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RESEARCH ARTICLE

UNSTEADY MHD FLOW OF NON-NEWTONIAN FLUID IN A PLANAR CHANNEL

*Dr. P. Sulochana

Professor in Mathematics, Intell Engineering College, Anantapuramu (A.P.)

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ABSTRACT

We consider MHD oscillating flow of an incompressible electrically conducting non-Newtonian fluid through porous medium in a pipe under the influence of uniform magnetic field and taking Hall current into account. An investigation is made to perceive the influences of magnetic field, permeability and Hall current on the flow of non-Newtonian fluid. The closed form analytical solution is obtained making use of Fourier transform technique.

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INTRODUCTION

Hartmann flow is a classical problem that has many important applications in magnetohrdrodynamics (MHD) generators and pumps. Hartmann (1937) first studied an incompressible viscous electrically conducting fluid flow between two infinite parallel non-conducting stationary disks under the action of a transverse magnetic field. Under different physical conditions it was considered by Hughes and Young (1966), Cowling (1957) and Pai (1962). The solutions for the velocity fields in closed form were studied (Sutton and Sherman, 1965; Alpher, 1961; Cramer and Pai, 1973) under different physical effects. Due to the growing use of non-Newtonian fluids material in many manufacturing and processing industries, considerable efforts have been directed towards understanding their flows. Many workers (Rajagopal, 1992; Rajagopal and Gupta, 1981; Ersoy, 1999) examined the non-Newtonian fluid in different geometries. The one-dimensional rate type viscoelastic Burgers' model (Burgers, 1935) has been used to characterize diverse viscoelastic materials; food products such as cheese, soil, asphalt, etc. Ravindran *et al*. (1973), Siddiqui *et al*. (?) and Rana *et al*. (2007) has studied the Burgers' fluid. Burgers (1935) proposed one-dimensional rate type visco-elastic model to describe the response of materials such as asphalt. Murali *et al*. (2004, 2003) have developed a fully three dimensional model that satisfies the invariance of frame indifference. It is a nonlinear model and, unlike the Maxwell model and the Oldroyd-B model, etc., it involves the stress being differentiated twice with respect to the Oldroyd upper convected derivative. This model can be linearlized by requiring that the elastic responses are sufficiently small from the appropriate natural configuration. Linearizing the model and restricting it to one dimension leads to model proposed by Burgers (1935). The extra stress and the symmetric part of the velocity gradient involved in this model have higher order Oldroyd derivatives. It turns out that in general we need additional boundary as well as initial conditions. This issue crops up with regards to several non-Newtonian fluid models. So the issue of whether the no-slip boundary condition is sufficient to have a well-posed problem is very important. The detailed critical review on the boundary conditions, the existence and uniqueness of the solution has been given by Rajagopal (1995, 1982), Rajagopal and Kaloni (1989) and Rajagopal *et al*.(1986). Rajagopal and Gupta (1984) augmented the boundary conditions and studied the flow of a second grade past a porous plate. Unfortunately, the above investigations do not include the Hall effect. The Hall term was ignored in applying Ohm's law as it has no remarkable effect for small and moderate values of the magnetic field. The recent trend, however, for the applications of MHD is towards a strong magnetic field, so that the influence of electromagnetic force is noticeable (Cramer and Pai, 1973). The study of MHD flows with Hall currents has important engineering applications in flight MHD, MHD generators and Hall accelerators. Hossain (1986), Hossain and Mohammad (1988), Attia and Syed-Ahmed (2004), Attia and Aboul-Hassan (2003) and Siddiqui *et al*. (2006, 2007) studied the Hall effects. Keeping the above

****Corresponding author: Dr. P. Sulochana,***
Professor in Mathematics, Intell Engineering College, Anantapuramu (A.P.)

mentioned facts in this paper, we have considered MHD oscillating flow of an incompressible electrically conducting Burger's fluid through porous medium in a pipe under the influence of uniform magnetic field and taking Hall current into account. The considered fluid model is a visco-elastic model and has been used to characterize food products such as cheese (Alpher, 1961), soil (Cramer and Pai, 1973), asphalt and asphalt mixes (Rajagopal, 1992; Rajagopal and Gupta, 1981) etc.

Formulation and solution of the problem

We consider MHD flow of an incompressible electrically conducting non-Newtonian fluid through a porous medium in pipe under the influence of uniform transverse magnetic field taking Hall current into account.

The constitutive equation for a Burger's fluid is

$$T = -pI + S,\ S + \lambda \frac{\delta S}{\delta t} = \mu\left(1 + \lambda_r \frac{\delta}{\delta t}\right) A_1 \tag{1}$$

In which T is the Cauchy stress tensor, p is the reaction stress due to constraint of incompressibility, S is the constitutively determined extra stress, A_1 is the first Rivlin-Ericksen tensor, λ the relaxation time, μ is the dynamic viscosity, $\lambda_r\left(<\lambda\right)$ is the retardation time and the upper convected derivative is

$$\frac{\delta S}{\delta t} = \frac{dS}{dt} - LS - SL^T \tag{2}$$

Where L is the velocity gradient.

In the following we consider an axially symmetric and fully developed flow of a Burger's fluid whose extra stress tensor and velocity field, in a system of cylindrical polar coordinates are of the form

$$S\left(r,t\right) = \begin{pmatrix} S_{rr} & S_{r\theta} & S_{rz} \\ S_{\theta r} & S_{\theta\theta} & S_{\theta z} \\ S_{zr} & S_{z\theta} & S_{zz} \end{pmatrix},\ V\left(r,t\right) = \begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix} \tag{3}$$

Where the initial condition $S\left(r,0\right)=0$ i.e. the fluid being at rest up to the moment at $t=0$ holds and the imposed oscillating pressure gradient is

$$\frac{\partial p}{\partial z} = P_0 e^{i\omega_0 t} \tag{4}$$

Where ω_0 is the oscillating frequency and P_0 is the amplitude. Moreover the z-axis acts as the axis of the cylinder and a uniform magnetic field B_0 is applied transversely to the axis of the circular cylinder. The magnetic Reynolds number is assumed to be very small, so that the induced magnetic field is negligible. There is no applied voltage so the electric field $E=0$ (Ersoy, 1999). If the Hall term is retained in generalized Ohm's law then the following expression holds (Burgers, 1935)

$$J + \frac{\omega_e \tau_e}{B_0}\left(J \times B\right) = \sigma\left[E + V \times B + \frac{1}{en_e}\nabla p_e\right] \tag{5}$$

In which B is the total magnetic field, ω_e is the cyclotron frequency of electrons, τ_e is the electron collision time, σ is the electrical conductivity, e is the electron charge, n_e is the number density of electron and p_e is the electron pressure. The ion-slip and thermoelectric effects are not included in Eq. (5). Further it is assumed that $\omega_e \tau_e \sim O\left(1\right)$ and $\omega_i \tau_i < 1$ where ω_i and τ_i are cyclotron frequency and collision time for ions respectively.

The no-slip boundary condition for the problem under consideration is

$$u\left(a,t\right) = 0 \tag{6}$$

Where

a is the radius of the cylinder.

By virtue of Eq. (3), the continuity equation is automatically satisfied and Eqs. (1) and (2) and balance of linear momentum gives $S_{rr} = S_{r\theta} = S_{\theta\theta} = S_{\theta z} = \partial p / \partial r = \partial p / \partial \theta = 0$ and (Ravindran *et al.*, 2004; Siddiqui *et al.*, ?; Rana *et al.*, 2007; Murali Kishnan and Rajagopal, 2004).

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(rS_{rz}\right) - \frac{\sigma B_0^2}{1-im}u - \frac{\nu}{k}u \tag{7}$$

$$\left(1 + \lambda\frac{\partial}{\partial t} + \beta\frac{\partial^2}{\partial t^2}\right) S_{rz} = \mu\left(1 + \lambda_r \frac{\partial}{\partial t}\right)\frac{\partial u}{\partial r} \tag{8}$$

$$S_{zz} + \lambda\left(\frac{\partial S_{zz}}{\partial t} - 2S_{rz}\frac{\partial u}{\partial r}\right) = -2\mu\lambda_r\left(\frac{\partial u}{\partial r}\right)^2 \tag{9}$$

Where

$m = \omega_e \tau_e$ is the Hall parameter, ν is the kinematic viscosity, k is the permeability of porous medium.

Elimination S_{rz} of from Eqs. (7) and (8) yields

$$\rho\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = -\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial p}{\partial z} + \mu\left(1+\lambda_r\frac{\partial}{\partial t}\right)\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right] - \left(\frac{\sigma B_0^2}{1-im} + \frac{\nu}{k}\right)\left(1 + \lambda\frac{\partial}{\partial t} + \beta\frac{\partial^2}{\partial t^2}\right)u \tag{10}$$

The above equations can be normalized using the following dimensionless parameters

$$u^* = \frac{u}{U_0},\ r^* = \frac{r}{a},\ t^* = \frac{t}{a^2/\nu},\ \omega_0^* = \frac{\omega_0}{\nu/a^2},\ Q_0 = \frac{P_0}{\left(\mu/a^2\right)U_0},$$

$$\lambda^* = \frac{\lambda}{a^2/\nu},\ \lambda_r^* = \frac{\lambda_r}{a^2/\nu},\ M^2 = \frac{\sigma B_0^2}{\left(\mu/a^2\right)},\ K = \frac{k}{a^2} \quad (11)$$

Where U_0 is the reference velocity and ν is the kinematic viscosity of the fluid. Accordingly, Eqs. (6) and (10) after neglecting the dimensionless mark $''*''$ for simplicity reduce to

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = -Q_0\left(1+\lambda\frac{\partial}{\partial t}\right)e^{i\omega_0 t} + \left(1+\lambda_r\frac{\partial}{\partial t}\right)\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right]$$
$$-\left(\frac{M^2}{1-im}+\frac{1}{K}\right)\left(1+\lambda\frac{\partial}{\partial t}\right)u$$
(12)

The corresponding boundary conditions are

$$u(\pm 1, t) = 0 \quad (13)$$

In order to solve the governing problem we define the temporal Fourier transform pair as

$$\psi(r,\omega) = \int_{-\infty}^{\infty} u(r,t)e^{-i\omega t}dt \quad (14)$$

$$u(r,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\psi(r,\omega)e^{i\omega t}d\omega \quad (15)$$

Taking Fourier transform to Eqs. (12) and (13) and then solving the resulting problem, we have the following general solution

$$\psi(r,\omega) = \frac{Q_0\left(1+i\lambda\omega_0\right)}{\xi^2\left(1+i\lambda_r\omega\right)}\left\{1-\frac{J_0(\xi r)}{J_0(\xi)}\right\}\delta(\omega-\omega_0) \quad (16)$$

Where $J_0(\cdot)$ is the zeroth-order Bessel function, $\delta(\cdot)$ is the dirac delta function and

$$\xi^2 = -\left(\frac{M^2}{1-im}+\frac{1}{K}+i\omega\right)\left[\frac{1+i\lambda\omega}{1+i\lambda_r\omega}\right]$$

The Fourier inversion of Eq. (16) after using the property of delta function gives

$$u(r,t) = \frac{Q_0\left(1+i\lambda\omega_0\right)}{\xi_0^2\left(1+i\lambda_r\omega_0\right)}\left\{1-\frac{J_0(\xi_0 r)}{J_0(\xi_0)}\right\}e^{i\omega_0 t} \quad (17)$$

Where

$$\xi_0 = \xi\big|_{\omega=\omega_0}$$

RESULTS AND DISCUSSION

We have considered the MHD oscillating flow of an incompressible electrically conducting non-Newtonian (Oldroyd-B) fluid through porous medium in a pipe under the influence of uniform magnetic field and taking Hall current into account. An investigation is made to perceive the influences of magnetic field, permeability and Hall current on the flow of Berger's fluid. The closed form analytical solution is obtained making use of Fourier transform technique. The computational work has been carried out for the governing flow through porous medium in a pipe making use of Mathematical software. This flow is governed by the non-dimensional parameters like, M Hartmann number, K permeability parameter, m Hall parameter, β the rheological parameter and ω the frequency of oscillation. For computational purpose we are fixing some parameters $\lambda = 5, \lambda_r = 1, Q_0 = -2, t = 1$. Special attention has been given to examine the velocity profiles for five different kinds of non-dimensional parameters, which are depict in the Figures (1-5).

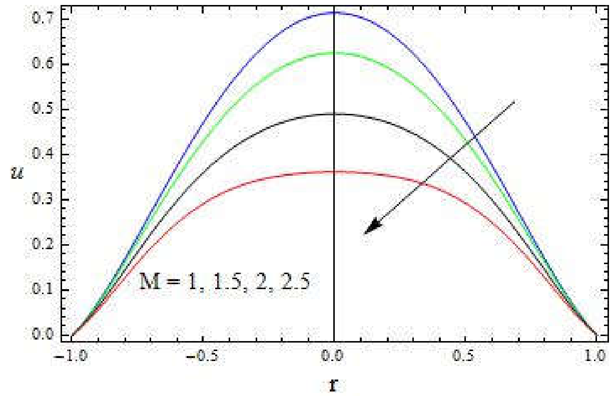

Fig.1. The velocity Profile against *M* with $K=1, m=1, \beta=1, \omega=1.2$

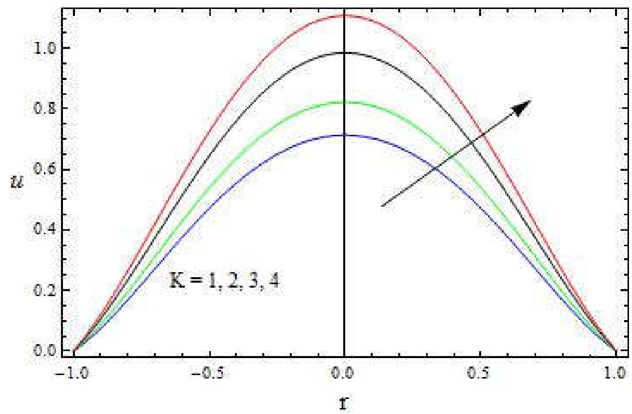

Fig.2. The velocity Profiles against *K* with
$M = 1.5, m = 1, \beta = 1, \omega = 1.2$

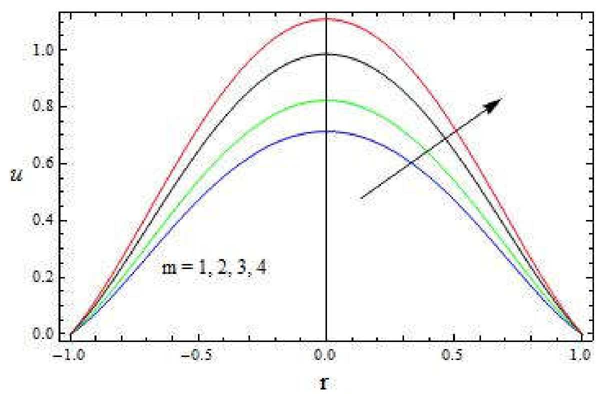

Fig.3. The velocity Profiles against *m* with
$M = 1.5, K = 1, \beta = 1, \omega = 1.2$

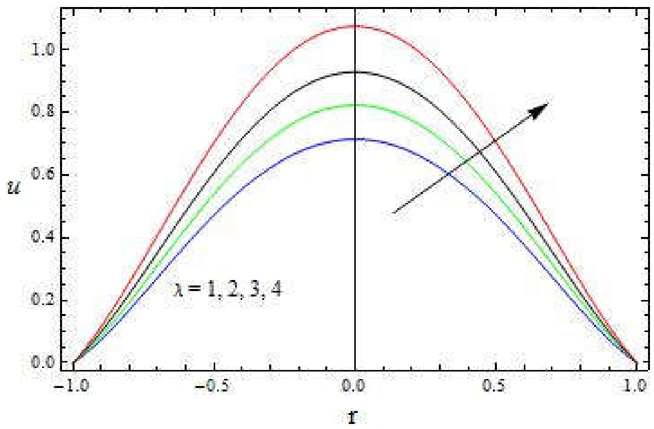

Fig.4. The velocity Profiles against λ with $M = 1.5, K = 1, m = 1, \omega = 1.2$

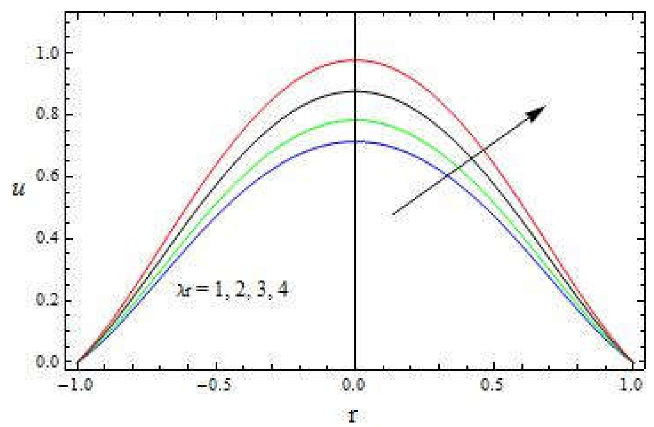

Fig.5. The velocity Profiles against λ_r with $M = 1.5, K = 1, m = 1, \beta = 1$

We noticed that from the Fig.1, in the presence of magnetic force, an increase of the magnitude in the magnetic parameter *M* reduces the velocity profile monotonically due to the effect of the magnetic force against the flow direction. This is in accordance to the fact that the magnetic field is responsible to reduce the velocity. From the Figure (2), the velocity enhance with increasing the permeability parameter *K* throughout the fluid region. We also observe that lower the permeability lesser the fluid speed in the entire fluid medium. Similar behavior is observed with increasing Hall parameter *m*. Figure 3 shows that the increase of Hall parameter *m* for fixed magnetic parameter *M* increases the velocity profiles. Moreover, the velocity increases. Further, when the magnetic Reynolds number is very small, the flow pattern with Hall effect is remarkably analogous to that for the non-conducting flow. Obviously, the supposition of very small magnetic Reynolds number will be legitimate for flow of liquid metals or slightly ionized gas. Also from the Figures (4 & 5) appears that the velocity is an increasing function of the λ, λ_r of the Oldroyd-B fluid.

Conclusion

We have considered the MHD oscillating flow of an incompressible electrically conducting non-Newtonian (Oldroyd-B) fluid through porous medium in a pipe under the influence of uniform magnetic field and taking Hall current into account. The conclusions are made as the following. An increase of the magnitude in the magnetic parameter *M* reduces the velocity profiles monotonically due to the effect of the magnetic force against the flow direction.

1. The velocity enhance with increasing the permeability parameter *K* or Hall parameter *m* throughout the fluid region.
2. Lower the permeability lesser the fluid speed in the entire fluid medium.
3. The solution of Oldroyd-B fluid only contributes if there is a pressure gradient of the oscillatory nature.

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