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RESEARCH ARTICLE

GENERALIZED INVERSE SEMIGROUPS OF GREEN'S FUZZY RELATIONS

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ABSTRACT

Theories of fuzzy relations occupy a central position in the field of research and development of mathematics. Recently regular semigroup has become very important and offers valuable results by applying fuzzy properties of many of its concepts. This study is an extension of the work "On Green's Fuzzy Orthodox Semigroups". An important class of regular semigroups is an orthodox semigroup and some special classes of orthodox semigroups are a 'Generalized inverse semigroups'. It endeavours to find out some important results on fuzzy generalized inverse semigroup. In the field of regular semigroups fuzziness becomes very important. So the researcher's are very much interested in finding out many results, connecting special classes of fuzzy regular semigroups, the set of its inverses, the set of its idempotents, their correspondence and to establish theorems. Using the composition of fuzzy relations we get quotient group of semigroup of Green's fuzzy relations. In the ideal theory a semigroup S is simple when the only ideal of S is itself. In this sense the definitions fuzzy simple, fuzzy self-simple and fuzzy anti-simple are described using the new concept of Green's fuzzy equivalence classes. Using two proportions to get a quotient class of a generalized inverse semigroup from a Green's fuzzy anti-simple fuzzy congruence, on a generalized inverse semigroup, the study concludes by establishing four theorems with three equivalent conditions on a regular semigroup.

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INTRODUCTION

The preliminaries of the ideal theory and Green's relations are very important in basic concepts relating to inverse semigroups. Much of the well known materials are used directly by characterizing fuzzy property.

Definition 1.1. Regular elements of a semigroup

An element 'a' in a semigroup S is called regular, if there exists an element x in S such that $a\,x\,a = a$ (Clifford and Preston, 1967).

Definition 1.2 Regular semigroup

A semigroup is called a regular semigroup if all its elements are regular (Clifford and Preston, 1961).

Definition 1.3. Inverse of an element in a semigroup

An element a' in S is an inverse of a in S if $aa'\,a = a$ and $a'\,aa' = a'$. The set of inverse elements of an element $a \in S$ is denoted by $V_{(a)}$.

Definition 1.4. Inverse semigroup

A semigroup S is called an inverse semigroup if every element a in S posses a unique inverse. A semigroup S has at most one identity element. If S has no identity element we can adjoin an extra identity denoted by say 1 on the set S. Then for all $x \in S$. $1 = 1.x = x$; $1.1 = 1$. Such a semigroup obtained from S by adjoining an identity if necessary is denoted by S'.

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That is S'= **Idempotent in a semigroup** - An element 'e' in a semigroup is said to be an idempotent if e.e = e.

Definition 1.5 Regular band

A semigroup B is said to a band if all its elements are idempotents. If xyx = $x \forall x, y \in$ B, then B is called a regular band. This introduces fuzzy congruence (Kim and Bae, 1997; Kuroki, 1997) Green's $\hat{\mathcal{L}}$ and $\hat{\mathcal{R}}$ relations (Green, 1951; Hariprakash, 2016) to describe fuzzy $\hat{\mathcal{L}}$-class containing 'a' denoted by $\hat{\mathcal{L}}a$ and fuzzy $\hat{\mathcal{R}}$-class containing 'a' denoted by $\hat{\mathcal{R}}a$. (Hariprakas, 2016). In particular $\hat{\mathcal{L}}a(x) = \hat{\mathcal{L}}(a, x)$; $\hat{\mathcal{R}}a(x) = \hat{\mathcal{R}}(a, x)$ for all $x \in S$.

$\hat{\mathcal{L}}a = \{(a, x) \in S \times S : \hat{\mathcal{L}}(a, x) > 0\}$
$\hat{\mathcal{R}}a = \{(a, x) \in S \times S : \hat{\mathcal{R}}(a, x) > 0\}$

Moreover in the cited paper (Hariprakas, 2016), defined fuzzy equivalence classes $S/\hat{\mathcal{L}}$, $S/\hat{\mathcal{R}}$, $S/\hat{\mathcal{D}}$, $S/\hat{\mathcal{H}}$ by characterizing (fuzzify) simple semigroup, using fuzzy property.

Definition 1.6. L-fuzzy self simple and L - fuzzy simple semigroups

Let $\hat{\mathcal{L}}$ is a fuzzy congruence $\hat{\mathcal{L}}$- Green's relation on a semigroup S. $\hat{\mathcal{L}}$ is said to be $\hat{\mathcal{L}}$- fuzzy self simple if for any $a \in S$, $\mathbf{L}_a(a)$ =1 and **L**-fuzzy simple if for any $a \in S$.
$\hat{\mathcal{L}}a(x) = 1$. $\forall x \in S$. If $\mathbf{L}_a(x) = 1 \Rightarrow x = a$. where $\hat{\mathcal{L}}a(x) = 1$for some and not for all $x \in S$, then S is **L**- fuzzy antisimple.

Definition 1.7. $\hat{\mathcal{R}}$-fuzzy self simple and $\hat{\mathcal{R}}$-fuzzy simple semigroups.

A semigroup S is R -fuzzy self simple if $\hat{\mathcal{R}}_a(a) = 1 \ \forall \ a \in S$ and $\hat{\mathcal{R}}$-fuzzy simple if $\hat{\mathcal{R}}_a(x) = 1 \ \forall a \in S$, where $\hat{\mathcal{R}}$ is a fuzzy congruence $\hat{\mathcal{R}}$-Green relation on S.
If $\hat{\mathcal{R}}_a(x) = 1 \Rightarrow x = a$ where $\hat{\mathcal{R}}_a(x) = 1$ for some and not for all $x \in S$ then S is fuzzy antisimple.

2. Fuzzy Generalized Inverse Semigroups

In the field of research, the study of regular semigroup becomes very important and offers valuable results by applying fuzzy property on these concepts. An important class of regular semigroups constitutes orthodox semigroups, and a special class of orthodox semigroups is called generalized inverse semigroups. [10]. In this section we find out fuzzy generalized inverse semigroups.

Definition. 2.1 (Fuzzy generalized inverse semigroup) A generalized inverse semigroup in which a membership function is defined is called a generalized inverse semigroup.

Proposition 2.2. If $\hat{\mathcal{L}}$ is a fuzzy congruence on a generalized inverse semigroup S and is $\hat{\mathcal{L}}$-fuzzy antisimple, $S/\hat{\mathcal{L}}$ is also a generalized inverse semigroup.
Proof. Given S is a generalized inverse semigroup. That is, S is an orthodox semigroup and the set E_s of its idempotents is normal. That is, for any $e,f,g,h \in E_s$, $efgh = egfh$. Since S is an orthodox semigroup and $\hat{\mathcal{L}}$-fuzzy antisimple $S/\hat{\mathcal{L}}$ is an orthodox semigroup (Hariprakash, 2016). Let $E_{S/\hat{\mathcal{L}}}$ be the set of all idempotents in $S/\hat{\mathcal{L}}$. Any element in $E_{S/\hat{\mathcal{L}}}$ and hence in $S/\hat{\mathcal{L}}$ is of the form $\hat{\mathcal{L}}_a$ where $a \in S$. That is,

$E_{S/\hat{\mathcal{L}}} = \{\hat{\mathcal{L}}_a$: is an idempotent in $S/\hat{\mathcal{L}}\}$
$= \{\hat{\mathcal{L}}_a$: a is an idempotent in $S\}$ (Hariprakash, 2016).

Let $\hat{\mathcal{L}}_a, \hat{\mathcal{L}}_b, \hat{\mathcal{L}}_c, \hat{\mathcal{L}}_d \in E_{S/\hat{\mathcal{L}}}$. Then a, b, c, d are idempotents in S. That is, $abcd \in S$. Since S is a generalized inverse semigroup.

$abcd = acbd$.
We have

$\hat{\mathcal{L}}_a * \hat{\mathcal{L}}_b * \hat{\mathcal{L}}_c * \hat{\mathcal{L}}_d = \hat{\mathcal{L}}_{abcd}$
$= \hat{\mathcal{L}}_{acbd}$
$= \hat{\mathcal{L}}_a * \hat{\mathcal{L}}_c * \hat{\mathcal{L}}_b * \hat{\mathcal{L}}_d$.

That is, $E_{S/\hat{\mathcal{L}}}$ is normal. Hence, S/ $\hat{\mathcal{L}}$ is a generalized inverse semigroup.

Here S/ $\hat{\mathcal{L}}$ is called $\hat{\mathcal{L}}$-fuzzy generalized inverse semigroup.

Proposition 3.3. If $\hat{\mathcal{R}}$ is a fuzzy congruence on a generalized inverse semigroup and *S* is $\hat{\mathcal{R}}$-fuzzy antisimple, $S/\hat{\mathcal{R}}$ a generalized inverse semigroup.

Proof

Result follows from proposition 2.2, using the property of $\hat{\mathcal{R}}$.

Here $S/\hat{\mathcal{R}}$ is called $\hat{\mathcal{R}}$-fuzzy generalized inverse semigroup.

Theorem 2.4. If *S* is a regular semigroup, the following statements are equivalent.

(a) $S/\hat{\mathcal{L}}$ is a $\hat{\mathcal{L}}$-fuzzy orthodox semigroup.

(b) For any $\hat{\mathcal{L}}_a$, $\hat{\mathcal{L}}_b$, in $S/\hat{\mathcal{L}}$ if $\hat{\mathcal{L}}_{a'}$ is an inverse of $\hat{\mathcal{L}}_a$ and $\hat{\mathcal{L}}_{b'}$ is an inverse of $\hat{\mathcal{L}}_b$, $\hat{\mathcal{L}}_{b'}\hat{\mathcal{L}}_{a'}$ is an inverse of $\hat{\mathcal{L}}_a\hat{\mathcal{L}}_b$.

(c) If $\hat{\mathcal{L}}_e \in E_{S/\hat{\mathcal{L}}}$ its inverse also belongs to $E_{S/\hat{\mathcal{L}}}$.

Proof

Assume (a). Let $\hat{\mathcal{L}}_a$, $\hat{\mathcal{L}}_b \in S/\hat{\mathcal{L}}$ and $\hat{\mathcal{L}}_{a'}$ an inverse of $\hat{\mathcal{L}}_a$ and $\hat{\mathcal{L}}_{b'}$ an inverse of $\hat{\mathcal{L}}_b$. First we can show that $\hat{\mathcal{L}}_{a'a}$ and $\hat{\mathcal{L}}_{bb'}$ are idempotent.

We have

$$\hat{\mathcal{L}}_{a'a} * \hat{\mathcal{L}}_{a'a} = \hat{\mathcal{L}}_{a'a}\hat{\mathcal{L}}_{a'a} = \hat{\mathcal{L}}_{a'a}$$

and

$$\hat{\mathcal{L}}_{bb'} * \hat{\mathcal{L}}_{bb'} = \hat{\mathcal{L}}_{bb'bb'} = \hat{\mathcal{L}}_{bb'}.$$

Hence, $\hat{\mathcal{L}}_{a'a}$ and $\hat{\mathcal{L}}_{bb'}$ are idempotents in $S/\hat{\mathcal{L}}$.

Since S/ $\hat{\mathcal{L}}$ is an orthodox semigroup $\hat{\mathcal{L}}_{a'a} * \hat{\mathcal{L}}_{bb'}$ is an idempotent.

That is,

$$(\hat{\mathcal{L}}_{a'a} * \hat{\mathcal{L}}_{bb'}) * (\hat{\mathcal{L}}_{a'a} * \hat{\mathcal{L}}_{bb'}) = \hat{\mathcal{L}}_{a'a} * \hat{\mathcal{L}}_{bb'} \text{ or } (\hat{\mathcal{L}}_{a'a} * \hat{\mathcal{L}}_{bb'})^2 = \hat{\mathcal{L}}_{a'a} * \hat{\mathcal{L}}_{bb'}.$$

Similarly,

$$(\hat{\mathcal{L}}_{bb'} * \hat{\mathcal{L}}_{a'a})^2 = \hat{\mathcal{L}}_{bb'} * \hat{\mathcal{L}}_{a'a}.$$

Again,

$$(\hat{\mathcal{L}}_a * \hat{\mathcal{L}}_b) * (\hat{\mathcal{L}}_{b'} * \hat{\mathcal{L}}_{a'}) * (\hat{\mathcal{L}}_a * \hat{\mathcal{L}}_b) = \hat{\mathcal{L}}_a * \hat{\mathcal{L}}_{a'} * \hat{\mathcal{L}}_a * \hat{\mathcal{L}}_b * \hat{\mathcal{L}}_{b'} * \hat{\mathcal{L}}_{a'} * \hat{\mathcal{L}}_a * \hat{\mathcal{L}}_b * \hat{\mathcal{L}}_{b'} * \hat{\mathcal{L}}_b$$
$$= \hat{\mathcal{L}}_a * (\hat{\mathcal{L}}_{a'} * \hat{\mathcal{L}}_a * \hat{\mathcal{L}}_b * \hat{\mathcal{L}}_{b'})^2 * \hat{\mathcal{L}}_b$$

$= \hat{\mathcal{L}}_a * \hat{\mathcal{L}}_{a'} * \hat{\mathcal{L}}_a * \hat{\mathcal{L}}_b * \hat{\mathcal{L}}_{b'} * \hat{\mathcal{L}}_b$
$= \hat{\mathcal{L}}_a * \hat{\mathcal{L}}_b.$

That is,

$(\hat{\mathcal{L}}_a * \hat{\mathcal{L}}_b) * (\hat{\mathcal{L}}_{b'} * \hat{\mathcal{L}}_{a'}) * (\hat{\mathcal{L}}_a * \hat{\mathcal{L}}_b) = \hat{\mathcal{L}}_a * \hat{\mathcal{L}}_{b'}(1)$

Also

$(\hat{\mathcal{L}}_{b'} * \hat{\mathcal{L}}_{a'}) * (\hat{\mathcal{L}}_a * \hat{\mathcal{L}}_b) * (\hat{\mathcal{L}}_{b'} * \hat{\mathcal{L}}_{a'})$
$= \hat{\mathcal{L}}_{b'} * \hat{\mathcal{L}}_b * \hat{\mathcal{L}}_{b'} * \hat{\mathcal{L}}_{a'} * \hat{\mathcal{L}}_a * \hat{\mathcal{L}}_b * \hat{\mathcal{L}}_{b'} * \hat{\mathcal{L}}_{a'} * \hat{\mathcal{L}}_a * \hat{\mathcal{L}}_{a'}$
$= \hat{\mathcal{L}}_{b'} * (\hat{\mathcal{L}}_b * \hat{\mathcal{L}}_{b'} * \hat{\mathcal{L}}_{a'} * \hat{\mathcal{L}}_a)^2 \hat{\mathcal{L}}_{a'}$
$= \hat{\mathcal{L}}_{b'} * \hat{\mathcal{L}}_b * \hat{\mathcal{L}}_{b'} * \hat{\mathcal{L}}_{a'} * \hat{\mathcal{L}}_a * \hat{\mathcal{L}}_{a'}$
$= \hat{\mathcal{L}}_{b'} * \hat{\mathcal{L}}_{a'}.(2)$

From (1) and (2), (b) follows

Assume (b). Given $\hat{\mathcal{L}}_e \in E_{S/\hat{\mathcal{L}}}$. That is $\hat{\mathcal{L}}_e$is an idempotent in $S/\hat{\mathcal{L}}$.

Let $\hat{\mathcal{L}}_x$ n be the inverse of $\hat{\mathcal{L}}_e$. Then $\hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_x = \hat{\mathcal{L}}_x$ and $\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e = \hat{\mathcal{L}}_x$.

Now,
$(\hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e) * (\hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e) = (\hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_x) * \hat{\mathcal{L}}_e = \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e.$

and

$(\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_x) * (\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_x) = (\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e) * \hat{\mathcal{L}}_x = \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_x.$

That is, $\hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e$ and $\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_x$ are idempotents in $S/\hat{\mathcal{L}}$. So, $(\hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e)^{-1} = \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e$. (Idempotents has its own inverse).

Then

$[(\hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e) * (\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_x)]^{-1} = [\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_x]^{-1} * [\hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e)^{-1}$
$= (\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_x) * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e$
$= \hat{\mathcal{L}}_e (\hat{\mathcal{L}}_x)^2 \hat{\mathcal{L}}_e.$

That is,

$[\hat{\mathcal{L}}_x * (\hat{\mathcal{L}}_e)^2 * \hat{\mathcal{L}}_x]^{-1} = \hat{\mathcal{L}}_e * (\hat{\mathcal{L}}_x)^2 * \hat{\mathcal{L}}_e.$

But $(\hat{\mathcal{L}}_e)^2 = \hat{\mathcal{L}}_e$. Then

$(\hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_x)^{-1} = \hat{\mathcal{L}}_e * (\hat{\mathcal{L}}_x)^2 * \hat{\mathcal{L}}_e.$

That is $(\hat{\mathcal{L}}_x)^{-1} = \hat{\mathcal{L}}_e * (\hat{\mathcal{L}}_x)^2 * \hat{\mathcal{L}}_e$. That is, $\hat{\mathcal{L}}_e * (\hat{\mathcal{L}}_x)^2 * \hat{\mathcal{L}}_e$ is the inverse of $\hat{\mathcal{L}}_x$.

That is,

$\hat{\mathcal{L}}_x = \hat{\mathcal{L}}_x [\hat{\mathcal{L}}_e (\hat{\mathcal{L}}_x)^2 \hat{\mathcal{L}}_e] \hat{\mathcal{L}}_x$

$= (\hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_x) * (\hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_x)$
$= \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_x = (\hat{\mathcal{L}}_x)^2$.

Then $\hat{\mathcal{L}}_x$ is an idempotent. Hence when $\hat{\mathcal{L}}_e$ is an idempotent in S/$\hat{\mathcal{L}}$, its inverse $\hat{\mathcal{L}}_x$ is also an idempotent in $S/\hat{\mathcal{L}}$. That is, $\hat{\mathcal{L}}_e \in E_{S/\hat{\mathcal{L}}}$ implies $(\hat{\mathcal{L}}_e)^{-1} \in E_{S/\hat{\mathcal{L}}}$.

Assume (c). Let $\hat{\mathcal{L}}_e$, $\hat{\mathcal{L}}_f$ are two idempotents in S/$\hat{\mathcal{L}}$. Then $\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f \in S/\hat{\mathcal{L}}$ (Since $S/\hat{\mathcal{L}}$ is a semigroup). Let $\hat{\mathcal{L}}_x$ be an inverse of $\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f$.

Then

$(\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f) * \hat{\mathcal{L}}_x * (\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f) = \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f$(3)
$\hat{\mathcal{L}}_x * (\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f) * \hat{\mathcal{L}}_x = \hat{\mathcal{L}}_x$.(4)

Now, we have

$(\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f) * (\hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e) * (\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f) = \hat{\mathcal{L}}_e * (\hat{\mathcal{L}}_f)^2 * (\hat{\mathcal{L}}_x) * (\hat{\mathcal{L}}_e)^2 * \hat{\mathcal{L}}_f = \hat{\mathcal{L}}_e * (\hat{\mathcal{L}}_f)^2 * \hat{\mathcal{L}}_x * (\hat{\mathcal{L}}_e)^2 * \hat{\mathcal{L}}_f$
$= (\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f) * \hat{\mathcal{L}}_x * (\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f)$
$= \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f$, by (3)

Also,

$(\hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e) * (\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f) * (\hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e) = \hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * (\hat{\mathcal{L}}_e)^2 * \hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e = (\hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e) * (\hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e)$
$= \hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * (\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f) * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e = \hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e$, from (4)

Hence $\hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e$ is inverse $\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f$

Moreover,

$(\hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e) * (\hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e) = \hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * (\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f) * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e = \hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e$

Hence $\hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e$ is an idempotent. That is, $\hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e \in E_{S/\hat{\mathcal{L}}}$.

Then by (c) its inverse $\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f \in E_{S/\hat{\mathcal{L}}}$.

Hence, the product of two idempotent in S/$\hat{\mathcal{L}}$ is an idempotents.

That is S/$\hat{\mathcal{L}}$ is an orthodox semigroup.

Theorem 2.5. If S is a regular semigroup, the following statements are equivalent.

(a) $S/_L$ is an L-fuzzy orthodox semigroup.
(b) For any a,b $\in S$ if $\hat{\mathcal{L}}_{a'} \in V(\hat{\mathcal{L}}_a), \hat{\mathcal{L}}_{b'} \in V(\hat{\mathcal{L}}_b)$, $\hat{\mathcal{L}}_{b'} * \hat{\mathcal{L}}_{a'} \in V(\hat{\mathcal{L}}_a * \hat{\mathcal{L}}_b)$ (c) If e $\in E_S$ and e' $\in V$(e), $\hat{\mathcal{L}}_{e'} \in E_{S/\hat{\mathcal{L}}}$ where $\hat{\mathcal{L}}_{e'} \in V(\hat{\mathcal{L}}_e)$.

Proof

Result follows from theorem 2.4.

Theorem 2.6. If S is a regular semigroup, the following statements are equivalent.

(a) $S/\hat{\mathcal{R}}$ is a $\hat{\mathcal{R}}$-fuzzy orthodox semigroup

(b) For any $\hat{\mathcal{R}}_a$, $\hat{\mathcal{R}}_b$ in $S/\hat{\mathcal{R}}$ if $\hat{\mathcal{R}}_{a'}$ is an inverse of $\hat{\mathcal{R}}_a$ and $\hat{\mathcal{R}}_{b'}$ is an inverse of $\hat{\mathcal{R}}_b$, $\hat{\mathcal{R}}_{b'} * \hat{\mathcal{R}}_{a'}$ is an inverse of $\hat{\mathcal{R}}_a * \hat{\mathcal{R}}_b$.

(c) If $\hat{\mathcal{R}}_e \in E_{S/\hat{\mathcal{R}}}$, its inverse also belongs to $E_{S/\hat{\mathcal{R}}}$.

Proof

Result follows from theorem 2.4 by applying the property of $\hat{\mathcal{R}}$ instead of $\hat{\mathcal{L}}$.

Theorem 2.7. If S is a regular semigroup, the following statements are equivalent.

(a) $S/\hat{\mathcal{R}}$ is a $\hat{\mathcal{R}}$-fuzzy orthodox semigroup.

(b) For any $a, b \in S$ if $\hat{\mathcal{R}}_{a'} \in V(\hat{\mathcal{R}}_a)$, $\hat{\mathcal{R}}_{b'} \in V(\hat{\mathcal{L}}_b)$, $\hat{\mathcal{R}}_{b'} \hat{\mathcal{R}}_{a'} \in V(\hat{\mathcal{R}}_a \hat{\mathcal{R}}_b)$

(c) If $e \in E_S$ and $e' \in V(e)$, $\hat{\mathcal{R}}_{e'} \in E_{S/\hat{\mathcal{R}}}$ where $\hat{\mathcal{R}}_{e'} \in V(\hat{\mathcal{R}}_e)$.

Proof

Result follows from theorem 2.6.

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