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RESEARCH ARTICLE

REVERSE SQUARE SUM LABELING OF FAMILY OF GRAPHS

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ABSTRACT

A (p,q)-graph G is called square sum (Acharya, 2011) if there exists a objective function $f : V(G) \to \{0,1,2---(p-1)\}$ such that the induced function $f^* : E(G) \to N$ defined by $f(uv) = [f(u)]^2 + [f(v)]^2$ for every edge $uv \in E(G)$ is injective. In this paper, we define Reverse square sum if $f^{-1} : V(G) \to \{p-1, p-2, ---2, 1, 0\}$. The sub-division graph $S(G)$ is the graph obtained by inserting a vertex a *w* of degree 2 into every edge *uv* of G such that *uw* and *vw* are two edges. We study here Reverse square sum labelling in the context of Arbitrary Super subdivision of a graph .We show that the graph obtained by arbitrary super subdivision of Paths, Stars, Tadpoles, Trees, Grid graphs, Armed crowns and Cycles are Reverse Square sum.

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INTRODUCTION

All the graphs we consider in this paper are finite and simple, without loops or multiple edges. Now, for most of the graph theory terminology utilized here, the authors refer the reader to Chartrand and Lesnaik (1986): however, to make this paper reasonably self-contained , we mention that for a graph G, we denote the vertex set and edge set of G by *V(G)* and *E(G)* respectively. Moreover $|V(G)| = p$ and $|E(G)| = q$. Acharya and Germina (2011) defined a square sum labelling of a (p,q)-graph G is said to be square sum if there exists a bijection $f : V(G) \longrightarrow \{0,1,2---(p-1)\}$ such that induced function $f^* : E(G) \to N$ defined by $f(uv) = [f(u)]^2 + [f(v)]^2$ for every edge $uv \in E(G)$ is injection. In this paper we define reverse square sum labelling of a graph G. A (p,q)-graph G is said to be Reverse square sum if there exists a bijection $f^{-1} : V(G) \to \{p-1, p-2, ---2, 1, 0\}$ such that an induced function $f^* : E(G) \to N$ defined by $f(uv) = [f(u)]^2 + [f(v)]^2$ for ∀ edge $uv \in E(G)$ is injective.

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Ife=uv is an edge of G and w is not a vertex of G then 'e' is subdivided when it is replaced by the edges uw and vw .If every edge of G is subdivided the resulting graph is the 'subdivision graph S (G)'. A graph H is called an arbitrary super subdivision of G, if H is obtained from G by replacing every edge e_i of G by complete bipartite graph K_{2,m_i} (for some $m_i, 1 \le i \le q$ in such a way that the end vertices of each e_i are identified with the two vertices of partite set with cardinality 2 of K_{2,m_i} after removing the edge e_i from graph G.

If m_i is varying arbitrarily for each edge e_i then the super subdivision is called arbitrary super subdivision and is denoted by SS(G) (Katheresan and Amutha, 2004; Sethuraman and Selvaraj, 2001). In this paper , we prove that the graph obtained by arbitrary super subdivision of Paths, Stars, Grid graphs, Trees, Tadpoles, Armed crowns and Cycles are Reverse Square sum.Tadpole $T(n,l)$ is a graph in which path p_i is attached to any one vertex of cycle C_n. Armed Crown $C_n \odot P_m$ is graph in which the path P_m is attached to every cycle C_n. One point union of m cycles of length n denoted as $C_n^{(m)}$ is the graph obtained by identifying one vertex of each cycle.

Theorem 1.1:

Arbitrary super subdivision of path P_n is reverse square sum.

Proof: suppose P_n has the vertex $\{v_1, v_2, v_3, ---, v_n\}$ and edge set $\{v_1v_2, v_2v_3, ---, v_{n-1}v_n\}$. Construction of arbitrary super subdivision of P_n is same as replacing each edge $e_i = v_iv_{i+1}, 1 \le i \le n-1$ of P_n by a complete bipartite graph K_{2,m_i} where m_i, is a positive integer. Let u_{ij} be any vertex of the partite set with cardinality $m_i, 1 \le i \le n-1, 1 \le j \le m_i$. Then

$\left|V\left(SS\left(P_n\right)\right)\right| = n+m$ Where $m = m_1 + m_2 + ---- + m_{n-1}$

Define, $f : V\left(SS^{-1}\left(P_n\right)\right) \to \{m+n-1, m+n-2, ---, 1, 0\}$

By, $f(v_1) = m+n-1$

$f(v_{i+1}) = f(v_i) - m_{i+1}, \quad 1 \le i \le n-1$

$f(u_{ij}) = f(v_i) - j, \quad 1 \le i \le n-1, 1 \le j \le m_i$

One can easily verify that,

$f(v_1) > f(u_{11}) > f(u_{12}) > -- > f(u_{1m_1}) >$

$f(v_2) > f(u_{21}) > f(u_{22}) > -- > f(u_{2m_2}) > -- >$

$f(v_{n-1}) > f(u_{(n-1)m_1}) > f(u_{(n-1)m_2}) > -- > f(u_{(n-1)m_{n-1}}) > f(v_n)$

The edge labels of $SS^{-1}(P_n)$ can be arranged in a decreasing order and hence are distinct.

Theorem 1.2: Arbitrary super subdivision of path $K_{1,n}$ is reverse square sum.

Proof: Let $V\left(K_{1,n}\right) = \{v_1, v_2, v_3, ---, v_n\} \cup \{V_0\}$ and edge set $E\left(K_{1,n}\right) = \{v_0v_1, v_0v_2, ---, v_0v_n\}$

Construction of arbitrary super subdivision of $K_{1,n}$ is same as replacing each edge $e_i, 1 \le i \le n$ of $K_{1,n}$ by a complete bipartite graph K_{2,m_i} where m_i is a positive integer. Let u_{ij} be any vertex of the partite set with cordinality m_i.

$1 \le i \le n, 1 \le j \le m_i$,. Then $\left|V\left(SS\left(K_{1,n}\right)\right)\right| = m+n+1$

Where $m = m_1 + m_2 + --- + m_n$

Define $f : \left|V\left(SS^{-1}\left(K_{1,n}\right)\right)\right| \to \{m+n, m+n-1, ---, 1, 0\}$

By $f\left(v_0\right) = 0$

$f\left(v_i\right) = m+i, 1 \le i \le n \qquad f\left(u_{ij}\right) = m_1 + m_2 + ---- + m_{i-1} + j, \; 1 \le i \le n, 1 \le j \le m_i$

$E(G) = \{v_0v_{ij} / 1 \le i \le n, 1 \le j \le m_i\} \cup \{v_iv_{ij} / 1 \le i \le n, 1 \le j \le m_i\}$

Clearly, $f(v_1) < f(v_2) < --- < f(v_n)$ and

$f(v_0) < f(u_{11}) < f(u_{12}) < --- < f(u_{1m_1}) <$

$f(u_{21}) < f(u_{22}) < f(u_{nm_2}) -- < f(u_{n1}) <$

$f(u_{n2}) < --- < f(u_{nm_n})$

Hence the edge labels of $SS^{-1}\left(K_{1,n}\right)$ are distinct and can be arranged in an decreasing order.

Theorem 1.3: Arbitrary super subdivision of cycle C_n is reverse square sum.

Proof: Let $\{v_1, v_2, v_3, --- v_n\}$ be vertices of the cycle C_n.

Let $e_i = v_iv_{i+1}, \; 1 \le i \le n-1$ be the edges of C_n with $e_n = v_nv_1$.

Construction of arbitrary super subdivision of C_n is same as replacing each edge $e_i, 1 \le i \le n$, of C_n by a complete bipartite graph K_{2,m_i} where m_i is a positive integer. Let u_{ij} be any vertex of the positive set with cardinality m_i $1 \le i \le n, 1 \le j \le m_i$.

Now $\left|V\left(SS\left(P_n\right)\right)\right| = n+m$ where $m = m_1 + m_2 + --- + m_n$

Consider two cases,

Case I: n is even

Define $f : V\left(SS^{-1}\left(C_n\right)\right) \to \{m+n-1, m+n-2, ---, 1, 0\}$ by

$f(v_1) = m+n-1$

$f(u_{1i}) = f(v_1) - i, \quad 1 \le i \le m, \; f(u_{ni}) = f(u_{1m_1}) - i$

$1 \le i \le m_n$

$f(v_2) = f(u_{nm_n}) - 1 \;\; f(v_n) = f(v_2) - 1$

$f(u_{2i}) = f(v_n) - i, \;\; 1 \le i \le m_2$

$f(u_{(n-1)i}) = f(u_{2m_2}) - i, \;\; 1 \le i \le m_{n-1}$

$f(v_3) = f(u_{n-1m_{n-1}}) - 1 \;\; f(v_{n-1}) = f(v_3) - 1$

$f(u_{3i}) = f(v_{n-1}) - i,\ 1 \le i \le m_3$

$f(u_{(n-2)i}) = f(u_{3m_3}) - i,\ \ 1 \le i \le m_{n-2}$

$f\left(v_{\left(\frac{n}{2}+1\right)i}\right) = f\left(u_{\left(\frac{n}{2}m_{\frac{n}{2}}\right)}\right) - i,\ 1 \le i \le m_{\frac{n}{2}+1}$

$f\left(u_{\left(\frac{n}{2}+1\right)}\right) = f\left(u_{\left(\frac{n}{2}+1 m_{\frac{n}{2}+1}\right)}\right) - 1$ In the above vertex labelling no two of the edge labels are same as the edge labels are in descending order.

Case II: n is odd.

Define $f : V\left(SS^{-1}\left(C_n\right)\right) \to \{m+n-1, m+n-2, ---, 1, 0\}$

by $f(v_1) = m + n - 1$

$f(u_{1i}) = f(v_1) - i\ ,\quad 1 \le i \le m_1,$

$f(u_{ni}) = f(u_{1m_1}) - i\ ,\quad 1 \le i \le m_n$

$f(v_2) = f(u_{nm_n}) - 1$

$f(v_n) = f(v_2) - 1$

$f(u_{2i}) = f(v_n) - i,\quad 1 \le i \le m_2$

$f(u_{(n-1)i}) = f(u_{2m_2}) - i,\ \ 1 \le i \le m_{n-1}$

$f(v_3) = f(u_{n-1m_{n-1}}) - 1$

$f(v_{n-1}) = f(v_3) - 1$

$f(u_{3i}) = f(v_{n-1}) - i,\quad 1 \le i \le m_3$

$f(u_{(n-2)i}) = f(u_{3m_3}) - i\ , 1 \le i \le m_{n-2}$

$f\left(u_{\left\lceil\frac{n}{2}\right\rceil i}\right) = f\left(u_{\left\lceil\frac{n}{2}+1\right\rceil}\right) - i\ \ , 1 \le i \le m_{\left\lceil\frac{n}{2}\right\rceil}$

In the above labelling of the vertices, no two of the edge labels are same as the edge labels are in decreasing order. Generally it is sufficient to check the labelling of edges with end vertices labels $\left(a, a+x\right)$ and $\left(a+1, a+y\right)$ where $a = f\left(v_{\left\lceil\frac{n}{2}\right\rceil}\right), x > y$ if

$a^2 + (a+x)^2 = (a+1)^2 + (a+y)^2$

Substituting $x - y = c$ in the simplification of above equation

We get $c^2 + 2ac + ayc = 2a + 1$ has no integer root (solution) for any positive integer value of c. Hence all the edges labels are distinct.

Theorem 1.4: Arbitrary super subdivision of Tadpole $T(n, l)$ is reverse square sum.

Proof: Let $\{v_1, v_2, v_3, --- v_n\}$ be the vertex set of cycle C_n of length n.

Let $e_i = v_i v_{i+1}, 1 \le i \le n-1$ be the edges of C_n with $e_n = v_n v_1$ and P_l be the path with vertices $v_{\left\lceil\frac{n}{2}+1\right\rceil}, v_{n+1}, v_{n+2}, --- v_{n+l-1}$ where $v_{\left\lceil\frac{n}{2}+1\right\rceil}$ is the vertex common to both C_n and P_l. Arbitrary super subdivision of C_n obtained by replacing each edge e_i of cycle C_n by complete bipartite graph K_{2,m_i} where m_i is any positive integer. Let u_{ij} be any vertex of the partite set with cardinality m_i, $1 \le i \le n, 1 \le j \le m_i$. The edge $v_{\left\lceil\frac{n}{2}+1\right\rceil} v_{n+1}$ of P_l is replaced by $K_{2,m_{(n+1)}}$ and edge $v_{n+1} v_{(n+i+1)}$, $1 \le i \le l-2$ is replaced by $K_{2,m_{(n+i+1)}}$.

Here

$\left|V\left(SS\left(T\left(n,l\right)\right)\right)\right| = m + n + l - 1$

where $m = m_1 + m_2 + --- + m_n + m_{n+1} + -- m_{n+l-1}$.

Define $f : \left|V\left(SS^{-1}\left(T\left(n,l\right)\right)\right)\right| \to \{m+n+l-2, m+n+l-3, ---, 1, 0\}$

as follows. The vertices of $SS^{-1}\left(C_n\right)$ is labelled as in the proof of Theorem1.3.

Consider two cases

Case I: n is even.

Define

$f : \left|V\left(SS^{-1}\left(P_l\right)\right)\right| \to \{m+n+l-2, m+n+l-3, ---, m_1, m_2, ---, m_n, m_{n-1}\}$

by $f\left(v_{\left\lceil\frac{n}{2}+1\right\rceil}\right) = m + 2 - (m_1 + m_2 + --- m_n)$

$f\left(v_{n+1}\right) = m + 1 - (m_1 + m_2 + --- m_{n+1})$

$f\left(v_{n+i}\right) = m + n - m_{n+i} - f(v_{n+i-1}) - (m_{n+i} + m_{n+i-1} + --- + m_{n+i-(l-2)})$, $2 \le i \le l-1$

$f(u_{(n+1),j}) = m + 2 - (m_1 + m_2 + --- + m_n + j)$,

$1 \le j \le m_{(n+1)}$

$f(u_{(n+2),j}) = m + 1 - (m_1 + m_2 + --- + m_{n+1} + j)$,

$1 \le j \le m_{(n+2)}$

$$f\left(u_{(n+l-1),j}\right) = m - m_{n+1} - j - f\left(v_{\left\lceil \frac{n+l-1}{2} \right\rceil}\right),$$

$1 \le j \le m_{(n+l-1)}$

Case II: n is odd.

$$f\left(v_{\left\lceil \frac{n}{2}+1 \right\rceil}\right) = m + 2 - (m_1 + m_2 + - - -m_n) + m_{\left\lceil \frac{n}{2}+1 \right\rceil}$$

$$f\left(v_{n+1}\right) = m + 1 - (m_1 + m_2 + - - -m_{n+1}) + m_{\left\lceil \frac{n}{2}+1 \right\rceil}$$

$$f\left(v_{n+i}\right) = m + n - l - 2 - f(v_{n+i-l}) - (m_{n+i} + m_{n+i-1} + - - - + m_{n+i-(l-2)})$$

$2 \le i \le l - 1$

$$f\left(u_{(n+1),j}\right) = m + 2 - (m_1 + m_2 + - - - + m_n + j),$$

$1 \le j \le m_{(n+1)}$

$$f\left(u_{(n+2),j}\right) = m + 1 - (m_1 + m_2 + - - - + m_{n+1} + j) + m_{\left\lceil \frac{n}{2} \right\rceil},$$

$1 \le j \le m_{(n+2)}$

$$f\left(u_{(n+l-1),j}\right) = m - m_{n+1} - j - f\left(v_{\left\lceil \frac{n+l-1}{2} \right\rceil}\right),$$

$1 \le j \le m_{(n+l-1)}$

In the above labelling no two edges labels are same and the edge labelling is in decreasing order.

Theorem 1.5: Arbitrary super subdivision of Tree T is reverse square sum.

Proof: Let T be a tree with n vertices. Construction of arbitrary super subdivision of T is same as replacing each edge e_i, $1 \le i \le n-1$, of T by a complete bipartite graph K_{2,m_i} where m_i is any positive integer. Let $m = m_1 + m_2 + - - - - + m_{n-1}$ Then $\left|V\left(SS\left(T\right)\right)\right| = m + n$. Let $v_{(0,0)}$ be the vertex with minimum eccentricity in T. Choose $v_{(0,0)}$ as the root vertex. Let k denote the height of T. Let n_i be the number of vertices adjacent to $v_{(0,0)}$ named as first level vertices say $v_{(1,1)}, v_{(1,2)}, - - - - v_{(1,n_1)}$. These vertices are at a distance 1 from $v_{(0,0)}$. Denote the vertices adjacent to $v_{(1,1)}, v_{(1,2)}, - - - - v_{(1,n_1)}$ by $v_{(2,1)}, v_{(2,2)}, - - - - v_{(2,n_2)}$ in such a manner that vertices adjacent to $v_{(1,i)}$ are labelled first in an increasing order when compared to that $v_{(i,j)}$,$1 \le i \le j \le n_i$ .These vertices are at a distance 2 from $v_{(0,0)}$ and named second level vertices. Proceeding like this, let $v_{(k,1)}, v_{(k,2)}, - - - - v_{(k,n_k)}$ denote the vertices in k^{th} level.

Define $f : V\left(SS^{-1}\left(T\right)\right) \to \{m+n-1, m+n-2, ---, 1, 0\}$

by $f\left(v(0,0)\right) = m+n-1$, $1 \le i \le n_1$ $f\left(v(1,i)\right) = m+n-1-i$,

$1 \le i \le n_2$ $f\left(v(3,i)\right) = (m+n-1) - (n_1 + n_2 + i))$,$1 \le i \le n_3$

$f\left(v(k,i)\right) = (m+n-1) - (n_1 + n_2 + - - - - + n_{k-1} + i)), 1 \le i \le n_k$

With the above vertex labelling no two of the edges labels are same as the edge labels are in a decreasing order.

Theorem 1.6: Arbitrary super subdivision of grid graph $P_n x P_l$ is reverse square sum.

Proof: Let v_{ij} be the vertex of $P_n x P_l$, $1 \le i \le n, 1 \le j \le l$. Arbitrary super subdivision of $P_n x P_l$ is obtained by replacing every edge of grid graph with K_{2,m_j} where m_i is any positive integer and we denote the resultant graph by G. Let $m = \sum_{i=1}^{2nl-l-n} m_i$. Let u_j be any vertex of partite set of cardinality m_i, $1 \le i \le n$ .Here $\left|V\left(G\right)\right| \le nl$

Define, $f^{-1} : V(G) \to \{m+nl-1, m+nl-2, ---, 1, 0\}$ as follows,

Let $f\left(V_{(1,1)}\right) = m + nl - 1$

Start from V_{11} of G, apply BFS algorithm and label the vertices by $m + nl - m, m + nl - 2, ---, 1, 0$ in descending order, the order in which they are visited. With vertex labelling no two of the edge labels are same as the edge labels are in decreasing order.

An immediate consequence of theorem 1.6 we have the following

Corollary1.6.1: Arbitrary super subdivision of ladder L_n, $SS^{-1}(L_n)$ is reverse square sum.

Theorem 1.7: Arbitrary super subdivision of armed crown $C_n \odot P_l$ is reverse square sum.

Proof: Let cycle C_n has the vertex set $\{v(1,1), v(2,1), ---v(n,1)\}$ And $v_{(i,j)}, 2 \le i \le n, 1 \le j \le l$ be the vertices of paths. Arbitrary super subdivision of $C_n \odot P_l$ is obtained by replacing every edge of $C_n \odot P_l$ with K_{2,m_i} where m_i is any positive integer.

Let $m = \sum_{i=1}^{mn} m_i$ and u_j be any vertex of partite set of cardinality m_i, $1 \le i \le m$ Here $\left|V\left(SS\left(C_n \odot P_l\right)\right)\right| = m + nl$

Define $f : V\left(SS^{-1}\left(C_n \odot P_l\right)\right) \to \{m+nl-1, ---, 1, 0\}$ by $f\left(V_{(1,1)}\right) = m + nl - 1$ Start from $V_{(1,1)}$ of $SS^{-1}\left(C_n \odot P_l\right)$, apply BFS algorithm and label the vertices as $m + nl - 1, ---, 1, 0$ in descending order, the order in which they are visited. With the above vertex labelling, no two of the edge labels are same as the edge labels are in a descending order.

Theorem 1.8: Construction of an arbitrary super subdivision of $C_n^{(l)}$ is reverse square sum.

Proof: Arbitrary super subdivision of $C_n^{(l)}$ is same as replacing each edge of $C_n^{(l)}$ by K_{2,m_i} where m_i is any positive integer and we denote this graph by G. Let $m = \sum m_i$. Here $\left|V\left(SS\left(C_n^{(l)}\right)\right)\right| = m + l(n-1) + 1$. Denote the common vertex of cycles by $v_{(0,0)}$. Let n_1 be the number of vertices adjacent to $v_{(0,0)}$ named as first level say $v_{(1,1)}, v_{(1,2)}, ---, v_{(1,n_1)}$ in clockwise direction. These vertices are at the distance 1 from $v_{(0,0)}$ denoting the vertices adjacent by $v_{(2,1)}, v_{(2,2)}, ---, v_{(2,n_2)}$ in such a manner that vertices adjacent to $v_{(1,i)}$ are labelled first in a decreasing order when compared to that of $v_{(1,j)}, 1 \le j \le n_i$. These vertices are at a distance 2 from $v_{(0,0)}$ and named second level vertices. Proceeding like this, let $v_{(k,1)}, v_{(k,2)}, ---, v_{(k,n_k)}$ denote the vertices in the last level.

Define, $f : V\left(SS^{-1}\left(C_n^{(l)}\right)\right) \to \{m + l(n-1), ---, 1, 0\}$ by

$f(v_{(0,0)}) = m + l(n-1)$ $f(v_{(1,i)}) = m + l(n-1) - i, 1 \le i \le n_1$

$f(v_{(2,i)}) = m + l(n-1) - (n_1 + i), 1 \le i \le n_2$

$f(v_{(3,i)}) = m + l(n-1) - (n_1 + n_2 + i), 1 \le i \le n_3$

$f(v_{(k,i)}) = m + l(n-1) - (n_1 + n_2 + ---- + n_{k-1} + i),$

$1 \le i \le n_k$

Case I: When n is even, with the above vertex labelling no two of the edge labels are same, as the edge labels are in an decreasing order.

Case II: When n is odd, it is enough to check the labelling of edges with end vertices having labels $(a, a+x)$ and $(a+1, a+y)$ where $a = f(V_{n-1})$, $i = 1, 3, ---, 2l-1, x > y$ .If $a^2 + (a+x)^2 = (a+1)^2 + (a+y)^2$ then $k^2 + 2ak + 2yk = 2a + 1$ where $x - y = c$ has no integer solution for any positive integer value of c.

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