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RESEARCH ARTICLE

A COMPREHENSIVE ANALYSIS OF BET, BET-XT, BET-FI AND BET-NG INDICES USING THE JOINT SYMMETRIC AND ASYMMETRIC ARMA-GARCH MODELS

[1,*]Andreea-Cristina Petrică and [2]Stelian STANCU

[1]Doctoral School of Economic Cybernetics and Statistics, The Bucharest University of Economic Studies, Romania

[2]Informatics and Economic Cybernetics Department, Bucharest University of Economic Studies, Romania, Centre for Industrial and Services Economics, Romanian Academy

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ABSTRACT

This paper focuses on investigating the volatility on the Romanian stock market by employing the joint symmetric and asymmetric ARMA - GARCH models on four of Bucharest Stock Exchange' own indices which reflect only the evolution of market prices: Bucharest Exchange Trading Index (BET), Bucharest Exchange Trading Extended Index (BET-XT), Bucharest Exchange Trading – Investment Funds (BET-FI) and Bucharest Exchange Trading Energy & Related (BET-NG). We estimated ARCH, GARCH, EGARCH and GJR-GARCH models using the maximum likelihood method under the assumption of Gaussian distributed innovation terms. The empirical results show that in three cases out of four the volatility reacted asymmetrically to the good and bad news. The predominant model turned out to be EGARCH model and this is because it does not require any constraint on the parameters since the positivity of the conditional variance is automatically satisfied.
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INTRODUCTION

A characteristic trait of analyzing the volatility represented by conditional variance of financial instruments is that it is not directly observable. This led to the development of conditional heteroscedastic processes whose purpose consists in accuracy in predicting the volatility. Starting with Autoregressive Conditionally Heteroscedastic process ($ARCH$) introduced by Engle (1982) where conditional variance is considered time-varying with respect to the innovation terms, Bollerslev (1986) develops the Generalized Autoregressive Conditionally Heteroscedastic model ($GARCH$) which has a pliable lag structure than $ARCH$.Since the processes mentioned above fail regarding the phenomenon of leverage effect because of the symmetric distribution, many nonlinear extensions of $GARCH$ processes have been developed to allow for asymmetric effects of positive and negative innovations, in which we mention the Exponential $GARCH$ process (Nelson (1991)) and Threshold $ARCH$ process ($TARCH$ introduced by Zakoian (1994) or $GJR\quad GARCH$ introduced by Glosten, Jagannathan and Runkle (1993)). The last two processes differ in their specification, $TARCH$ uses the conditional standard deviation, while $GJR\quad GARCH$ process uses conditional variance.

***Corresponding author: Andreea Petrica,**
Doctoral School of Economic Cybernetics and Statistics, The Bucharest University of Economic Studies, Romania

Making a brief review of recent empirical studies, we mention the following scientific papers: Alberg, Shalit and Yosef (2008) investigate Tel Aviv Stock Exchange Indices (TA100 and TA25) using various asymmetric $GARCH$ models and distributions (normal, Student's t and asymmetric Student's t distribution). The results suggest that $AR(1)\quad EGARCH(1,1)$ model gives better forecasts than $GJR\quad GARCH$ and $APARCH$ models. Analyzing the Khartoum Stock Exchange's volatility, Elsheikh and Zakaria (2011) present the better fit of the asymmetric $GARCH$ models by comparing the symmetric and asymmetric $GARCH$ models. This result is due to the the presence of leverage effect. Begu, Spătaru and Marin (2012) estimate the daily returns of RON/EUR exchange rates by making use of the conditional heteroscedastic processes: $ARCH$, $GARCH$, $EGARCH$and $TGARCH$. After comparing the forecasting performance of those four processes, they found that the most promising model for characterizing the dynamic behavior of RON/EUR exchange rate returns is $AR(4) - GARCH(1,1)$.Pele (2012) analyzes the behavior of BET Index using an $AR(1)\quad GARCH(1,1)$ model which can be a tool for estimating the probability of stock market crashes, and finds out that stable distributions improve the prediction of an extreme event. Ali (2013) uses seven asymmetric $GARCH$ models for environmental stochastic processes and finds out that the outputs for $EGARCH$, $TGARCH$, $AVGARCH$ and

$NGARCH$ are similar. However, for capturing the response of the pathogen indicator it seems that $TGARCH$ fits the data better than the other models. Using $GARCH$ methodology on the dollar-adjusted daily return of the BET Index, Damianova (2014) finds evidence of market inneficiency regarding the Bucharest Stock Exchange. The analysis takes into account four Bucharest Stock Exchange' own indices which reflect only the evolution of market prices, respectively: Bucharest Exchange Trading Index (BET), Bucharest Exchange Trading Extended Index (BET-XT), Bucharest Exchange Trading – Investment Funds (BET-FI) and Bucharest Exchange Trading Energy & Related (BET-NG). As calculation methodology, all BSE indices are price indexes free float capitalization-weighted, with maximum limits for weights component companies. Liquidity represents the main criterion for selecting the companies in the index and the coefficient for liquidity is given by the following equation:

$$Coefficient_{liquidity} = \frac{\sum_{\substack{i=1,N \\ j=\{1,3,6,9,12\}}} Av_{i,j} \times j}{31} \qquad (1)$$

where $Av_{i,j}$ represents the weight of symbol i in the total turnover for the symbols of the regulated market during the time j (1, 3, 6, 9, or 12 months). Note that only transactions made on the segment 'Regular' of the market are taken into account and that starting with 2015, besides liquidity it applies also criteria relating to transparency and quality reporting issuers and their communication with investors.

BET is the first index developed by the BSE (Release date: 09.19.1997, Number of Companies: 10, Index Value: 1000 points) and represents the benchmark index for the local capital market reflecting the evolution of ten most liquid companies listed on BSE regulated market, except for financial investment companies (SIFs). It is possible that their number might increase in the future due to the listing on BSE of new representing companies for sectors of national economy and the recording of relevant events with impact on the listed companies. BET Index is a free float capitalization-weighted price index, the maximum weight of a symbol is 20%. BET-XT Index reflects the price evolution of the most traded/liquid 25 Romanian companies listed on BSE regulated market, including the financial investment companies (SIFs). It was launched on July 1st, 2008 with a starting value of 1000 points, calculated retroactively from January 2nd, 2007. Being the third index developed by BSE, BET-FI is the first sector index and reflects the evolution of financial investment companies (SIFs) and other similar institutions. It was launched on October 31st, 2000, with a starting value of 1000 points. BET-NG is a sector index reflecting the evolution of the whole sector and all companies listed on the regulated market of BSE which main activity is energy and related utilities. The maximum weight of a symbol in the index is 30% and the number of composing companies is variable. BET-NG Index was launched on July 1st, 2008, with a starting value of 1000 points, calculated retroactively from January 2nd, 2007.

MATERIALS AND METHODS

The aim of this section is to give the comprehensive framework of the Autoregressive Conditionally Heteroscedastic ($ARCH$) and its generalization ($GARCH$) models which represent the main methodologies of modeling volatility of stock market. All these models will be used in the next section to investigate the volatility clustering, persistence and the asymmetric responses of the conditional variance to positive and negative shocks (good news and bad news) known as leverage effect. The structure of these models consists in two equations that must be estimated simultaneous: the conditional mean and the conditional variance. In this paper we will focus on the description of the second equation.

Symmetric Models

The main characteristic of symmetric models is that they treat the shocks as being symmetric. This means that shocks affect the conditional variance in the same way whether they are positive or negative, so they have the same impact on volatility.

The Autoregressive Conditionally Heteroscedastic Model (ARCH)

In the Autoregressive Conditionally Heteroscedastic process, $ARCH(q)$, the conditional variance may be specified as:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \quad + \alpha_q \varepsilon_{t-q}^2 = \omega + \sum_{i=1}^{q} (\alpha_i \varepsilon_{t-i}^2) (2)$$

with $\omega > 0$ and $\alpha_i \geq 0$, for $i = \overline{1,q}$. So, it is described by its past q squared innovations.

Or in terms of the lag operator (L), the equation (2) becomes:

$$\sigma_t^2 = \omega + (\alpha_1 L + \alpha_2 L^2 + \quad + \alpha_q L^q) \varepsilon_t^2 \qquad (3)$$

Xekalaki and Degiannakis (2010, p.10)give the following formal definition for the $ARCH$ process $\{\varepsilon_t\}$:

$$\varepsilon_t(\theta) = z_t \sigma_t(\theta),$$
$$z_t \sim f(w; 0,1) i.i.d.$$
$$\sigma_t^2(\theta) = g(\sigma_{t-1}(\theta), \sigma_{t-2}(\theta), \dots;\ \varepsilon_{t-1}(\theta), \varepsilon_{t-2}(\theta), \dots;\ v_{t-1}, v_{t-2}, \dots), (4)$$

Where:

$\{\varepsilon_t(\theta)\}$ - the error process to be modeled;
$\{z_t\}$ - a sequence of i.i.d. random variables with the density function $f(.)$, having zero mean and unit variance;
w - the vector of the parameters of f to be estimated;
$\sigma_t^2(\theta)$–the conditional varianceof ε_t, $\sigma_t^2(\theta) > 0$;
v_t - a vector of predetermined variables included in I_t;
$g(.)$ - a linear or non-linear functional form of I_{t-1};
I_{t-1} - the filtration information until time t or the information set at time t 1.

The link between autoregressive processes and autoregressive conditionally heteroscedastic processes is given by the fact that the $ARCH(q)$ model can be interpreted as an autoregressive process in the squared innovations, see Xekalaki (2010, p.20). As we will see later, in our analysis the $ARCH(q)$ model represents the return of Bucharest Stock Exchange Indices series $\{x_t\}$ with the process $\{\varepsilon_t\}$. Considering that $\{x_t\}$ incorporates more complex structures, we will deal with $ARMA(r,s)$ $ARCH(q)$ models and rewrite the above equations such as:

$$x_t = \theta_1 x_{t-1} + \theta_2 x_{t-2} + \quad + \theta_r x_{t-r} + \varepsilon_t + \varphi_1 \varepsilon_{t-1} + \varphi_2 \varepsilon_{t-2} + \quad + \varphi_s \varepsilon_{t-s}$$
$$\varepsilon_t = z_t \sigma_t$$

$$\sigma_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + \ldots + \alpha_q\varepsilon_{t-q}^2 \quad (5)$$

Therefore, x_t has a conditionally varying mean, μ_t, arising from an $ARMA(r,s)$ process. The conditional mean of x_t given the information at time $t-1, I_{t-1}$, is $E(y_t|I_{t-1}) = E_{t-1}(y_t) = \mu_t$.

The innovation process to be modeled for the conditional mean is given by $\varepsilon_t = x_t - \mu_t = x_t - \theta_1 x_{t-1} - \theta_2 x_{t-2} - \ldots - \theta_r x_{t-r} - \varphi_1\varepsilon_{t-1} - \varphi_2\varepsilon_{t-2} - \ldots - \varphi_s\varepsilon_{t-s}$ and represents a shock in the return series of the Bucharest Stock Exchange Indices. The conditional variance of ε_t given the information at time $t-1, I_{t-1}$, is $Var(y_t|I_{t-1}) = V_{t-1}(y_t) \equiv E_{t-1}(\varepsilon_t^2) = \sigma_t^2$. To ensure that σ_t^2 is positive we have to impose some restrictions with respect to the parameters in the conditional variance equation: $\omega > 0$ and $\alpha_i \geq 0$ for $i = 1, 2, \ldots, q$. If ARCH LM Test indicates that there is an $ARCH$ effect in the squared innovations, we can use the $PACF$ of ε_t^2 which is a useful tool to determine the order q of the process. So, having the following equation of the conditional variance:

$$\sigma_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + \ldots + \alpha_q\varepsilon_{t-q}^2 \quad \ldots\ldots (6)$$

and rewriting the equation above as an autoregressive process of order q, we get:

$$\varepsilon_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + \ldots + \alpha_q\varepsilon_{t-q}^2 + u_t \quad \ldots\ldots (7)$$

where u_t is a sequence of martingale difference, $u_t = \varepsilon_t^2 - \sigma_t^2$, $\{u_t\}$ is an uncorrelated series with zero mean. The disadvantage is given by the sample size, if we deal with a small sample the $PACF$ of the squared residuals may not be effective. On the other hand, if there is significant $PACFs$ at higher order lags it means that an $ARCH$ process with higher order must be used. This leads us to use a $GARCH$ model instead of $ARCH$ model.

The Generalized Autoregressive Conditionally Heteroscedastic Model (GARCH)

The $ARCH(q)$ models of higher order are difficult to estimate because they often produce negative estimations of α coefficients. This problem is solved by $GARCH$ models which turn the autoregressive process from $ARCH$ model into an $ARMA$ process by adding a moving average process. Bollerslev (1986) introduces a more general class of the processes such that to enable a more flexible lag structure. The extension of the Autoregressive Conditionally Heteroscedastic model to the Generalized Autoregressive Conditionally Heteroscedastic model is similar to the extension of the autoregressive processes to the autoregressive moving average processes, which allows a more parsimonious description in many situations. The mathematical representation of the $GARCH$ process differs from the $ARCH$ process by introducing news on past volatility. In the Generalized Autoregressive Conditionally Heteroscedastic process, $GARCH(p,q)$, the conditional variance may be specified as:

$$\sigma_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + \ldots + \alpha_q\varepsilon_{t-q}^2 + \beta_1\sigma_{t-1}^2 + \beta_2\sigma_{t-2}^2 + \ldots + \beta_p\sigma_{t-p}^2 = \omega + \sum_{i=1}^{q}\left(\alpha_i\varepsilon_{t-i}^2\right) + \sum_{j=1}^{p}\left(\beta_j\sigma_{t-j}^2\right) \quad \ldots\ldots (8)$$

where:
ω – constant term;
α_i – coefficients of $ARCH$ terms i.e. news about past volatility;
β_j–coefficients of $GARCH$ terms i.e. the persistence of volatility;
p – the number of lagged conditional variance terms (σ^2);
q – the number of lagged innovation terms (ε^2),

with $\omega > 0$, $\alpha_i \geq 0$, for $i = \overline{1,q}$, $\beta_j \geq 0$, for $j = \overline{1,p}$ and $\sum_{i=1}^{q}\alpha_i + \sum_{j=1}^{p}\beta_j < 1$ to ensure that the conditional variance is weak-stationary.If we obtain a large value of α_i, it means that the volatility is sensitive to market shock's, otherwise the volatility is insensitive.In case of a large β_j coefficient, we are dealing with a persistent volatility.

If we rewrite the equation (8) in terms of the lag operator (L), we get:

$$\sigma_t^2 = \omega + (\alpha_1 L + \alpha_2 L^2 + \ldots + \alpha_q L^q)\varepsilon_t^2 + \left(\beta_1 L + \beta_2 L^2 + \ldots + \beta_p L^p\right)\sigma_t^2 \quad 9$$

or equivalently,

$$\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2 \quad \ldots\ldots (10)$$

where $a(L)$ and $b(L)$ are polynomials of degrees q and p.[1]

Thus, the conditional variance of disturbances/innovations/ errors depends of the q - past squared innovations and p - lagged conditional variance. So, the $ARCH$ terms outline the volatility in previous periods measured as the lagged squared innovations from conditional mean equation, while the $GARCH$ terms indicate the persistance of past innovations impact on volatility.

Asymmetric Models

The main characteristic of asymmetric models is that they treat the shocks as being asymmetric, which means that negative shocks (bad news) affect more the volatility than the positive ones (good news). The difference between symmetric and asymmetric models is that the first category can not explain the leverage effect observed in financial time series. This fact led to introducing the asymmetric models that can capture this phenomenon.

The Exponential Generalized Autoregressive Conditionally Heteroscedastic Model (EGARCH)

Nelson (1991) introduces an extension to the $GARCH$ model, the so-called Exponential $GARCH$ model ($EGARCH$), which is able to allow for asymmetric effects of positive and negative shocks. The model has two main advantages compared to the models specified above, respectively:

- Capacity of modeling the asymmetrical effect;
- Expressing the conditional variance in terms of logarithm leads to non-negative volatility as well as relaxing the constrains of the coefficients.

Dutta (2014, p.102) gives the following representation for the conditional variance of an $EGARCH(p,q)$ model:

$$log(\sigma_t^2) = \omega + \sum_{i=1}^{q}\alpha_i\frac{|\varepsilon_{t-i}| + \delta_i\varepsilon_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{p}\beta_j\,log\,(\sigma_{t-j}^2) \quad (11)$$

[1] $\alpha(L) = \alpha_1 L + \alpha_2 L^2 + \ldots + \alpha_q L^q$ and $\beta(L) = \beta_1 L + \beta_2 L^2 + \ldots + \beta_p L^p$.

where:

δ_i - the asymmetric response parameter(leverage parameter). A non-zero coefficient indicates that the effect is asymmetric, while a negative value of the coefficient shows that the volatility rises more after negative shocks than after positive ones.
β_j - the persistence parameter. A large value of the parameter indicates that the variance moves slowly through time.

If $\varepsilon_{t-1} > 0$ i.e. positive shocks (good news), then the total impact of $\varepsilon_{t-i}$is $(1+\delta_i)\ |\varepsilon_{t-i}|$, otherwise the total impact is $(1\quad \delta_i)\ |\varepsilon_{t-i}|$ and the leverage effect is represented by the parameter δ_i. Thus, $\delta_i \neq 0$ leads us to the conclusion that the effect is asymmetric. So, the $EGARCH$ process models the logarithm of conditional variance as a function of the p - lagged logarithm of conditional variances and the absolute innovations from the past.

The Threshold Autoregressive Conditionally Heteroscedastic Model (TARCH and GJR GARCH)

Zakoian (1994) introduces another asymmetric $GARCH$ model to handle leverage effects called the Threshold $ARCH$ model, which is similar to $GJR\quad GARCH$ model introduced by Glosten, Jagannathan, and Runkle (1993). The difference between those two models is given by using the specification on the conditional standard deviation instead of conditional variance.

Therefore, the conditional variance for the $GJR\quad GARCH$ model has the following equation:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q}(\alpha_i + \gamma_i \quad Id(\varepsilon_{t-i}))\quad \varepsilon_{t-i}^2 + \sum_{j=1}^{p}\beta_j\sigma_{t-j}^2 \qquad (12)$$

where Id - represents the indicator function,

$$Id(\varepsilon) = \begin{cases} 0, & if \varepsilon \geq 0 \\ 1, & ot\ \ erwise \end{cases}.$$

Similarly, the conditional variance for the $TARCH$ model has the following representation:

$$\sigma_t = \omega + \sum_{i=1}^{q}(\alpha_i^+ \quad \varepsilon_{t-i}^+ + \alpha_i^- \quad \varepsilon_{t-i}^-) + \sum_{j=1}^{p}\beta_j \quad \sigma_{t-j} \qquad (13)$$

where$\varepsilon^+ = \begin{cases} \varepsilon, & if \varepsilon > 0 \\ 0, & ot\ \ erwise \end{cases}and\varepsilon^- = \begin{cases} \varepsilon, & if \varepsilon < 0 \\ 0, & ot\ \ erwise \end{cases}$.

RESULTS AND DISCUSSION

This section points out and illustrates the empirical results obtained by fitting thejoint symmetric and asymmetric$ARMA\quad GARCH$models to the continuously compounded daily returnsofthe tested stock indices, respectively: BET, BET-XT, BET-FI and BET-NG.The data were collected from Bloomberg data base and consist of 4655 daily observations of the BET Index (22 September 1997 - 29 July 2016), 2050 daily observations of the BET-XT Index (10 June 2008 - 29 July 2016), 3927 daily observations of the BET-FI Index (31 October 2000 - 29 July 2016), and 2050 daily observations of the BET-NG Index (10 June 2008 - 29 July 2016). For working with stationary series we use for all indices percentage daily returns calculated as the first difference in logarithm of closing prices of the index of successive days ($Return_t = log\left(\frac{Price_t}{Price_{t-1}}\right)\quad 100$). The pattern of daily price and percentage return series for BET, BET-XT, BET-FI and BET-NG during the analyzed periods are presented in Figures 1-4.Next, to investigate stationarity we apply Augmented Dickey-Fuller Test for both series – prices and returns – and the results are summarized in Table 1. As can be seen, in case of the price indices results indicate non-stationarity, while in case of the daily percentage returns the null hypothesis of a unit root can be rejected at all conventional confidence levels (1%, 5% and 10%).Table 2 shows that the daily returns of all four BSE Indices are not normally distributed.

Skewness different from zero indicates asymmetry (0.237, 0.618, 0.557 and 0.075 i.e. the distribution has a long left tail), while Kurtosis above 3 indicates a probability distribution with fat-tails (16.210, 12.972, 17.608 and9.995i.e. the distribution of returns is leptokurtic) which imply an additional risk. Furthermore, the Jarque-Bera statistics presents higher values and rejects the hypothesis of normality for all returns (at 1% confidence level).Standard deviation indicates that the most volatile index is BET-FI ($Std.Dev. = 2.310$), followed by BET ($Std.Dev. = 1.782$), while the less volatile index is BET-NG ($Std.Dev. = 1.693$). We can also see that for BET and BET-FI the average daily return is positive (0.041 and0.085), while for BET-XT and BET-NG– is negative (0.015 and 0.024).Based on Box-Jenkins methodology, we employ the Autoregressive Moving Average models for the conditional mean in order to find the adequate model. The results are presented in Table 3.

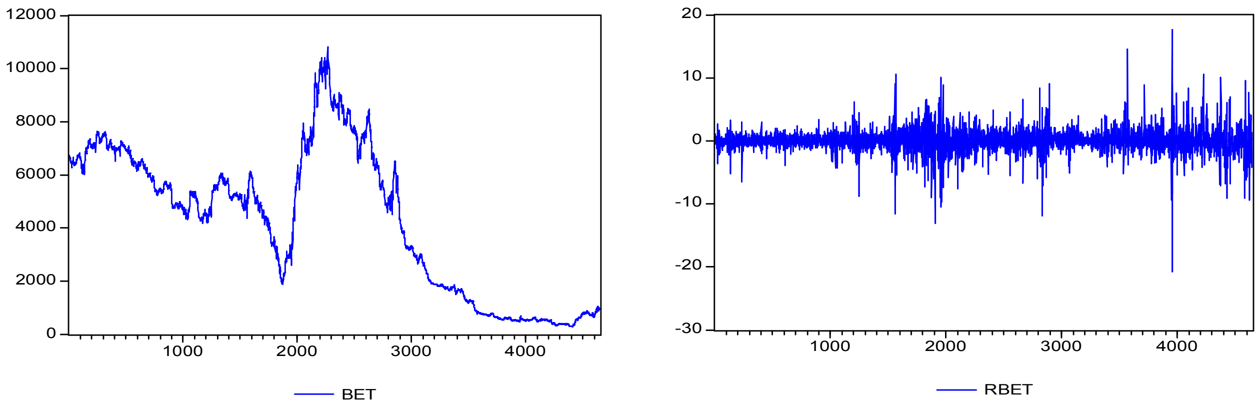

Fig. 1. Daily prices and returns for the BET Index (22 September 1997 - 29 July 2016)

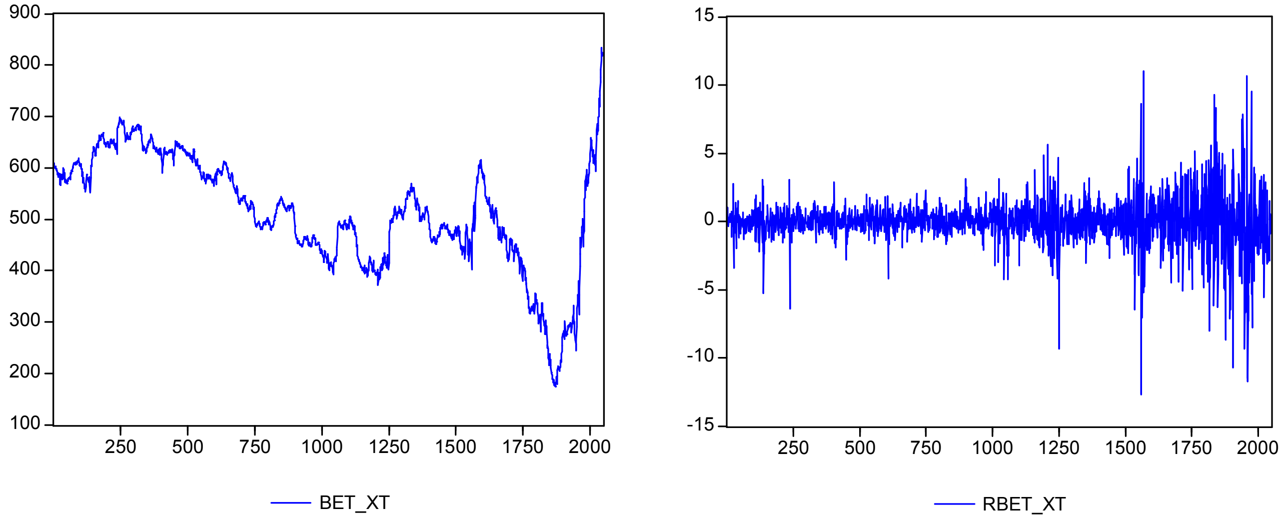

Fig. 2. Daily prices and returns for the BET-XT Index (10 June 2008 - 29 July 2016)

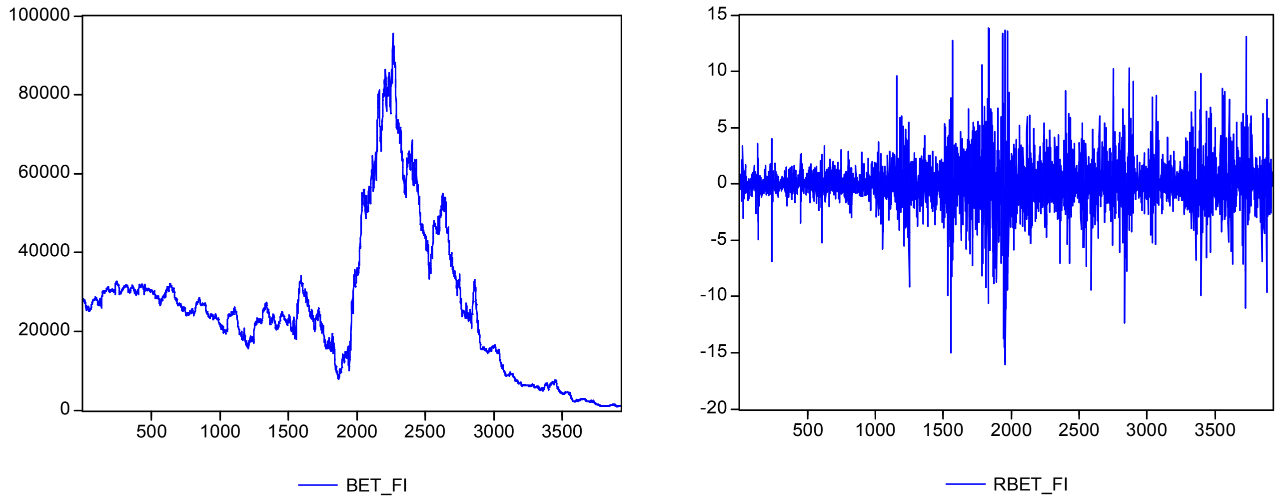

Fig. 3. Daily prices and returns for the BET-FI Index (31 October 2000 - 29 July 2016)

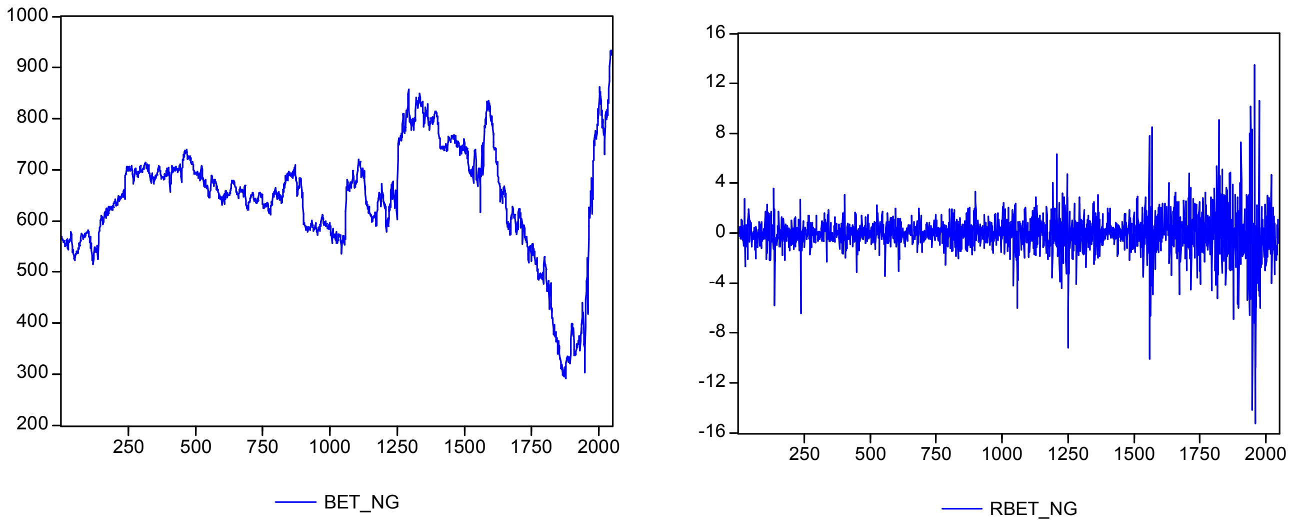

Fig. 4. Daily prices and returns for the BET-NG Index (10 June 2008 - 29 July 2016)

Table 1. Augumented Dickey-Fuller Test output

Index	ADF Test (Probability)	Critical Values		
		1%	5%	10%
BET	-0.803 (0.8177)	-3.432	-2.862	-2.567
RBET	-61.962 (0.0001)	-3.432	-2.862	-2.567
BET-XT	-0.542 (0.8805)	-3.433	-2.863	-2.567
RBET-XT	-41.143 (0.0000)	-3.433	-2.863	-2.567
BET-FI	-0.882 (0.7944)	-3.432	-2.862	-2.567
RBET-FI	-55.618 (0.0001)	-3.432	-2.862	-2.567
BET-NG	-1.214 (0.6704)	-3.433	-2.863	-2.567
RBET-NG	-41.460 (0.0000)	-3.433	-2.863	-2.567

Source: Authors' calculations. Note: The results presented above correspond to the ADF Test including a constant, but results also holds for the other two options – "constant and trend", and "none".

Table 2. Descriptive Statistics of the BSE Indices Daily Returns

Index	Mean	Min.	Max.	Std. Dev.	Skewness	Kurtosis	Jarque-Bera (Prob.)
RBET	0.041	-20.770	17.625	1.782	-0.237	16.210	33885.02 (0.000000)
RBET-XT	-0.015	-12.687	11.024	1.733	-0.618	12.972	8620.79 (0.000000)
RBET-NG	-0.024	-15.257	13.455	1.693	-0.557	17.608	18325.06 (0.000000)
RBET-FI	0.085	-16.076	13.826	2.310	-0.075	9.995	8008.78 (0.000000)

Source: Authors' calculations.

Table 3. The adequate ARMA models for the conditional mean of the BSE Indices Daily Returns

Index	The adequate ARMA model	Adjusted R-squared	Akaike info criterion	ARCH Test
RBET	MA(2)	0.009	3.983	569.155 (0.000000)
RBET-XT	ARMA(3,3)	0.029	3.911	228.776 (0.000000)
RBET-FI	ARMA(1,2)	0.017	4.496	334.643 (0.000000)
RBET-NG	ARMA(2,2)	0.019	3.875	300.059 (0.000000)

Source: Authors' calculations. Note: A probability of ARCH-LM Test less than 5% leads to null hypothesis rejection which means that there is ARCH effects.

Table 4. Estimation results of GARCH models for BET Index Daily Returns

	Variance Equation			
Coefficients	ARCH(2)	GARCH(2,3)	EGARCH(2,3)	GJR-GARCH(2,1)
ω	1.306261	0.001543	-0.065115	0.014413
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
α_1	0.488379	0.313258	0.404622	0.335391
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
α_2	0.187702	-0.300392	-0.311163	-0.232942
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β_1		1.354929	1.586224	0.915731
		(0.0000)	(0.0000)	(0.0000)
β_2		-0.215003	-0.704380	
		(0.0120)	(0.0000)	
β_3		-0.152060	0.114859	
		(0.0000)	(0.0166)	
δ_1			0.121237	
			(0.0000)	
δ_2			-0.121418	
			(0.0000)	
γ				-0.025974
				(0.0000)
		Akaike Information Criterion		
	3.682774	3.504679	3.490524	3.509086
		Log Likelihood		
	-8563.815	-8147.387	-8111.448	-8157.644
		ARCH-LM Test		
Obs. R-squared (Prob.)	0.023859 (0.877245)	0.047373 (0.827700)	0.309688 (0.577872)	0.521286 (0.470294)

Source: Authors' calculations.

As can be seen from Table 3, values of ARCH Test of 569.155, 228.776, 334.643 and300.059witha probability of zero indicate that the null hypothesis of ARCH Test is rejected, case in which we can run the ARCH family models. Thus, having established that the conditional variance varies over time, we proceed to model this heteroscedasticity by applying the symmetric and asymmetric$GARCH$ models.These models are estimated using the maximum likelihood method under the assumption of Gaussian distributed innovation terms (see Tables 4-7).

The volatility of the BET Index returns have been modeled using the following symmetric and asymmetric GARCH models:$MA(2)$ $ARCH(2)$, $MA(1)$ $GARCH(2,3)$, $MA(2)$ $EGARCH(2,3)$ and $MA(2)$ GJR $GARCH(2,1)$. Estimating BET Index returns using $MA(2)$ $GARCH(2,3)$ model, we found out that the coefficients of the conditional mean are not statistically significant at any confidence level (1%, 5%, and 10%), which led us choosing the $MA(1)$ process for estimating the predictable component.

Table 5. Estimation results of GARCH models for BET-XT Index Daily Returns

Variance Equation				
Coefficients	ARCH(2)	GARCH(1,2)	EGARCH(3,1)	GJR-GARCH(1,2)
ω	0.527213 (0.0000)	0.039632 (0.0000)	-0.167785 (0.0000)	0.028727 (0.0000)
α_1	0.722862 (0.0000)	0.269717 (0.0000)	0.400711 (0.0000)	0.319088 (0.0000)
α_2	0.372127 (0.0000)		-0.074551 (0.0620)	
α_3			-0.098714 (0.0095)	
β_1		0.495290 (0.0000)	0.991137 (0.0000)	0.558616 (0.0000)
β_2		0.244195 (0.0040)		0.231578 (0.0191)
δ_1			0.097018 (0.0000)	
δ_2			-0.034527 (0.2141)	
δ_3			0.014891 (0.5402)	
γ				-0.192599 (0.0000)
Akaike Information Criterion				
	3.333772	3.162049	3.144314	3.150222
Log Likelihood				
	-3400.449	-3223.776	-3201.634	-3210.677
ARCH-LM Test				
Obs. R-squared (Prob.)	2.808187 (0.093784)	0.079024 (0.778625)	0.010356 (0.918942)	0.054976 (0.814621)

Source: Authors' calculations.

Table 6. Estimation results of GARCH models for BET-FI Index Daily Returns

Variance Equation				
Coefficients	ARCH(5)	GARCH(2,2)	EGARCH(1,1)	GJR-GARCH(1,1)
ω	0.567724 (0.0000)	0.006453 (0.0000)	-0.235248 (0.0000)	0.035292 (0.0000)
α_1	0.399955 (0.0000)	0.287409 (0.0000)	0.353658 (0.0000)	0.193480 (0.0000)
α_2	0.217218 (0.0000)	-0.244914 (0.0000)		
α_3	0.280255 (0.0000)			
α_4	0.114947 (0.0000)			
α_5	0.155443 (0.0000)			
β_1		1.418534 (0.0000)		
β_2		-0.457421 (0.0000)		
δ			0.011390 (0.2101)	
γ				0.005145 (0.7689)
Akaike Information Criterion				
	4.030451	3.950244	3.949824	3.961825
Log Likelihood				
	-7899.760	-7743.353	-7743.529	-7767.082
ARCH-LM Test				
Obs. R-squared (Prob.)	0.332096 (0.564427)	0.515261 (0.472870)	13.99000 (0.000184)	7.992346 (0.004698)

Source: Authors' calculations.

In case of $MA(2)$ GJR $GARCH(2,1)$ model we have obtained significance at 10% confidence level. The empirical results (minimum $AIC = 3.490524$) show that the adequate process for modeling the conditional variance is $EGARCH(2,3)$ model. The a symmetrical $EGARCH(2,3)$ results summarized in Table 4 indicate that all the estimated coefficients are statistically significant at the 1% confidence level. The asymmetric (leverage) parameter δ_i, $i = 1, 2$, is different from zero ($\delta_1 = 0.121237, \delta_2 =$ 0.121418) which means that bad news imply an increased next period volatility than good news of the same sign. Regarding the persistence parameter β, which measures the persistence of shocks to conditional variance, we can see from Table 4 that it is very large (0.996703). This means that variance moves slowly through time. The results of ARCH-LM Test indicate that the conditional variance equation is well specified ($Obs.$ R $squared = 0.309688$ with a p-value of 0.577872 indicates that there is no ARCH effect left in the innovations). The volatility of the BET-XT Index returns have been modeled using the following symmetric and asymmetric GARCH models: $ARMA(3,3)$ $ARCH(2)$, $ARMA(3,3)$ $GARCH$ $(1,2)$, $ARMA(3,3)$ $EGARCH(3,1)$ and $ARMA(3,3)$ GJR $GARCH(1,2)$.

Table 7. Estimation results of GARCH models for BET-NG Index Daily Returns

	Variance Equation			
Coefficients	ARCH(5)	GARCH(2,2)	EGARCH(1,1)	GJR-GARCH(1,1)
ω	0.567724 (0.0000)	0.006453 (0.0000)	-0.235248 (0.0000)	0.035292 (0.0000)
α_1	0.399955 (0.0000)	0.287409 (0.0000)	0.353658 (0.0000)	0.193480 (0.0000)
α_2	0.217218 (0.0000)	-0.244914 (0.0000)		
α_3	0.280255 (0.0000)			
α_4	0.114947 (0.0000)			
α_5	0.155443 (0.0000)			
β_1		1.418534 (0.0000)		
β_2		-0.457421 (0.0000)		
δ			0.011390 (0.2101)	
γ				0.005145 (0.7689)
		Akaike Information Criterion		
	4.030451	3.950244	3.949824	3.961825
		Log Likelihood		
	-7899.760	-7743.353	-7743.529	-7767.082
		ARCH-LM Test		
Obs. R-squared (Prob.)	0.332096 (0.564427)	0.515261 (0.472870)	13.99000 (0.000184)	7.992346 (0.004698)

Source: Authors' calculations.

Based on the assumption of 5% confidence level, the following estimated parameters of $EGARCH(3,1)$ model are not statistically significant, respectively: $\alpha_2 = 0.074551(p \quad value = 0.0620)$, $\delta_2 = 0.034527$ $(p \quad value = 0.2141)$ and $\delta_3 = 0.014891$ $(p \quad value = 0.5402)$.AIC indicates that the adequate process for modeling the conditional variance is $GJR \quad GARCH(1,2)$ model. Analyzing the asymmetrical $GJR \quad GARCH(1,2)$ results, summarized in Table 5, we see that all the estimated coefficients are statistically significant at the 5% confidence level. For a leverage effect, the asymmetric parameter γ should be grater than zero. In our caseγ is negative, $\gamma = 0.192599$, but the $GJR \quad GARCH$ model is still admissible provided that the condition for non-negativity is fulfilled $(\alpha_1 + \gamma = 0.126489 > 0)$. The results of ARCH-LM Test indicate that the conditional variance equation is well specified $(Obs. \; R \quad squared = 0.054976$ with a p-value of 0.814621 indicates that there is no ARCH effect left in the innovations).The volatility of the BET-FI Index returns have been modeled using the following symmetric and asymmetric $GARCH$ models: $ARMA(1,2) \quad ARCH(5)$, $ARMA(1,2) \quad GARCH(2,2)$, $ARMA(1,2) \quad EGARCH(1,1)$ and $ARMA(1,2) \quad GJR \quad GARCH(1,1)$. Estimating BET-FI Index returns using asymmetric $GARCH$ models, we found out that for both $EGARCH(1,1)$ model and $GJR \quad GARCH(1,1)$ model the asymmetric parameteris not statistically significant at any confidence level (1%, 5%, and 10%). Furthermore, modeling the conditional variance using $GARCH(2,2)$ model, the $ARCH$ term α_2 and the GARCH termβ_2 are both less than zero and the persistence coefficient measured as $\sum_i \alpha_i + \sum_j \beta_j$ is grater than one (1.003608) meaning that the conditional variance is explosive instead of weak stationary. It implies than we cannot model BET-FI Index returns using $GARCH$ model. In case of $ARCH(5)$ model, restrictions for positive conditional variance are fulfilled since$\alpha_i \geq 0$, for all$i = \overline{1,5}$. The results of ARCH-LM Test presented in Table 6 show that there is no ARCH effect left in the innovations $(Obs. \; R \quad squared = 0.332096$with a p-value of 0.564427).

The volatility of the BET-NG Index returns have been modeled using the following symmetric and asymmetric $GARCH$ models: $ARMA(2,2) \quad ARCH(4)$,$ARMA(2,2) \quad GARCH\ (2,2)$, $ARMA(2,2) \quad EGARCH(1,1)$ $ARMA(2,2)$ and $GJR \quad GARCH(2,2)$. Table 7 points out that $ARMA(2,2) \quad EGARCH(1,1)$ is the most adequate model for estimating the conditional variance (having the minimum AIC, which is 3.162398). The asymmetrical $EGARCH(1,1)$ results summarized in Table 7 indicate that all the estimated coefficients are statistically significant at the 1% confidence level. The presence of leverage effects in the returns of BET-NG is confirmed by the non-zero asymmetric (leverage) parameter δ which has a value of 0.078629. In terms of volatility persistence, Table 7 shows a very large value of the estimated persistence parameter $(\beta = 0.976939)$. This imply a slowly decreasing of the rises in the conditional variance due to shocks. Likewise, ARCH-LM Test indicate that the conditional variance equation is well specified as there is no ARCH effect left in the innovations $(Obs. \; R \quad squared = 3.023597$ with a p-value of 0.082061).

Conclusion

In this paper, we analyze and compare the joint symmetric and asymmetric$ARMA \quad GARCH$ models applied to the continuously compounded daily returns ofthe following Romanian stock indices: BET, BET-XT, BET-FI and BET-NG. In doing so, we estimated $ARCH$, $GARCH$, $EGARCH$ and $GJR \quad GARCH$ models using the maximum likelihood method under the assumption of Gaussian distributed innovation terms for each index returns series over the following periods: 22 September 1997 - 29 July 2016 (BET Index), 10 June 2008 - 29 July 2016 (BET-XT Index), 31 October 2000 - 29 July 2016 (BET-FI Index) and 10 June 2008 - 29 July 2016 (BET-NG Index).The empirical results show that asymmetric $GARCH$ models perform better in case of BET, BET-XT and BET-NG, while for BET-FI we found evidence that $ARCH$ model fits better. The volatility of the BET Index returns is modeled using the asymmetrical $EGARCH(2,3)$model where we found that the lever age coefficient δ_i, $i = 1, 2$, is different from zero ($\delta_1 = 0.121237, \delta_2 = \ 0.121418$) meaning that bad news imply an increased next period volatility than good news of the same sign and, on the other hand - the persistence of shocks to conditional variance is very large (0.996703)i.e. variance moves slowly through time. In case of the BET-XT Index returns, the volatility is estimated through the asymmetrical $GJR \quad GARCH(1,2)$model where the leverage coefficient is negative, $\gamma = \ 0.192599$, but the model is still admissible provided that the condition for non-negativity is fulfilled ($\alpha_1 + \gamma = 0.126489 > 0$). The empirical results obtained for modeling the BET-FI Index returns are very interesting taking into account that the estimated coefficients for both $EGARCH(1,1)$ and $GJR \quad GARCH(1,1)$are not statistically significant at any confidence level (1%, 5%, and 10%). Moreover, modeling the conditional variance using $GARCH(2,2)$ model we have found out that the restrictions required to ensure a positive and weak-stationary conditional variance are not fulfilled.Thus, the explosive volatility leads to modeling BET-FI Index returns using$ARCH$ model instead of $GARCH$ model, where the restrictions for positive conditional variance are fulfilled. The energy sector index, BET-NG, captures both volatility clustering and leverage effects. Therefore, its conditional variance is modeled using the asymmetrical $EGARCH(1,1)$ whose estimated coefficients are statistically significant at the 1% confidence level. The presence of leverage effects in the returns of BET-NG is confirmed by the positive asymmetric (leverage) parameter ($\delta = 0.078629$), while the persistence parameter ($\beta = 0.976939$) implies a slowly decrease of the rises in the conditional variance due to shocks. Moreover, the ARCH-LM Test for all symmetric and asymmetric $GARCH$ models indicatesthat there is no ARCH effect left in the innovations. The empirical results show that in three cases out of four, the volatility reacted asymmetrically to the good and bad news. The predominant model turned out to be $EGARCH$ model and this is because it does not require any constraint on the parameters since the positivity of the conditional variance is automatically satisfied.

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