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RESEARCH ARTICLE

GEOMETRIC MEAN LABELING OF SUBDIVISION ON TRIANGULAR SNAKES

***1Somasundaram, S., 2Sandhya, S.S. and 3Viji, S. P.**

[1]Department of Mathematics, M. S. University, Tirunelveli – 627012
[2]Department of Mathematics, Sree Ayyappa College for Women, Chunkankadai - 629003
[3]Department of Mathematics, K.N.S.K. College of Engineering, Therekalputhoor – 629006

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ABSTRACT

A Graph G = (V, E) with p vertices and q edges is said to be a Geometric mean graph if it is possible to label the vertices $x\in V$ with distinct labels f(x) from 1,2…..,q+1 in such way that when each edge e=uv is labeled with $f(e=uv)=\lceil\sqrt{f(u)f(v)}\rceil$ or $\lfloor\sqrt{f(u)f(v)}\rfloor$, then the resulting edge labels are distinct. In this case, f is called Geometric mean labeling of G. In this paper, we investigate the Geometric mean labeling behaviour of subdivision on Triangular Snakes.

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INTRODUCTION

All graphs considered here will be finite undirected and simple. Let V(G) and E(G) will denote the vertex set and edge set of a graph G. The cardinality of the vertex set of a graph G is denoted by p and the cardinality of its edge set is denoted by q. For all detailed survey of graph labeling we refer to Gallian [1]. For all other standard terminology and notations we follow Harary [2]. The concept of Mean labeling on Subdivision was introduced in [3] and Harmonic mean labeling was introduced in [4]. The concept of Geometric mean labeling was introduced and the basic results proved in [5]. We investigate the Geometric mean labeling behaviour of S(G) for some standard graphs G. The definitions and other informations which are useful for the present investigation are given below.

Definition 1.1: A graph G= (V,E) with p vertices and q edges is said to be a Geometric mean if it is possible to label the vertices $x\in V$ with distinct labels f(x) from 1,2….,q+1 in such a way that when each edge e=uv is labeled with $f(e=uv)=\lceil\sqrt{f(u)f(v)}\rceil$ (or) $\lfloor\sqrt{f(u)f(v)}\rfloor$, then the resulting edge labels are distinct. In this case f is called Geometric mean labeling of G.

Corresponding author: Somasundaram, S.,
Department of Mathematics, M. S. University, Tirunelveli – 627012

Definition 1.2: If e=uv is an edge of G and w is a vertex not in G then e is said to be subdivided when it is replaced by the edges uw and wv. The graph obtained by subdividing each edge of graph G is called the subdivision graph of G is called the the subdivision graph of G and is denoted by S(G).

Definition 1.3: A Triangular Snake T_n is obtained from a path $v_1v_2….v_n$ by joining v_i and v_{i+1} to a new vertex w_i for $1\le i\le n-1$. That is, every edge of a path is replaced by a Triangle C_3.

Definition 1.4: An Alternate Triangular Snakes $A(T_n)$ is obtained from a path $u_1u_2…..u_n$ by joining u_i and u_{i+1} (alternatively) to a new vertex v_i.

That is, every alternate edge of a path is replaced by C_3.

Theorem 1.5[5]: Any path is a Geometric mean graph.

Theorem 1.6[5]: Any cycle is a Geometric mean graph.

Main Results

Theorem: 2.1 Subdivision of Triangular Snake is a Geometric mean graph.

Proof:

Let T_n be a Triangular snake and $u_1 u_2 \ldots u_n$ be path of T_n.
Let $S(T_n) = T_N$ be a graph obtained by subdivide the edges of T_n.

Here we consider the following cases

Case (i)

Let T_N be a graph which is obtained by subdividing each edge of P_n.
Let $t_1, t_2 \ldots\ldots t_{n-1}$ be the vertices which subdivide the edges u_i and u_{i+1}.

Define a function $f: V(T_N) \rightarrow \{1,2\ldots\ldots q+1\}$ by
$f(u_i) = 4i-3, 1 \leq i \leq n$
$f(t_i) = 4i-2, 1 \leq i \leq n-1$
$f(v_i) = 4i, 1 \leq i \leq n-1$
Edges are labeled with
$f(u_it_i) = 4i-3, 1 \leq i \leq n-1$
$f(u_iv_i) = 4i-2, 1 \leq i \leq n-1$.
$f(t_iu_{i+1}) = 4i-1, 1 \leq i \leq n-1$
$f(v_iu_{i+1}) = 4i, 1 \leq i \leq n-1$

The labeling pattern is shown in the following figure.

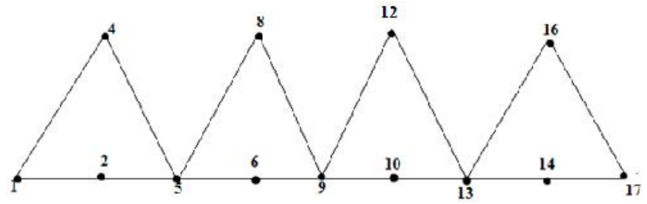

Figure 1.

From the above labeling pattern, we get the edge labels are all distinct. Thus f provides a Geometric mean labeling for T_N

Case (ii): Let T_N be a graph obtained by subdividing the edges u_iv_i and $u_{i+1} v_i$.

Let x_i and y_i be the new vertices which subdivide the edges u_iv_i and v_iu_{i+1}, $1 \leq i \leq n-1$

Define a function $f: V(T_N) \rightarrow \{1,2\ldots\ldots q+1\}$ by
$f(u_i) = 5i-4, 1 \leq i \leq n$
$f(v_i) = 5i-2, 1 \leq i \leq n-1$
$f(x_i) = 5i-3, 1 \leq i \leq n-1$
$f(y_i) = 5i, 1 \leq i \leq n-1$

Then the edges are labeled with,
$f(u_ix_i) = 5i-4, 1 \leq i \leq n-1$
$f(x_iv_i) = 5i-3, 1 \leq i \leq n-1$
$f(u_iu_{i+1}) = 5i-2, 1 \leq i \leq n-1$
$f(v_iy_i), = 5i-1, 1 \leq i \leq n-1$
$f(y_i u_{i+1}) = 5i, 1 \leq i \leq n-1$

In the above labeling pattern, f is a Geometric mean labeling of T_N and the labeling pattern shown in the following figure.

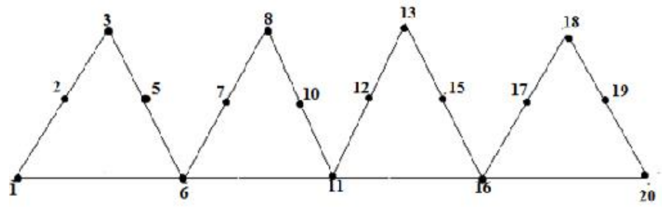

Figure 2.

Case(iii): Subdividing all the edges of T_n.

Let T_N be graph which is obtained by subdividing all the edges of T_n.

Let x_i, y_i and t_i be the new vertices which are subdividing the edges u_iv_i, v_iu_{i+1} and u_iu_{i+1}, $1 \leq i \leq n-1$.

Define a function
$f: V(G) \rightarrow \{1,2\ldots..q+1\}$ by
$f(u_i) = 6i-5, 1 \leq i \leq n$
$f(v_i) = 6i-2, 1 \leq i \leq n-1$
$f(x_i) = 6i-3, 1 \leq i \leq n-1$
$f(y_i) = 6i, 1 \leq i \leq n-1$
$f(t_i) = 6i-4, 1 \leq i \leq n-1$
Edges are labeled with
$f(u_it_i) = 6i-5, 1 \leq i \leq n-1$
$f(u_ix_i) = 6i-4, 1 \leq i \leq n-1$
$f(x_iv_i) = 6i-3, 1 \leq i \leq n-1$
$f(t_iu_{i+1}) = 6i-2, 1 \leq i \leq n-1$
$f(v_iy_i) = 6i-1, 1 \leq i \leq n-1$
$f(y_iu_{i+1}) = 6i, 1 \leq i \leq n-1$

From the above labeling pattern, f is a Geometric mean labeling of T_N. The labeling pattern is shown in the following figure.

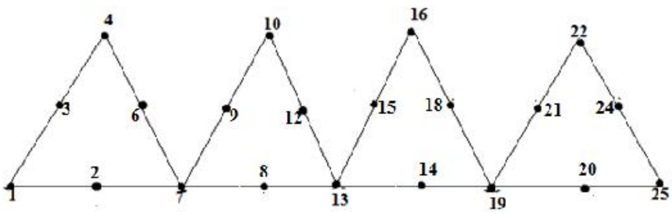

Figure 3.

From all the above cases, we conclude that subdivision of Triangular snake is a Geometric mean graph.

Theorem 2.2: Subdivision of any Alternate Triangular Snake is a Geometric mean graph.

Proof: Let $A(T_n)$ be the Alternative Triangular Snake

Let $S(A(T_n)) = D(T_N)$ be the graph which is obtained by subdividing all the edges of $A(T_n)$. Here we consider the following cases.

Case (i): If the triangle starts from u_1.

Let $S(A(T_n)) = S(T_N)$ be the graph which is obtained by subdividing all the edges of $A(T_n)$. Let t_i, x_i, and y_i be the new vertices which subdivide the edges u_iu_{i+11}, u_iv_i, and v_iu_{i+1} $1 \le i \le n$. Then we need to considered two subcases

Subcase(i) (a): If n is odd, then

Define a function f: $V(A(T_N)) \rightarrow \{1,2\ldots.q+1\}$ by
$f(u_i) = 4i-3, \forall i = 1,3,5\ldots.n$
$f(u_i) = 4i-1, \forall i = 2,4,6\ldots..n-1.$
$f(t_i) = 4i+2, \forall i = 1,3,5\ldots\ldots n-1.$
$f(t_i) = 4i, \forall i = 2,4,6\ldots\ldots n-1.$
$f(v_i) = 8i-5, \forall i = 1,2,3\ldots\ldots\frac{n-1}{2}$
$f(x_i) = 8i-6, \forall i = 1,2,3\ldots\ldots\frac{n-1}{2}$
$f(y_i) = 8i-3, \forall i = 1,2,3\ldots\ldots\frac{n-1}{2}$

From the above labeling pattern, we get the edges labels are all distinct. Thus f provides a Geometric mean labeling for $A(T_N)$. The labeling pattern is shown in the following figure.

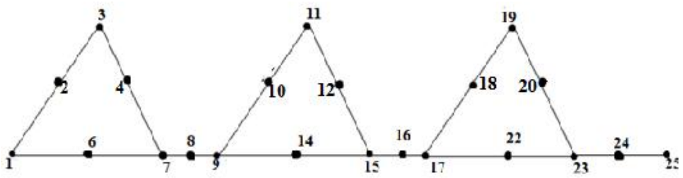

Figure 4.

Subcase (i) (b): If n is even

Define a function $f:V(T_N) \rightarrow \{1,2\ldots.q+1\}$by
$f(u_i) = 4i-3, \forall i = 1,3,5\ldots.n-1.$
$f(u_i) = 4i-1, \forall i = 2,4,6\ldots..n.$
$f(t_i) = 4i+2, \forall i = 1,3,5\ldots\ldots n-1.$
$f(t_i) = 4i, \forall i = 2,4,6\ldots\ldots n-1.$
$f(v_i) = 8i-5, \forall i = 1,2,3\ldots\ldots\frac{n}{2}$
$f(x_i) = 8i-6, \forall i = 1,2,3\ldots\ldots\frac{n}{2}$
$f(y_i) = 8i-3, \forall i = 1,2,3\ldots\ldots\frac{n}{2}$

Then the edge labels are all distinct. Hence the mapping f is a Geometric mean labeling of $A(T_N)$. The labeling pattern shown in the following figure.

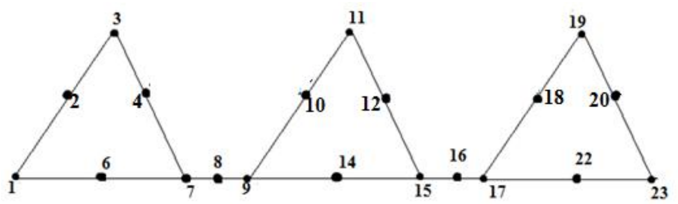

Figure 5.

Case (ii): If the triangle starts from u_2.

Let $S(A(T_n) = A(T_N))$ be the graph obtained by subdividing all the edge of $A(T_n)$.

Let t_i, x_i,y_i be the new vertices which are subdivide the edges u_iu_{i+1} $u_{i+1}v_i$ and v_iu_{i+2} $1 \le i \le n-1$. Then we consider two subcases

Subcase (i) (a): If n is odd.

Define a function f: $V(A(T_N)) \rightarrow \{1,2\ldots.q+1\}$ by
$f(u_i) = 4i-3, \forall i = 1,3,5\ldots.n$
$f(u_i) = 4i-5, \forall i = 2,4,6\ldots..n-1.$
$f(t_i) = 4i-2, \forall i = 1,3,5\ldots\ldots n-1.$
$f(t_i) = 4i, \forall i = 2,4,6\ldots\ldots n-1.$
$f(v_i) = 8i-3, \forall i = 1,2,3\ldots\ldots\frac{n-1}{2}$
$f(x_i) = 8i-4, \forall i = 1,2,3\ldots\ldots\frac{n-1}{2}$
$f(y_i) = 8i-2, \forall i = 1,2,3\ldots\ldots\frac{n-1}{2}$

From the above labeling pattern, we get the edges labels are all distinct. Thus f is a Geometric mean labeling of $A(T_N)$ and the labeling pattern is displaced below.

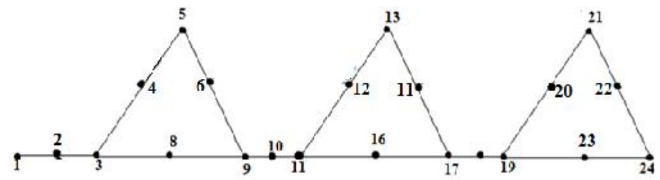

Figure 6.

Subcase (ii) (b): If n is even

Define a function f: $V(A(T_N)) \rightarrow \{1,2\ldots.q+1\}$ by
$f(u_i) = 4i-3, \forall i = 1,3,5\ldots.n-1$
$f(u_i) = 4i-5, \forall i = 2,4,6\ldots..n.$
$f(t_i) = 4i-2, \forall i = 1,3,5\ldots\ldots n-1.$
$f(t_i) = 4i, \forall i = 2,4,6\ldots\ldots n-1.$
$f(v_i) = 8i-3, \forall i = 1,2,3\ldots\ldots\frac{n-2}{2}$
$f(x_i) = 8i-4, \forall i = 1,2,3\ldots\ldots\frac{n-2}{2}$
$f(y_i) = 8i-2, \forall i = 1,2,3\ldots\ldots\frac{n-2}{2}$

From the above labeling pattern, we get the edges labels are all distinct. Thus f is a Geometric mean labeling of $A(T_N)$ and the labeling pattern is displaced below.

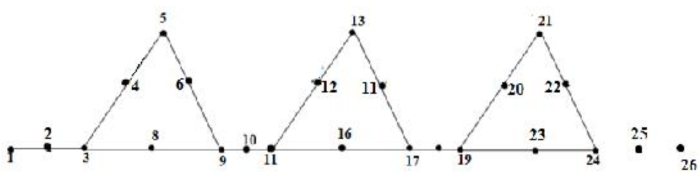

Figure 7.

From all the above cases, we conclude that Subdivision of Alternate Triangular Snakes are Geometric mean graphs.

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