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RESEARCH ARTICLE

VACUUM SOLUTION OF BIANCHI TYPE-I MODEL IN $f(R)$ THEORY OF GRAVITY

[1]Ladke, L. S., [2]Jaiswal, V. K. and *[3]Hiwarkar, R. A.

[1]Nilkanthrao Shinde Science and Arts College, Bhandravati, India
[2]Priyadarshini J. L. College of Engineering, Nagpur
[3]Guru Nanak Institute of Engineering & Technology, Nagpur

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ABSTRACT

The present paper deals with the study of vacuum solution of Bianchi-type-I model in the metric version of $f(R)$ theory of gravity. Using the special form of deceleration parameter, we find the solution of Einstein's field equation. The function $f(R)$ is also evaluated for the model and the physical properties of this model have been discussed.

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INTRODUCTION

The $f(R)$ theory of gravity is an alternative theory of gravity which has received much attention in recent years due to its cosmologically important models. The $f(R)$ theory of gravity considered as most suitable theory. This theory shows the unification of early-time inflation and late-time acceleration because the present a higher order curvature invariant consist as functions of the Ricci scalar (Nojiri and Odintsov 2007). It has been observed that a natural gravitational alternative to dark energy is provided by this modified theory of gravity, so $f(R)$ gravity as modified theory is a desirable candidate to overcomes the issue such as dark energy problems as well as singularity occurred in general theory of relativity (Nojiri and Odintsov 2008). In the $f(R)$ gravity there are two approaches to find out the solutions of modified Einstein's field equation. The first approach is called metric approach and second one is the Palitini approach.

Bianchi type-I space-time, which is generalization of the open universe in FRW cosmology play a significant character in the discussion of large scale structure and realization of the actual picture of the universe. Many authors have shown keen interest in exploring different issues in the $f(R)$ theory of gravity (Weyl 1919, Eddington 1922, Buchdahl 1970 etc.) Recently, Sharif and Shamir (2009) have solved the field equations of modified $f(R)$ theory of gravity and obtained the static plane symmetric space-time using the metric approach with the assumption of constant scalar curvature. Reddy D.R.K. et. al. (2014) presented the work on vacuum solutions of Bianchi Type-I and V models in the $f(R)$ gravity with a special form of deceleration parameter. Some Bianchi type cosmological models in $f(R)$ gravity obtained by Farasat Shamir (2010). Moreover, Antonio De Felice (2010) given review about various applications of $f(R)$ theory of cosmology and gravity. It is believed that the early universe may not have been exactly uniform. In theoretical cosmology inhomogeneous and anisotropy model of the universe plays an important role. The existence of anisotropic in early phase of universe is an interesting phenomenon to investigate.

***Corresponding author: Hiwarkar, R. A**
Guru Nanak Institute of Engineering & Technology, Nagpur

Anil kumar Yadav (2011) studied some anisotropic dark energy models in Bianchi type-V space time. Bianchi type models are among simplest models with anisotropy background. Vacuum and non vacuum solutions of Bianchi type–I and V space times in metric $f(R)$ gravity have been explored by Sharif *et al*. (2009) and Shamir M. F. (2010). Akarsu *et al*. (2011) have investigated the Bianchi type –I anisotropic DE model with constant deceleration parameter. Sharif and Kausar (2011) investigated non vacuum solution of a Bianchi type –VI universe by considering the isotropic and anisotropic fluid as the source of dark matter and energy. Yadav & Yadav (2011a), Yadav *et al*. (2011b) have studied anisotropic DE models with a variable EoS parameter. Kumar and Singh (2007) solved the field equations in the presence of perfect fluid using a Bianchi type-I space time in general relativity. A bianchi type–III string cosmology with bulk viscosity has been studied by Xing–Xiang (2005). Wang (2005) explored string cosmological models in Kantowski – Sachs space time.

With the motivation of above research work, In present paper, we obtained the vacuum solution of Bianchi type-I space-time in the framework of $f(R)$ theory of gravity using special form of deceleration parameter. The physical properties have been discussed. Lastly we summarize and conclude the results.

Field equations in $f(R)$ theory of gravity

The $f(R)$ theory of gravity is one of the modified theory of gravity which is a generalization of general theory of relativity. The action for $f(R)$ theory of gravity is given by

$$S=\int\left(\frac{1}{16\pi G}f(R)+L_m\right)\sqrt{-g}\; d^4x, \tag{1}$$

where $f(R)$ is a general function of Ricci scalar R and L_m is the matter Lagrangian.

Now by varying the action S with respect to g_{ij}, we obtain the field equations in the $f(R)$ theory of gravity as

$$F(R)R_{ij}-\frac{1}{2}f(R)g_{ij}-\nabla_i\nabla_j F(R)+g_{ij}\Box F(R)=kT_{ij}, \tag{2}$$

where $F(R)\equiv\dfrac{df(R)}{dR}$, $\Box\equiv\nabla^i\nabla_i$, ∇_i is the covariant derivative and T_{ij} is the standard matter energy momentum tensor. If we take $f(R)=R$, the field equations (2) reduce to the field equations of general theory of relativity which is proposed by Einstein.

Contracting the above field equations(2), we have

$$F(R)R-2f(R)+3\Box F(R)=kT, \tag{3}$$

For vacuum, we have

$$F(R)R-2f(R)+3\Box F(R)=0, \tag{4}$$

This gives an important relation between $F(R)$ and $f(R)$ which may be used to simplify the field equations and to evaluate $f(R)$.

From (4), we get

$$f(R)=\frac{1}{2}\left[3\Box F(R)+F(R)R\right], \tag{5}$$

Now we consider the metric

$$ds^2 = dt^2 - A^2(t)dr^2 - B^2(t)[d\theta^2 + l^2(\theta)d\phi^2], \tag{6}$$

where $l^2(\theta) = \begin{cases} \theta^2, & \text{when k} = 0 \text{ (Bianchi I model)}, \\ \sin h^2\theta, & \text{when k} = -1 \text{ (Bianchi III model)}, \\ \sin^2\theta, & \text{when k} = 1 \text{ (Kantowski-Sachs model)}. \end{cases}$

Bianchi Type-I Model

The line element of Bianchi type-I space-time is given by

$$ds^2 = dt^2 - A^2(t)dr^2 - B^2(t)[d\theta^2 + \theta^2 d\phi^2], \tag{7}$$

where A, B are cosmic scale factors.

The corresponding Ricci scalar R becomes

$$R = 2\left[\frac{\ddot{A}}{A} + \frac{2\ddot{B}}{B} + \frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2}\right], \tag{8}$$

where over dot denotes derivative with respect to time t.

Using equations (2) and (5), the vacuum field equations take the form

$$\frac{F(R)R_{ij} - \nabla_i \nabla_j F(R)}{g_{ij}} = \frac{1}{4}[F(R)R - \Box F(R)], \tag{9}$$

Since the metric (7) depends only on t, one can view equation (9) as the set of differential equations for A, B, C and $F(R)$. It follows from the equation (9) that the combination

$$A_i = \frac{F(R)R_{ii} - \nabla_i \nabla_i F(R)}{g_{ii}}, \tag{10}$$

is not depend on the index i, and hence $A_i - A_j = 0$ for all i and j.

Thus, $A_0 - A_1 = 0$ gives

$$-2\frac{\ddot{B}}{B} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{F}}{AF} - \frac{\ddot{F}}{F} = 0, \tag{11}$$

Similarly $A_0 - A_2 = 0$ gives

$$-\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{\dot{B}\dot{F}}{BF} - \frac{\ddot{F}}{F} = 0, \tag{12}$$

The average scale factor and the volume scale factor are defined respectively as under

$$a = (AB^2)^{\frac{1}{3}}, V = a^3 = AB^2, \tag{13}$$

The mean Hubble parameter H is defined by

$$H=\frac{1}{3}[H_r+H_\theta+H_\phi]=\frac{1}{3}\frac{\dot{V}}{V}=\frac{\dot{a}}{a}, \tag{14}$$

where $H_r=\frac{\dot{A}}{A}$, $H_\theta=H_\phi=\frac{\dot{B}}{B}$ are the directional Hubble parameters in the directions of r,θ,ϕ axes respectively. The expansion scalar θ and the shear scalar σ^2 are defined as follows

$$\theta=u^i_{;i}=\frac{\dot{A}}{A}+\frac{2\dot{B}}{B}, \tag{15}$$

$$\sigma^2=\frac{1}{2}\sigma_{ij}\sigma^{ij}=\frac{1}{3}\left[\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right]^2, \tag{16}$$

where $\sigma_{ij}=\frac{1}{2}(\nabla_j u_i+\nabla_i u_j)-\frac{1}{3}g_{ij}\theta$. (17)

The mean anisotropy parameter $\overline{A}$ is given by

$$\overline{A}=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2=\frac{2\sigma^2}{3H^2}. \tag{18}$$

Solution of Field Equations

Equations (11) and (12) are two non-linear equations in three unknowns A, B and F. One additional constraint relating these parameters is used to obtain the deterministic solution of the system. So we used a physical condition that the expansion scalar θ is proportional to the shear scalar σ^2 (i.e. $\theta\ \alpha\ \sigma^2$). This condition leads to the following relation between cosmic scale factors A and B:

$$A=B^n, \tag{19}$$

The subtraction the equation (12) from equation (11) gives

$$\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{\dot{B}^2}{B^2}+\frac{\dot{A}\dot{F}}{AF}-\frac{\dot{B}\dot{F}}{BF}=0, \tag{20}$$

After solving (20),

$$\frac{A}{B}=d_1\exp\left[c_1\int\frac{dt}{a^3F}\right], \tag{21}$$

where d_1 and c_1 are constants of integration.

Singh and Debnath [2009] has defined a special form of deceleration parameters for FRW metric as

$$q = -\frac{\ddot{a}\,a}{\dot{a}^2} = -1 + \frac{\alpha}{1 + a^{\alpha}}\,, \qquad (22)$$

where $\alpha > 0$ is a constant.

From equation (14), we can obtained the mean Hubble parameter H as

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{\dot{a}}{a} = k(1 + a^{-\alpha}), \qquad (23)$$

On integrating equation (23), we calculated mean scale factor as

$$a = (e^{k\alpha t} - 1)^{\frac{1}{\alpha}}, \qquad (24)$$

Sharif and Shamir [2009] have established a result in the context of $f(R)$ gravity which show that

$$F \;\alpha\; a^{m}, \qquad (25)$$

Thus using power law relation between F and a , we have

$$F = l\,a^{m}, \qquad (26)$$

where l is the constant of proportionality, m is any integer (here taken as -2).

Using equations (24) and (26) for $k = 1,\ \alpha = 2$ in the equation (21), we obtained the scale factors as

$$A = r_1\,(e^{2t} - 1)^{\frac{1}{2}} \exp\left[s_1 \tan^{-1}(e^{2t} - 1)^{\frac{1}{2}} \right], \qquad (27)$$

$$B = r_2\,(e^{2t} - 1)^{\frac{1}{2}} \exp\left[s_2 \tan^{-1}(e^{2t} - 1)^{\frac{1}{2}} \right], \qquad (28)$$

where $r_1 = d_1^{\frac{2}{3}}, r_2 = d_1^{\frac{-1}{3}}, s_1 = \frac{2c_1}{3l},\ s_2 = \frac{-c_1}{3l}.$ (29)

From equations (27) and (28), the directional Hubble parameters in the directions of r, θ and ϕ axes are found to be

$$H_r = \frac{s_1}{(e^{2t} - 1)^{\frac{1}{2}}} + \frac{e^{2t}}{e^{2t} - 1}, \qquad (30)$$

$$H_\theta = H_\phi = \frac{s_2}{(e^{2t} - 1)^{\frac{1}{2}}} + \frac{e^{2t}}{e^{2t} - 1}, \qquad (31)$$

The mean Hubble parameter H becomes

$$H=\frac{e^{2t}}{e^{2t}-1}, \tag{32}$$

The volume scale factor is given by

$$V=(e^{2t}-1)^{\frac{3}{2}}, \tag{33}$$

The expansion scalar θ obtained as

$$\theta=\frac{3e^{2t}}{e^{2t}-1}, \tag{34}$$

From equations (33) and (34), we observe that the spatial volume is zero and the expansion scalar is infinite at $t=0$, which show that the universe starts evolving with zero volume at $t=0$.
The shear scalar σ^2 obtained as

$$\sigma^2=\frac{3s_2^2}{e^{2t}-1}, \tag{35}$$

The mean anisotropy parameter $\overline{A}$ turns out to be

$$\overline{A}=\frac{2s_2^2(e^{2t}-1)}{e^{4t}}, \tag{36}$$

If $t=0$, the model becomes isotropic otherwise the model will be anisotropic.
The time dependent deceleration parameter q is given by

$$q=\frac{2}{e^{2t}}-1, \tag{37}$$

At an early phase of cosmic evolution $t\to 0$, $q=1$ and at late phase of cosmic evolution $t\to\infty$, $q=-1$. We are concerned about early deceleration and late acceleration stage of the universe. So that at early time q is positive whereas at late time q takes a negative value which suggests that universe is undergoing decelerating at early stage to accelerating late stage which is conformity with the recent observational data.

The Ricci scalar for Bianchi type-I model is

$$R=2\left[\frac{6(s_2{}^2+2e^{2t})}{e^{2t}-1}\right], \tag{38}$$

The function of Ricci scalar, $f(R)$ is

$$f(R)=\frac{1}{2}\left[\frac{l}{e^{2t}-1}R-\frac{6l\,e^{2t}(e^{2t}-2)}{(e^{2t}-1)^3}\right], \tag{39}$$

which clearly indicates that $f(R)$ cannot be explicitly written in terms of R. However by inserting value of R, $f(R)$ can be written as a function of t which is true as R depends upon t.

$$f(R) = \frac{3l\left[s_2^{\,2}(e^{2t}-1)+e^{4t}\right]}{(e^{2t}-1)^2}. \tag{40}$$

Concluding Remark

In this paper, we have to studied vacuum solution of Bianchi-type-I model in the metric version of $f(R)$ theory gravity with the special form of deceleration parameter. We have taken the use of power law relation between $F(R)$ and a .

This cosmological model has initial singularity at $t=0$.
For this model, the expansion scalar θ , shear scalar σ and Hubble parameter H decrease with passage of time.
The cosmological parameters H_r, H_θ, H_ϕ, H and θ are all infinite at $t=0.$
The volume scale factor V vanishes at $t=0$ and increase with time. This show that at the initial epoch the universe starts with zero volume and expands uniformly.
For $t=0$ model is isotropic.
At an early phase of cosmic evolution universe is decelerating and at late phase of cosmic evolution universe is accelerating. This is confirmed from the recent observational data.

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