



RESEARCH ARTICLE

SHELL MODEL DESCRIPTION FOR SOME NUCLEI AROUND DOUBLE MAGIC NUCLEUS ¹³²Sn

Ali Khalaf Hasan AL-SINAAYID and *Fatema Hameed Obeed AL-FATLAWI

Department of Physics, College of Education for Girls, University of Kufa, Al-Najaf, Iraq

ARTICLE INFO

Article History:

Received 27th March, 2016
Received in revised form
10th April, 2016
Accepted 21st May, 2016
Published online 30th June, 2016

Key words:

Nuclear Shell Model, Energy Levels,
Modified Surface Delta Interaction.

ABSTRACT

The low lying Spectra and high spin states have been calculated for the isotopes (¹³⁴Te, ¹³⁴Sb and ¹³⁴Sn) in different model spaces. First set of calculations have been carried in (0g_{7/2} - 0h_{11/2}) valance space. The second set of calculations have been performed in (1f_{7/2} - 0i_{13/2}) valance space and the third set of calculations have been performed in (0g_{7/2}, 1f_{7/2}) valance space. Nuclear shell model results showed that modified surface delta interaction is quite successful in introducing the spectrum energies of these nuclei and it able to produce the energy levels of the ground band clearly. In this work, the total angular momentum and parity for uncertainty and indeterminate experimentally energy levels were determinate and assured.

Copyright©2016, Ali Khalaf Hasan AL-SINAAYID and Fatema Hameed Obeed AL-FATLAWI. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Citation: Ali Khalaf Hasan AL-SINAAYID and Fatema Hameed Obeed AL-FATLAWI, 2016. "Shell model description for some nuclei around double magic nucleus ¹³²Sn", International Journal of Current Research, 8, (06), 33276-33280.

INTRODUCTION

Originally, the nuclear shell model was adopted when an attempt to describe the nuclear system with the atomic shell structure which proved to be successful. The model describes the filling of orbits and completing shells by nucleons with increasing energy within the nuclear potential. Shells are filling in a manner consistent with Pauli exclusion. Each nucleon is treating individually as an independent orbiting particle in a central potential despite of the existence of strong interactions between nucleons (Glassmaker, 1998). However, every nucleon retains an individual set of quantum numbers and wave function. The spherical shell model gives us an understanding of the observed magic numbers in atomic nuclei (Casten, 2000). The magic numbers in nuclei has clearly demonstrated the nuclear shell structure associated with the independent -particle model for nuclei (Vesselin, 2002). In this model each closed- shell configuration provides a convenient first approximation and one can assume that the system under consideration consists of a closed-shell core plus valence particles in a valence shell (Sheline *et al.*, 1972).

Theory

The concept of an effective two-body interaction has proved to be very successful in describing a vast amount of the observable nuclear structure properties. The effective two-body interaction can be determined by one of three basic approaches:

1. The model-independent method, based on fits of two-body matrix elements to nuclear properties.
2. The potential method which assumes an explicit mathematical form for the two-body interaction, with a small number of parameters determined experimentally.
3. The G-matrix interaction, which a more fundamental approach where the two body interaction is constructed from a measured nucleon-nucleon phase shifts (Mkhize, 2007).

The anti-symmetric matrix elements of the modified surface delta interaction used in this work, has the form (Lawson, 1980; Taqi, A.H, 2010).

$$\langle j_1 j_2 | V_{MSDI} | j_3 j_4 \rangle = C_0 \times f_J(j_1, j_2) \times f_J(j_3, j_4) \times \{ [1 - (-1)^{J+T+\ell_3+\ell_4}] - K_J(j_1 j_2) K_J(j_3 j_4) [(1 + (-1)^T] + \{ [2T(2T + 1) - 3] B + C \} \delta_{12} \delta_{34}] \} \quad (1)$$

*Corresponding author: Fatema Hameed Obeed AL-FATLAWI
Department of Physics, College of Education for Girls, University of Kufa, Al-Najaf, Iraq.

Where

$$f_J(j_1, j_2) = (-1)^{j_2 + l_2} \sqrt{\frac{2(2j_1 + 1) \times (2j_2 + 1)}{2(2J + 1) \times (1 + \delta_{j_1, j_2})}} \times \left\langle j_1 j_2 \frac{1}{2} \left(-\frac{1}{2}\right) \middle| J 0 \right\rangle$$

$$f_J(j_3, j_4) = (-1)^{l_3 - j_4} \sqrt{\frac{2(2j_3 + 1) \times (2j_4 + 1)}{2(2J + 1) \times (1 + \delta_{j_3, j_4})}} \times \left\langle j_3 j_4 \frac{1}{2} \left(-\frac{1}{2}\right) \middle| J 0 \right\rangle,$$

$${}^{\circ}C_0 = (-1)^{n_1 + n_2 + n_3 + n_4} A_T, A_T = \begin{cases} A_0, \dots, \text{for } T = 0 \\ A_1, \dots, \text{for } T = 1 \end{cases}, A_0 \approx A_1 \approx B \approx \frac{25}{A} \text{ MeV} \quad (2)$$

where $A_0, A_1, B,$ and C are the strength parameters of the MSDI obtained from fitting to experimental spectra in various mass regions (Brussard and Glaudemans, 1977). The independent - particle Hamiltonian of a (Z - particle) system can be written in terms of two - particle interactions as:

$$H = \sum_{K=1}^Z T_K + \sum_{K=1}^Z \sum_{l=K+1}^Z W(r_K^{\rightarrow}, r_l^{\rightarrow}) \quad (3)$$

Where $W(r_K^{\rightarrow}, r_l^{\rightarrow})$ is the two - body interaction between the k th and l th nucleons. Inserting an average potential $\pm U$ (rk), the Hamiltonian becomes:-

$$H = \sum_{K=1}^Z [T_K + U(r_K)] \sum_{K=1}^Z \sum_{l=K+1}^Z W(r_K^{\rightarrow}, r_l^{\rightarrow}) - \sum_{K=1}^Z U(r_K) \quad (4)$$

Where the first term is identical to the independent - particle Hamiltonian, and the second and third account for the deviation from independent particle motion, known as the residual interaction. Separating the summations into core and valence contributions, eq. (4) can be re - written as (Coraggio *et al.*, 2009):

$$H = H_{Core} + H_1 + H_2 + V_{MSDI} \quad (5)$$

Where H_{Core} contains the interaction of nucleons making up the core, H_1 and H_2 are the single - particle contributions of particles (1) and (2), and V_{MSDI} describe the residual interaction between particles (1) and (2), as well as any interaction with core nucleons, by inserting (eq (5)) into Schrödinger equation yields expression for the energy as:-

$$E = E_{Core} + E_1 + E_2 + \langle \Phi_{J,T} | V_{MSDI} | \Phi_{J,T} \rangle \quad (6)$$

Here E_{Core} is the binding energy of the nucleus core and $\langle \Phi_{J,T} | V_{MSDI} | \Phi_{J,T} \rangle$ is the residual interaction which

needs to be defined by theory. It is important to note that the energy given by eq. (6) is for pure configurations only. In principle any close - lying state with the same total angular momentum (J) and total isospin (T) will mix. The mixed eigen states are giving by linear combinations of the unperturbed wave functions (Hasan and Hussain, 2013):

$$(\Psi_{J,T})_s = \sum_{K=1}^n a_{ks} (\phi_{JT})_s \quad (7)$$

Where (s) is the number of mixing configurations and takes values $(s=1, 2, \dots, n)$. The coefficients (a_{ks}) fulfill the condition (Caurier *et al.*, 2005):

$$\sum_{K=1}^n |a_{ks}|^2 = 1 \quad (8)$$

RESULTS AND DISCUSSION

In this study, we have taken a doubly magic isotope ^{132}Sn as a closed core situated away from the line of stability, and let the last two protons in ^{134}Te nucleus occupy the five levels $0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}$ and $0h_{11/2}$) and the last two neutrons in the ^{134}Sn nucleus occupy the six levels $(1f_{7/2}, 1f_{5/2}, 0h_{9/2}, 2p_{3/2}, 2p_{1/2}$ and $0i_{13/2})$ while in ^{134}Sb nucleus, which have one proton in orbital $(0g_{7/2})$ and one neutron in orbital $(1f_{7/2})$. A single proton energies that have taken from the experimental spectrum of ^{133}Sb isotope are $(0.962, 2.439, 2.791$ and $2.800) \text{ MeV}$ for $(1d_{5/2}, 1d_{3/2}, 2s_{1/2}$ and $0h_{11/2})$ respectively. While, A single neutrons energies that have taken from the experimental spectrum of ^{133}Sn isotope are $(2.004, 1.561, 0.853, 1.363,$ and $2.694) \text{ MeV}$ (Sonzogni, 2004).

^{134}Te Nucleus

In our calculations for this nucleus, the high spin levels scheme have been extended to 7.608 MeV excitation energy and spin 6_6^+ as shown in the table (1). Our found the levels $\{0_1^+, 2_1^+, 4_1^+$ and $6_1^+\}$ with energies $\{0, 1.119, 1.269$ and $1.406\} \text{ MeV}$ when comparison with experimental values $\{0, 1.279, 1.576$ and $1.691\} \text{ MeV}$. We expect the levels $\{1_1^+$ or $3\}$ and 0_2^+ at energies $\{2.196\}$ and $\{2.655\} \text{ MeV}$ while experimental values are $\{2.397\}$ and $\{2.682\} \text{ MeV}$ which were uncertain in spin and parity $(6^+$ and $3^+)$ respectively. The $\{2_3^+\}$ level at $\{3.048\} \text{ MeV}$ is correspond to experimental value $\{2.933\} \text{ MeV}$. The energies $\{4.025, 4.026, 4.586, 4.635, 4.996$ and $6.998\} \text{ MeV}$ which were so close to with experimental values $\{4.013, 4.269, 4.556, 4.563, 5.079$ and $7.050\} \text{ MeV}$ which were uncertainty in total angular momentum and parity. We found the $\{2_6^+, 2_8^+, 6_5^+$ and $6_6^+\}$ at $\{4.531, 6.178, 6.826$ and $7.608\} \text{ MeV}$ were in a good agreement with experimental

values {4.504, 5.986, 6.709 and 7.722} MeV respectively which were undetermined the total angular momentum and parity. There are several levels in the ranges: $\{4_2^+$ to $5_1^+\}$, $\{4_3^+$ to $3_3^+\}$, $\{3_5^+$ to $(6_4^-$ or $8_2^-)\}$, $\{0_3^+$, 7_3^- to $8_4^+\}$, $\{0_4^+$ to $4_8^-\}$, 10_1^+ and 0_5^+ , in

our calculations with energies $\{(2.057$ to $2.168)$, $(3.186$ to $3.983)$, $(4.636$ to $4.987)$, $(5.266, 6.227$ to $6.817)$, $(6.838$ to $6.932)$, $(6.999$ and $7.123)\}$ which were found without analogue experimental values and they were listed in Table (1).

Table 1. Comparison between calculated excitation energy levels with experimental data for ^{134}Te nucleus

$J^{\pi}_{cal.}$	$E_{cal.}$ (MeV).	$J^{\pi}_{Exp.}$	$E_{Exp.}$ (MeV)
0_1^+	0	0^+	0
2_1^+	1.119	2^+	1.279
4_1^+	1.269	4^+	1.576
6_1^+	1.406	6^+	1.691
4_2^+	2.057	—	—
2_2^+	2.081	—	—
5_1^+	2.168	—	—
$1_1^+, 3_1^+$	2.196	$(6)^+$	2.397
0_2^+	2.655	$(3)^+$	2.682
2_3^+	3.048	2^+	2.933
4_3^+	3.186	—	—
2_4^-	3.357	—	—
7_1^-	3.501	—	—
4_4^-	3.534	—	—
8_1^-	3.672	—	—
$5_2^+, 9_1^-$	3.673	—	—
3_2^+	3.674	—	—
6_2^-	3.667	—	—
4_5^+	3.891	—	—
5_3^-	3.936	—	—
4_6^+	3.965	—	—
3_3^+	3.983	—	—
$6_3^-, 2_5^+$	4.025	$(9)^+$	4.013
4_3^-	4.026	$(7)^+$	4.269
2_6^+	4.531	—	4.504
4_7^+	4.586	$(8)^+$	4.556
1_2^+	4.635	$(8)^+$	4.563
3_5^+	4.636	—	—
2_7^+	4.824	—	—
7_2^-	4.981	—	—
$6_4^-, 8_2^-$	4.987	—	—
3_6^+	4.996	$(9)^+$	5.079
0_3^+	5.266	—	—
2_8^+	6.178	—	5.986
7_3^-	6.227	—	—
2_9^+	6.386	—	—
1_3^+	6.474	—	—
5_4^-	6.603	—	—
8_3^-	6.646	—	—
8_4^+	6.817	—	—
6_5^+	6.826	—	6.709
0_4^+	6.838	—	—
4_8^-	6.932	—	—
2_{10}^+	6.998	$(14)^+$	7.050
10_1^+	6.999	—	—
0_5^+	7.123	—	—
6_6^+	7.608	—	7.722

Table 2. Comparison between calculated excitation energy levels with experimental data for ^{134}Sn nucleus

$J^{\pi}_{\text{cal.}}$	$E_{\text{cal.}}(\text{MeV})$	$J^{\pi}_{\text{Exp.}}$	$E_{\text{Exp.}}(\text{MeV})$	$J^{\pi}_{\text{cal.}}$	$E_{\text{cal.}}(\text{MeV})$	$J^{\pi}_{\text{Exp.}}$	$E_{\text{Exp.}}(\text{MeV})$
0_1^+	0	0^+	0	2_7^+	4.392	—	—
2_1^+	1.011	2^+	0.725	4_8^+	4.446	—	—
4_1^+	1.406	4^+	1.073	4_9^+	4.515	—	—
6_1^+	1.565	6^+	1.247	2_8^+	4.654	—	—
2_2^+	1.923	—	—	4_{10}^+	4.689	—	—
4_2^+	2.164	—	—	6_6^+	4.694	—	—
1_1^+	2.269	—	—	2_9^+	4.715	—	—
$5_1^+, 3_1^+$	2.322	—	—	8_3^-	4.786	—	—
3_2^+	2.575	(8^+)	2.508	2_{10}^-	4.819	—	—
0_2^+	2.649	—	—	3_8^+	4.836	—	—
4_3^+	2.685	—	—	6_7^+	4.984	—	—
5_2^-	2.746	—	—	3_9^-	5.012	—	—
6_2^-	.7572	—	—	$6_8^-, 8_4^+$	5.016	—	—
3_3^+	2.832	—	—	7_3^+	5.021	—	—
7_1^+	2.913	—	—	$2_{11}^+, 4_{11}^+, 1_6^+$	5.033	—	—
$4_4^+, 2_3^+, 8_1^+$	3.029	—	—	4_{12}^-	5.384	—	—
2_4^+	3.098	—	—	7_4^-	5.439	—	—
6_3^+	3.248	—	—	0_4^+	5.461	—	—
3_4^+	3.331	—	—	2_{12}^+	5.509	—	—
2_5^+	3.431	—	—	0_5^-	5.526	—	—
5_3^+	3.472	—	—	8_5^-	5.607	—	—
$2_2^+, 3_5^-, 1$	3.473	—	—	4_{13}^+	5.651	—	—
3_1^+	3.554	—	—	10_2^+	5.658	—	—
1_4^+	3.685	—	—	$3_{10}^-, 5_7^-, 6_9^-, 7_5^-$	5.723	—	—
6_4^+	3.763	—	—	9_2^-	5.900	—	—
5_4^+	3.787	—	—	2_{13}^+	6.038	—	—
4_5^+	3.831	—	—	7_6^-	6.095	—	—
7_2^-	3.882	—	—	5_8^-	6.137	—	—
3_6^+	3.955	—	—	4_{14}^-	6.166	—	—
4_6^-	3.992	—	—	$6_{10}^-, 8_6^-, 4_{15}^+$	6.167	—	—
2_6^+	4.026	—	—	0_6^+	6.714	—	—
9_1^-	4.033	—	—	12_1^+	6.769	—	—
5_5^-	4.094	—	—	8_7^+	6.870	—	—
$4_7^+, 6_5^-, 8_2^-, 10_1^-$	4.163	—	—	6_{11}^+	6.977	—	—
5_6^-	4.291	—	—	10_3^-	6.982	—	—
$3_7^+, 1_5^+$	4.326	—	—	11_1^-	7.071	—	—
0_3^+	4.379	—	—				

Table 3. Comparison between calculated excitation energy levels with experimental data for ^{134}Sb nucleus

$J^{\pi}_{\text{cal.}}$	$E_{\text{cal.}}(\text{MeV})$	$J^{\pi}_{\text{Exp.}}$	$E_{\text{Exp.}}(\text{MeV})$
1	0	(0)	0
8	0.404	(5^-)	0.441
2	0.519	(4^-)	0.555
6	0.594	(6)	0.617
4	0.606	—	—
3	0.622	—	—
5	0.725	—	—
7	0.778	—	—

¹³⁴Sn Nucleus

The comparison between the calculated values and experimental data (Sonzogni, 2004) has shown in Table (2). Recently, the levels of ¹³⁴Sn nucleus are extended to 2.508 MeV in the experimental data. The first, three calculated levels are found to be $\{0_1^+, 2_1^+, 4_1^+ \text{ and } 6_1^+\}$ with energies $\{0, 1.011, 1.406 \text{ and } 1.565\}$ MeV which were rather close to with experimental values as $\{0, 0.725, 1.073 \text{ and } 1.247\}$ MeV. We expect the level $\{3_2^+\}$ at $\{2.575\}$ MeV while experimental value is $\{2.508\}$ MeV uncertain in spin and parity (8^+). Another energy levels in the range $\{0_2^+ \text{ to } 11_1^-\}$ in our calculations which were undetermined the spins and energies in the experimental data, because the highest experimental level for this nucleus is $\{2.508\}$ MeV (Sonzogni, 2004), with spin and parity (11^-) while in this work the energies reached the values $\{7.071\}$ MeV which increased (64) extra level on experimental values. We found in the framework of shell model calculations (64) new energy levels were determined for (¹³⁴Sn) isotope. This investigation increases the theoretical Knowledge of all isotopes with respect to energy levels

¹³⁴Sb Nucleus

Experimental levels are available to 5.324 MeV excitation energy and spin (14^-) for this nucleus. Comparison of the calculated levels with experimental data (Saleem *et al.*, 2004) presented in table (3). The levels $\{1^-, 8^- \text{ and } 2^-\}$ are predicted by nuclear shell model with energies $\{0, 0.404 \text{ and } 0.519\}$ MeV, while experimental values are $\{0, 0.414 \text{ and } 0.555\}$ MeV uncertainty at $\{(0^-), 5^- \text{ and } 4^-\}$ spins and parity. We expect certain the level (6^-) to energy 0.617 MeV uncertainty experimentally. Our choosing for this nucleus in this shell, we found that the number of levels limited compared with experimental values.

REFERENCES

- Brussard, P.J.; Glaudemans, P. W. M. 1977. Shell Model Applications in Nuclear Spectroscopy; North Holland: Amsterdam.
- Casten, R.F. 2000. Nuclear Structure from a Simple Perspective, 2nd ed.; Oxford University, Press Inc:New York.
- Caurier, E.; Martinez, P.G.; Nowacki, F.; Poves, A.; Zuker, A.P. 2005. "The shell model as a unified view of nuclear structure", *Rev.Mod.Phys.*, 77,427.
- Coraggio, L.; Covello, A.; Gargano, A.; Itaco, N.; Kuo, T.T.S. 2009. "Shell model calculations and realistic effective interactions", *Prog.Part.Nucl.Phys.*, 62,143.
- Glassmaker, T. 1998. *Ann.Rev.of Nucl. Part. Sci.*, 48, 1.
- Hasan, A. K. and Hussain, F. M. 2013. *Adv. in Phys. Theor. and Appl.*, 20,128., 'Calculation of Energy Levels for Nuclei ³⁴S, ³⁴Ar, ³⁴Cl by using Surface Delta Interaction".
- Lawson, R. D. 1980. Theory of the Nuclear Shell Model; Clarendon Press, Oxford: New York.
- Mkhize, P.S. M.Sc, 2007. Department of physics, Western Cape University, WesternCape.
- Saleem, K.A.; Chaudhury, Z.; Bernard, D.; Browne, E.; Sonzogni, A. A. 2004. Nucl. Data Sheet, 103, 1.
- Sheline, R.; Ragnarsson, I.; Nilsson, S. 1972. *Phys. Lett. B.*, 41,115.
- Sonzogni, A. A. 2004. Nucl. Data Sheet, 103, 1.
- Taqi, A.H. 2010. Acta Phys. Polon B., 6,1130-1331, ".Particle-Particle and Hole-Hole Random Phase Approximation Calculations For ⁴²Ca and ³⁸Ca Nuclei".
- Vesselin, G.G. PhD, 2002. Department of physics and Astronomy, Sofia University, Sofia.
