. So their number would decrease and so on.

Example 4.2:

Let's consider the predator-prey model:

$$\begin{cases} \dot{x} = \frac{dx}{dt} = -x(1+x+y) \\ \dot{y} = \frac{dy}{dt} = y(1+y) \end{cases} \tag{25}$$

Which is equivalent to Emden –Fowler equation of astrophysics

$$(\xi^2\eta')' + \xi^\lambda \eta^n = 0 \tag{26}$$

where, $\lambda = n = 0$, with initial conditions:

$$x(0) = 2, \; y(0) = 3 \tag{27}$$

By applying the previous method subject to the initial condition, we have

$$\begin{aligned} x(t) &= 2 + K^{-1}\left[vK[-x - x^2 - xy]\right] \\ y(t) &= 3 + K^{-1}\left[vK[y + y^2]\right] \end{aligned} \tag{28}$$

If we assume an infinite series solution of the unknown functions:

$$\begin{aligned} \textstyle\sum_{n=0}^{\infty} x_n(t) &= 2 + K^{-1}\left[vK\left[-\textstyle\sum_{n=0}^{\infty} x_n(t) - \sum_{n=0}^{\infty} A_n - \sum_{n=0}^{\infty} \tilde{A}_n\right]\right] \\ \textstyle\sum_{n=0}^{\infty} y_n(t) &= 3 + K^{-1}\left[vK\left[\textstyle\sum_{n=0}^{\infty} y_n(t) + \sum_{n=0}^{\infty} \bar{A}_n\right]\right] \end{aligned} \tag{29}$$

Where $A_n, \widetilde{A}_n$ and $\bar{A}_n$ are Adomian polynomials that represent the nonlinear terms.

Now, applying the same procedure to get the recursive relation:

$$x_0(t) = 2$$
$$x_{n+1}(t) = K^{-1}[vK[-x_n(t) - A_n - \tilde{A}_n]]$$

And,

$$y_0(t) = 3$$
$$y_{n+1}(t) = K^{-1}[vK[y_n(t) + \bar{A}_n]]$$

Calculating the necessary terms:

$$A_0 = x_0^2 = 4, \tilde{A}_0 = x_0 y_0 = 6, \bar{A}_0 = y_0^2 = 9$$

Yielding,

$$x_1(t) = K^{-1}\left[vK\left[-x_0(t) - A_0 - \tilde{A}_0\right]\right] = -12t$$
$$y_1(t) = K^{-1}\left[vK\left[y_0(t) + \bar{A}_0\right]\right] = 12t$$

And,

$$A_1 = 2x_1x_0 = -48t \ ,\tilde{A}_1 = x_1y_0 + x_0y_1 = -12t, \quad \tilde{A}_1 = 2y_1y_0 = 72t$$
$$x_2(t) = K^{-1}\left[vK\left[-x_1(t) - A_1 - \tilde{A}_1\right]\right] = 36t^2$$
$$y_2(t) \ = K^{-1}\left[vK\left[y_1(t) + \bar{A}_1\right]\right] = 42t^2$$

Proceeding in a similar manner, we have:

$$A_2 = 2x_0x_2 + x_1^2 = 288t^2 \ , \tilde{A}_2 = x_0y_2 + x_1y_1 + x_2y_0 = 48t^2, \tilde{A}_2 = 2y_0y_2 + y_1^2 = 396t^2$$
$$x_3(t) = -124t^3$$
$$y_3(t) = 146t^3$$
$$\vdots$$

Therefore, the solutions are given by:

$$x(t) = 2 - 12t + 36t^2 - 124t^3 + \cdots \tag{31}$$

$$y(t) = 3 + 12t + 42t^2 + 146t^3 + \cdots \tag{32}$$

By taking values for t (time) and using Eqs. (30), (31), the values obtained for $x(t)$ and $y(t)$ are given in the following table (table 2).

Table 2. Predator-Prey model which is equivalent to Emden–Fowler equation of astrophysics

t	$x(t)$	$y(t)$
0	2.0000	3.0000
0.05	1.4745	3.7233
0.10	1.0360	4.7660
0.15	0.5915	6.2378
0.20	0.0480	8.2480

In Fig. 2, we can see that only the prey population progressively decreases and becomes extinct despite the inhesion of increasing predator population. This can be related to the effect of the prey death rate being greater than the conversion rate.

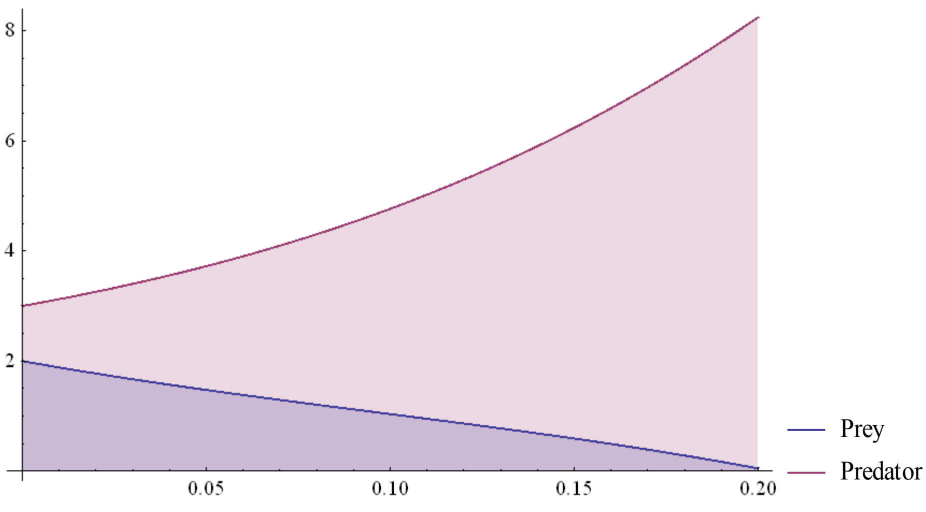

Fig. 2. Predator-Prey model which is equivalent to Emden–Fowler equation of astrophysics

Conclusion

In this paper, we present a mathematical model of the Prey-Predator system which is very important in applied sciences and we employ Kamal transform and Decomposition Method to obtain the approximate solutions of these equations. (KTADM) gives a new approach to the solution of these kinds of problems. We solve two examples and we use Mathematica Program to plot figures and to illustrate this technique and it is seen that the (KTADM) is very useful and effective method to get the approximate solutions. Thus, it can be applied to many complicated linear and nonlinear equations reliably.

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