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RESEARCH ARTICLE

PRE- ρ -COMPACT SPACE IN A TOPOLOGICAL SPACE

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ABSTRACT

In this paper pre-L-compact, pre-R-compact, pre-L-locally compact, pre-R-locally compact, sequentially pre-L-compact, sequentially pre-R-compact, countably pre-L-compact, countably pre-R-compact are introduced and the relationship between these concepts are studied.

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INTRODUCTION

Mashhour, M.E Abd El.Monsef and S.N.El-Deeb (Mashhour *et al*., 1982) introduced a new class of pre-open sets in 1982. R.Selvi and M.Priyadarshini introduced a new class of pre-L-open sets in 2016(October). In this paper pre-L-compact, pre-R-compact, pre-L-locally compact, pre-R-locally compact, sequentially pre-L-compact, sequentially pre-R-compact, countably pre-L-compact, countably pre-R-compact are defined and their properties are investigated.

PRELIMINARIES

Throughout this paper $f^{-1}(f(A))$ is denoted by A^* and $f(f^{-1}(\mathrm{B}))$ is denoted by B^*.

Definition 2.1

Let A be a subset of a topological space (X, τ). Then A is called pre-open if A $\subseteq$ int(cl(A)) and pre-closed if cl(int(A)) $\subseteq$ A; (Mashhour *et al*., 1982).

Definition 2.2.

Let f: (X, τ) $\rightarrow$(Y, σ) be a function. Then f is pre-continuous if f $^{-1}$(B) is open in X for every pre-open set B in Y. (Mashhour *et al*., 1982)

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Definition: 2.3.

Let f: (X, τ) $\rightarrow$(Y, σ) be a function. Then f is pre-open (resp. pre-closed) if f(A) is pre-open(resp. pre-closed) in Y for every pre-open(resp. pre-closed) set A in X. (Mashhour *et al*., 1982)

Definition: 2.4.

Let f: (X, τ)$\rightarrow$Y be a function. Then f is

1) p-L-Continuous if A^* is open in X for every pre-open set A in X.
2) p-M-Continuous if A^* is closed in X for every pre-closed set A in X. (Selvi and Priyadarshini, 2016)

Definition: 2.5.

Let f: X$\rightarrow$(Y, σ) be a function. Then f is

1) p-R-Continuous if B^* is open in Y for every pre-open set B in Y.
2) p-S-Continuous if B^* is closed in Y for every pre-closed set B in Y. (Selvi and Priyadarshini, 2016)

Definition: 2.6.

Let f: (X, τ) $\rightarrow$(Y, σ) be a function, then f is said to be

1) P-irresolute if $f^{-1}(V)$ is pre-open in X, whenever V is pre-open in Y.
2) P-resolute if $f(V)$ is pre-open in Y, whenever V is pre-open in X. (Mashhour *et al*., 1982)

Definition: 2.7.

Let (X,τ) is said to be
1) finitely p-additive if finite union of pre-closed set is pre-closed.
2) Countably p-additive if countable union of pre-closed set is pre-closed.
3) p-additive if arbitrary union of pre-closed set is pre-closed. (Mashhour *et al*., 1982)

Definition: 2.8.

Let (X,τ) be a topological space and x$\in$X .Every pre-open set containing x is said to be a p-neighbourhood of x. (Popa *et al*.,)

Definition: 2.9.

Let A be a subset of X. A point x$\in$X is said to be pre-limit point of A if every pre-neighbourhood of x contains a point of A other than x. (Malghan *et al*.,)

Definition: 2.10.

Let A be a subset of a topological space (X, τ), pre-closure of A is defined to be the intersection of all pre-closed sets containing A. It is denoted by pcl(A). (Erdal .Ekici and Migual calder, 2010)

Definition: 2.11.

Let A be a subset of X. A point x$\in$X is said to be pre-limit point of A if every pre-neighbourhood of x contains a point of A other than x. (Malghan *et al*.,)

Definition: 2.12.

A collection τ of subsets of X is said to have finite intersection property if for every sub collection {C1, C2...........Cn} of τ the intersection C1$\cap$C2$\cap$..........$\cap$Cn is non empty. (James and Munkers, 2010)

Definition: 2.13.

A collection $\{U_\alpha\}_{\alpha \in \Delta}$ of pre-open sets in X is said to be pre-open cover of X if X= $\cup_{\alpha\in\Delta} U_\alpha$. (Leelavathy, 2016)

Definition: 2.14.

A topological space (X, τ) is said to be pre-compact if every pre-open covering of X contains finite sub collection that also cover X. A subset A of X is said to be pre-compact if every covering of A by pre-open sets in X contains a finite subcover. (Leelavathy, 2016)

Definition: 2.15.

A subset A of a topological space (X, τ) is said to be countably pre-compact, if every countable pre-open covering of A has a finite subcover. (Leelavathy, 2016)

Example: 2.16.

Let (X, τ) be a countably infinite indiscrete topological space. In this space $\{\{x\}/ x \in X\}$ is a countable pre-open cover which has no finite subcover. Therefore it is not countably pre-compact. (Leelavathy, 2016)

Definition: 2.17.

A subset A of a topological space (X, τ) is said to be sequentially pre-compact if every sequence in A contains a subsequence which pre-converges to some point in A. (Leelavathy, 2016)

Definition: 2.18.

A topological space (X, τ) is said to be pre-locally compact if every point of X is contained in a pre-neighbourhood whose pre-closure is pre-compact. (Leelavathy, 2016)

Definition: 2.19

Let f: $(X,\tau) \to Y$ be a function and A be a subset of a topological space (X, τ). Then A is called

1) P-L-open if $A^* \subseteq \text{int}(\text{cl}(A^*))$
2) P-M-closed if $A^* \supseteq \text{cl}(\text{int}(A^*))$. (Selvi and Priyadarshini, 2016)

Definition: 2.20.

Let f: $X \to (Y,\sigma)$ be a function and B be a subset of a topological space (Y, σ). Then B is called

1) P-R-open if $B^* \subseteq \text{int}(\text{cl}(B^*))$
2) P-S-closed if $B^* \supseteq \text{cl}(\text{int}(B^*))$. (Selvi and Priyadarshini, 2016)

Example: 2.21.

Let X = {a, b, c, d} and Y = {1, 2, 3, 4}. Let τ = { Φ , X, {a}, {b}, {a, b}, {a, b, c}}. Let f: $(X,\tau) \to Y$ defined by f(a)=1, f(b)=2, f(c)=3, f(d)=4. Then f is p-L-open and p-M-Closed. (Selvi and Priyadarshini, 2016)

Example: 2.22.

Let X = {a, b, c, d} and Y = {1, 2, 3, 4}. Let σ ={ Φ ,Y, {1},{2},{1,2},{1,2,3} }. Let g : $X \to (Y,\sigma)$ defined by g(a)=1, g(b)=2, g(c)=3, g(d)=4. Then g is p-R-open and p-S-Closed. (Selvi and Priyadarshini, 2016)

Definition: 2.23.

Let f: $(X,\tau) \to (Y,\sigma)$ be a function, then f is said to be

1) P-L-irresolute if $f^{-1}(f(\mathrm{A}))$ is pre-L-open in X, whenever A is pre-L-open in X.
2) P-M-irresolute if $f^{-1}(f(\mathrm{A}))$ is pre-M-closed in X, whenever A is pre-M-closed in X.
3) P-R-resolute if $f(f^{-1}(\mathrm{B}))$ is pre-R-open in Y, whenever B is pre-R-open in Y.
4) P-S-resolute if $f(f^{-1}(\mathrm{B}))$ is pre-S-closed in Y, whenever B is pre-S-closed in Y.(Selvi and Priyadarshini, 2016)

Definition: 2.24.

Let (X,τ) is said to be
1) finitely p-M-additive if finite union of p-M-closed set is p-M-closed.
2) Countably p-M-additive if countable union of pre-M-closed set is pre-M-closed.
3) p-M-additive if arbitrary union of pre-M-closed set is pre-M-closed. (Selvi and Priyadarshini, 2016)

Definition: 2.25.

Let (X,τ) be a topological space and $x \in X$ .Every pre-L-open set containing x is said to be a p-L-neighbourhood of x.(Selvi and Priyadarshini, 2016)

Definition: 2.26

Let A be a subset of X. A point $x \in X$ is said to be pre-L-limit point of A if every pre-L-neighbourhood of x contains a point of A other than x.(Selvi and Priyadarshini, 2016)

3. Pre- ρ -compact space

Definition: 3.1

(i) A collection $\{U_\alpha\}_{\alpha \in \Delta}$ of pre-L-open sets in X is said to be pre-L-open cover of X if $X=\bigcup_{\alpha\in\Delta} U_\alpha$.

(ii) A collection $\{U_\alpha\}_{\alpha \in \Delta}$ of pre-R-open sets in X is said to be pre-R-open cover of X if $X=\bigcup_{\alpha\in\Delta} U_\alpha$.

Definition: 3.2.

(i) A topological space (X,τ) is said to be pre-L-compact if every pre-L-open covering of X contains finite sub collection that also cover X. A subset A of X is said to be pre-L-compact if every covering of A by pre-L-open sets in X contains a finite subcover.

(ii) A topological space (X,τ) is said to be pre-R-compact if every pre-R-open covering of X contains finite sub collection that also cover X. A subset A of X is said to be pre-R-compact if every covering of A by pre-R-open sets in X contains a finite subcover.

Theorem: 3.3.

A topological space (X,τ) is

1) pre-L-compact $\Rightarrow$ compact 2) Any finite topological space is pre-L-compact.

Proof:

1) Let $\{A_\alpha\}_{\alpha\in\Omega}$ be an open cover for X. Then each A_α is pre-L- open.

Since X is pre-L-compact, this open cover has a finite subcover. Therefore (X,τ) is compact.

2) Obvious since every pre-L-open cover is finite.

Example: 3.4.

Let (X,τ) be an infinite indiscrete topological space. In this space all subsets are pre-L-open. Obviously it is compact. But $\{x\}x \in X$ is a pre-L-open cover which has no finite subcover. So it is not pre-L-compact. Hence compactness need not imply pre-L-compactness.

Theorem: 3.5 A pre-M-closed subset of pre-L- compact space is pre –L-compact.

Proof:

Let A be a pre-M-closed subset of a pre-L-compact space (X,τ) and {U α } $\alpha \in \Delta$ be a pre –L-open cover for A , then {{U α } $\alpha \in \Delta$, {X-A}} is a pre-L-open cover for X . Since X is pre-L-compact, there exists $\alpha_1, \alpha_2 .... \alpha_n \in \Delta$ such that $X = U\alpha_1 \cup U\alpha_2 ...... \cup U\alpha_n \cup (X - A)$ Therefore $A \subseteq U\alpha_1 \cup U\alpha_2 ...... \cup U\alpha_n$ which proves A is pre-L-compact.

Remark: 3.6.

The converse of the above theorem need not be true as seen in the following example (3.7).

Example: 3.7.

Let X = {a, b, c, } and Y = {1, 2, 3, }. Let f: (X, τ) $\rightarrow$Y defined by f(a)=1, f(b)=2, f(c)=3. Let X={a,b,c} τ ={ ϕ ,{a},X}-open set, closed set-{ ϕ, X, {b, c}}.

Here PLO(X) = { ϕ, X, {a} {a,b},{a,c}} is pre-L-compact ,A={a,c} is Pre-L-compact but not pre-M-closed

Theorem: 3.8.

A topological space (X,τ) is pre-L-compact if and only if for every collection τ Of pre-M-closed sets in X having finite intersection property, $\bigcap_{c\in\tau} C$ of all elements of τ is non empty.

Proof:

Let (X,τ) be pre-L-compact and τ be a collection of pre-M-closed sets with finite intersection property. Suppose $\bigcap_{c\in\tau} C = \phi$ then $\bigcup_{c\in C}(X - C) = X$. Therefore $\{X - C\}_{c\in C}$ is a pre-L-open cover for X. Then there exists C_1, C2,......Cn $\in$ τ such that $\bigcup_{i=1}^{n}(X - C_i)$ =X

Therefore $\bigcap_{i=1}^{n} C_i = \phi$ which is a contradiction. Therefore $\bigcap_{c\in\tau} C \neq \phi$

Conversly assume the hypothesis given in the statement .To prove X is pre-L-compact.

Let {U α } $\alpha \in \Delta$ be a pre-L-open cover for X .

then $\bigcup_{\alpha\in\Delta} U_\alpha = X \Rightarrow \bigcap_{\alpha\in\Delta}(X - U_\alpha) = \phi$ By hypothesis $\alpha_1, \alpha_2 .... \alpha_n$, there exists such that $\bigcap_{i=1}^{n}(X - U_{\alpha_i}) = \phi$. Therefore $\bigcup_{i=1}^{n} U_{\alpha_i} = X$. Therefore X is pre-L-compact.

Corollary: 3.9.

Let (X,τ) be a pre-L-compact space and let $C_1 \supseteq C_2 \supseteq ...... \supseteq C_n \supseteq C_{n+1}....$ be a nested sequence of non empty pre-M-closed sets in X. then $\bigcap_{n\in Z^+} C_n$ is non empty.

Proof:

Obviously $\{C_n\}_{n\in Z^+}$ finite intersection property. By theorem (3.8) $\bigcap_{n\in Z^+} C_n$ is non empty.

Theorem: 3.10.

Let (X,τ), (Y, σ) be two topological space and f: (X, τ) $\rightarrow$(Y, σ) be a bijection then

1) f is pre- continous and X is pre –L-compact $\Rightarrow$ Y is compact.
2) f is pre –L-irresolute and X is pre- L-compact $\Rightarrow$ Y is pre-L- compact .
3) f is continous and X is pre-L-compact $\Rightarrow$ Y is compact.
4) f is strongly irresolute and X is compact $\Rightarrow$ Y is pre- L-compact.
5) f is pre –L-open and Y is pre- L-compact $\Rightarrow$ X is compact .
6) f is open and Y is pre-L- compact $\Rightarrow$ X is compact .
7) f is pre- R-resolute and Y is pre-R-compact $\Rightarrow$ X is pre- R-compact .

Proof:

1)Let $\{U_\alpha\}$ $\alpha \in \Delta$ be a open cover for Y.

Therefore $Y = \cup U_\alpha$. Therefore $X = f^{-1}(Y) = \cup f^{-1}(U_\alpha)$.

Then $\{f^1(U_\alpha)\}$ $\alpha \in \Delta$ is a pre-L- open cover for X.

Since X is pre-L- compact,

there exists $\alpha_1, \alpha_2 .... \alpha_n$ such that $X = \cup f^{-1}(U_{\alpha_i})$. Therefore $Y = f(X) = \cup(U_{\alpha_i})$.

Therefore Y is compact.

Proof of (2) to (4) are similar to the above.

5)Let $\{U_\alpha\}$ $\alpha \in \Delta$ be a open cover for X. then $\{f(U_\alpha)\}$ is a pre-L-open cover for Y.

Since Y is pre-L-compact ,there exists $\alpha_1, \alpha_2 .... \alpha_n$ such that $Y = \cup f(U_\alpha)$

Therefore $X = f^{-1}(\mathrm{Y}) = \cup_{\alpha\in\Delta}(U_\alpha)$. Therefore X is compact.

Proof of (6) and (7) are similar.

Remark: 3.11 From (3) and (6) it follows that" Pre-L- compactness " is a pre-L- topological property.

Theorem: 3.12 (Generalisation of Extreme Value theorem)

Let f: X $\rightarrow$ Y be pre-L-continuous where Y is an ordered set in the ordered topology. If X is pre-L-compact then there exists c and d in X such that f(c) $\leq$ f(x) $\leq$ f(d) for every x $\in$ X.

Proof:

We know that pre-L-continuous image of a pre-L-compact space is compact By theorem(3.10). Therefore A=f(X) is compact. Suppose A has no largest element then $\{(-\infty, a) / a \in A\}$ form an open cover for A and it has a finite subcover.

Therefore $A \subseteq (-\infty, a_1) \cup (-\infty, a_2) \cup ..... \cup (-\infty, a_n)$. Let $a = \max_i a_i$.

Then $A \subseteq (-\infty, a)$ which is a contradiction to the fact that $a \in A$

Therefore A has a largest element M. Similarly it can be proved that it has the smallest element m.

Therefore $\exists$ c and d in X $\ni$ f(c) = m, f(d) = M and f(c) $\leq$ f(x) $\leq$ f(d) $\forall$ x $\in$ X.

4. Countably pre- ρ -compact space

Definition: 4.1.

(i) A subset A of a topological space (X, τ) is said to be countably pre-L-compact, if every countable pre-L-open covering of A has a finite subcover.

(ii) A subset A of a topological space (X,τ) is said to be countably pre-R-compact, if every countable pre-R-open covering of A has a finite subcover.

Example: 4.2.

Let (X,τ) be a countably infinite indiscrete topological space.

In this space $\{\{x\}/ x\in X\}$ is a countable pre-L-open cover which has no finite subcover. Therefore it is not countably pre-L-compact.

Remark: 4.3.

1) Every pre-L-compact space is countably pre-L-compact. It is obvious from the definition.
2) Every countably pre-L compact space is countably compact.

It follows since open sets are pre-Lopen.

Theorem: 4.4.

In a countably pre-L-compact topological space, every infinite subset has a pre-L-limit point.

Proof:

Let (X,τ) be countably pre-L-compact space. Suppose that there exists an infinite subset A which has no pre-L-limit point. Let $B=\{a_n / n\in N\}$ be a countable subset of A.

Since B has no pre-L-limit point of B, there exists a pre-L-neighbourhood U_n of a_n such that $B\bigcap U_n=\{a_n\}$. Now $\{U_n\}$ is a pre-L-open cover for B .Since B^c is pre-L-open , $\{B^c,\{U_n\}_{n\in Z^+}\}$ is a countable pre-L-open cover for X. But it has no finite sub cover, which is a contradicition, since X is countably pre-L-compact .Therefore every infinite subset of X has a pre-L-limit point.

Corollary: 4.5.

In a pre-L-compact topological space every infinite subset has a pre-L-limit point.

Proof:

It follows from the theorem (4.4), since every pre-L-compact space is countably pre-L-compact.

Theorem: 4.6

A pre-M-closed subset of countably pre-L-compact space is countably pre-L-compact.

Proof:

Let X is a pre-L-compact space and B be a pre-M-closed subsets of X.

Let $\{A_i / i=1,2,3,\ldots\infty\}$ be a countable pre-L-open cover for B. Then $\{\{A_i\},X-B\}$ where $i=1,2,3,\ldots\infty$ is a pre-L-open cover for X. Since X is countably pre-L-compact, there exists $i_1,i_2,i_3\ldots.i_n \ni (X-B)\bigcup_{k=1}^{n} A_{ik}=X$.
Therefore $B=\bigcup_{k=1}^{n} A_{ik}$ and this implies B is countably pre-L-compact.

Definition: 4.7.

In a topological space (X,τ) a point $x\in X$ is said to be a pre-L-isolated point of A if there exists a pre-L-open set containing x which contains no point of A other than x.

Theorem: 4.8.

A topological space (X,τ) is countably pre-L-compact if and only if for every countable collection τ of pre-L-closed sets in X having finite intersection property , $\bigcap_{c\in C} C$ of all elements of τ is non empty.

Proof:

It is similar to the proof of theorem(3.8).

Corollary: 4.9.

X is countably pre-L-compact if and only if every nested sequence of pre-M-closed non empty sets $C1 \supset C2 \supset$ ..... .has a non empty intersection.

Proof:

Obviously $\{C_n\}_{n\in Z^+}$ has finite intersection property. By theorem (4.8) $\bigcap_{n\in Z^+} C_n$ is non empty.

5. Sequentially pre- ρ L-compact space

Definition: 5.1.

(i) A subset A of a topological space (X,τ) is said to be sequentially pre-L-compact if every sequence in A contains a subsequence which pre-L-converges to some point in A.
(ii) A subset A of a topological space (X,τ) is said to be sequentially pre-R-compact if every sequence in A contains a subsequence which pre-R-converges to some point in A.

Theorem: 5.2.

Any finite topological space is sequentially pre-L-compact.

Proof:

Let (X,τ) be a finite topological space and $\{x_n\}$ be a sequence in X. In this sequence except finitely many terms all other terms are equal. Hence we get a constant subsequence which pre-L-converges to the same point .

Theorem: 5.3.

Any infinite indiscrete topological space is not sequentially pre-L-compact.

Proof:

Let (X,τ) be infinite indiscrete topological space and $\{x_n\}$ be a sequence in X.

Let x$\in$X be arbitrary.Then U={x} is pre-L- open and it contains no point of the sequence except x.

Therefore $\{x_n\}$ has no subsequence which pre-L-converges to x. Since x is arbitrary, X is not sequentially pre-L-compact.

Theorem: 5.4.
A finite subset A of a topological space (X,τ) is sequentially pre-L-compact.

Proof:

Let $\{x_n\}$ be an arbitrary sequence in X. Since A is finite, at least one element of the sequence say x0 must be repeated infinite number of times. So the constant subsequence x_0, x_0 ….. must pre-L-converges to x_0.

Remark: 5.5.

Sequentially pre-L-compactness implies sequentially compactness, since all open sets are pre-L-open .But the inverse implication is not true as seen from (5.6).

Example: 5.6.

Let (X,τ) be an infinite indiscrete space is sequentially compact but not sequentially pre-L-compact.

Theorem: 5.7.

Every sequentially pre-L-compact space is countably pre-compact.

Proof:

Let (X,τ) be sequentially pre-L-compact. Suppose X is not countably pre-L-compact. Then there exists countable pre-open cover $\{U_n\}_{n\in Z^+}$ which has no finite sub cover. Then $X=\bigcup_{n\in Z^+}U_n$. Choose $X_1\in U_1, X_2\in U_2-U_1, X_3\in U_3-\bigcup_{i=1,2}U_i \ldots\ldots X_n\in U_n-\bigcup_{i=1}^{n}U_i$. This is possible since {Un} has no finite sub cover. Now $\{x_n\}$ is a sequence in X. Let $x\in X$ be arbitrary .then $x\in Uk$ for some K .By our choice of $\{x_n\}$, xi $\notin$ Uk for all $i\geq k$. Hence there is no subsequence of $\{x_n\}$ which can pre-L-converge to x. Since x is arbitrary the sequence $\{x_n\}$ has no pre-L-convergent subsequence which is a contradiction. Therefore X is countably pre-L-compact.

Theorem: 5.8.

Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be a bijection, then

1) f is pre-R-resolute and Y is sequentially pre-R-compact $\Rightarrow$ X is sequentially pre-R-compact.
2) f is pre-L-irresolute and X is sequentially pre-compact $\Rightarrow$ Y is sequentially pre-L-compact.
3) f is continuous and X is sequentially pre-L-compact $\Rightarrow$ Y is sequentially pre-L-compact.
4) f is strongly pre-L-continous and X is sequentially pre-L-compact $\Rightarrow$ Y is sequentially pre-L-compact.

Proof:

1) Let $\{x_n\}$ be a sequence in X .Then $\{f(x_{nk})\}$ is a sequence in Y. It has a pre –R-convergent subsequence $\{f(x_{nk})\}$ such that $\{f(x_{nk})\}\xrightarrow{pre} y_0$ in Y. Then there exists $x_0\in X$ such that $f(x_0)=y_0$. Let U be pre-R -open set containing x0 then f(U) is a pre-R-open set containing y0 .Then there exists N such that f $\in$ f(U) for all k $\geq$N.

Therefore $f^{-1}\circ f(x_{nk})\in f^{-1}\circ f(U)$. Therefore $x_{nk}\in U$ for all $k\geq N$.

This proves that X is sequentially pre-R-compact. Proof for (2) to (4) is similar to the above.

Remark: 5.9.

From theorem (5.8), (1) and (2) it follows that “Sequentially compactness” is a pre- ρ -topological property.

6. pre- ρ -locally compact space

Definition: 6.1.

A topological space (X,τ) is said to be pre-L-locally compact if every point of X is contained in a pre-L-neighbourhood whose pre-L-closure is pre-L-compact.

Theorem: 6.2.

Any pre-L-compact space is pre-L-locally compact.

Proof:

Let (X, τ) be pre-L-compact, Let x ∈ X then X is pre-L-neighbourhood of x and pcl(X)=X which is pre-L-compact.

Remark: 6.3.

The converse need not be true as seen in the following example (6.4)

Example: 6.4.

Let (X, τ) be an infinite indiscrete topological space. it is not pre-L-compact. But for every x ∈ X , {x} is a pre-L-neighbourhood and $\{\overline{x}\} = \{x\}$ is pre-L-compact.

Therefore it is pre-L-locally compact.

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