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RESEARCH ARTICLE

# G^^*S-CONTINUOUS MAPS IN TOPOLOGICAL SPACES

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**ABSTRACT**

In this paper, we introduce the concepts of g^*s-continuity and g^*s-irresoluteness mappings and their characterizations.



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## INTRODUCTION

The concept of generalized closed set of a topological was introduced by N. Levine in 1970 (Levine, 1970). These sets were also considered by W.Dunham and N. Levine in 1980 (Dunham and Levine, 1980) and by W. Dunham in 1982 (Dunham and Levinec, 1982). Since then new concepts have been introduced, studied, investigated and developed in the field of generalized closed sets by various authors. In 1991, K. Balachandran, H. Maki and P. Sundaram (Balachandran *et al.,* 1991) defined a new class of mappings called generalized continuous mappings which contains the class of continuous mappings. S. Pious Missier and M. Anto studied and investigated the basic properties of $\hat{g}^*$s-closed sets (Pious Missier and Anto) by generalizing the semi closed sets using $\hat{g}$-open sets. Based on $\hat{g}^*$s-closed sets, we continue the study of the associated functions, namely, $\hat{g}^*$s-irresolute and $\hat{g}^*$s-continuous functions.

### Preliminaries

Definition 2.1 (Levine, 1963) A subset A of a topological space (X,$\tau$) is called semi open if A ⊆ cl(int(A)). A subset A of a topological space (X,$\tau$) is called semi closed if $A^c$ is the complement of A.
Definition 2.2 (Veerakumar, 2001) A subset A of a topological space (X,$\tau$) is called a $\hat{g}$- closed set if cl(A) ⊆ U whenever A ⊆ U and U is semi open.
Definition 2.3 (Veerakumar, 2006) A subset A of a topological space (X,$\tau$) is called a $\hat{g}^*$-closed set if cl(A) ⊆ U whenever A ⊆ U and U is $\hat{g}$-open.
Definition 2.4 A function f: (X, $\tau$) → (Y,$\sigma$) is continuous if $f^{-1}$(U) is closed in X for each closed set U in Y.
Definition 2.5 (Balachandran *et al.,* 1991) A function f: (X, $\tau$) → (Y,$\sigma$) is g-continuous if $f^{-1}$(U) is g-closed in X for each closed set U in Y.
Definition 2.6 (Levine, 1970) A function f: (X, $\tau$) → (Y,$\sigma$) is irresolute if $f^{-1}$(U) is semi closed in X for each semi closed set U in Y.

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Definition 2.7 (Veerakumar, 2001) A function f: $(X, \tau) \rightarrow (Y, \sigma)$ is $\hat{g}$- irresolute if $f^{-1}(U)$ is $\hat{g}$- closed in X for each $\hat{g}$- closed set U in Y.
Definition 2.8 (Crorrly and Hilderbrand, 1972) A function f: $(X, \tau) \rightarrow (Y, \sigma)$ is pre semi closed if $f(V)$ is semi closed in Y for each semi closed set V in X.
Definition 2.9 (Devi *et al.,* 1993) A topological space $(X, \tau)$ is called a $T_b$ space if every gs-closed set is closed.
Definition 2.10 (Devi *et al.,*1993) A topological space $(X, \tau)$ is called a $T_d$ space if every gs-closed set is g-closed.
Definition 2.11 (Levine, 1970) A topological space $(X, \tau)$ is called a $T_{\frac{1}{2}}$ space if every gs-closed set is g-closed.
Lemma 2.12 If f: $(X, \tau) \rightarrow (Y, \sigma)$ is irresolute, then for every subset B of Y, $scl(f^{-1}(B)) \subseteq f^{-1}(scl(B))$.

**Proof:** Let $x \in scl(f^{-1}(B))$ .Suppose that V is any semi open set of Y containing f(x).Then $x \in f^{-1}(V)$.Since f is irresolute, $f^{-1}(V)$ is semi open set of X and $f^{-1}(V) \cap f^{-1}(V) \neq \emptyset$.

$\Rightarrow f^{-1}(V \cap B) \neq \emptyset$.

$\Rightarrow V \cap B \neq \emptyset$.

$\Rightarrow f(x) \in scl(B)$.

$\Rightarrow x \in f^{-1}(f(x)) \subseteq f^{-1}(scl(B))$.

$\Rightarrow scl(f^{-1}(B)) \subseteq f^{-1}(scl(B))$.

Notations used:

(i) $\hat{g}^*sC(X, \tau)$ denotes the class of all $\hat{g}^*$s-closed sets in $(X, \tau)$.
(ii) $\hat{g}^*sO(X, \tau)$ denotes the class of all $\hat{g}^*$s-open sets in $(X, \tau)$.
(iii) scl(A) denotes semi closure of A
(iv) sint(A) denotes semi interior of A.

3. $\widehat{g}^*$s-continuous maps.

Definition 3.1(Pious Missier and Anto) A subset A of a topological space $(X, \tau)$ is called a $\hat{g}^*$s-closed set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$-open.
Definition 3.2 A function f: $(X, \tau) \rightarrow (Y, \sigma)$ is $\hat{g}^*$s -continuous if $f^{-1}(U)$ is $\hat{g}^*$s-closed in X for each closed set U in Y.
Definition 2.3 (Veerakumar, 2001) A function f: $(X, \tau) \rightarrow (Y, \sigma)$ is $\hat{g}^*$s - irresolute if $f^{-1}(U)$ is $\hat{g}^*$s - closed in X for each $\hat{g}^*$s - closed set U in Y.

Example 3.4 Let $(X, \tau)$ and $(Y, \sigma)$ be two topological spaces where X = Y = { a, b, c, d } with $\tau = \{\emptyset, X, \{a\}, \{a, b, c\}, \{a, d\}\}$ and $\sigma = \{\emptyset, X, \{a\}, \{a, b\}, \{a, b, c\}\}$.Then $\tau^c = \{\emptyset, X, \{b, c, d\}, \{d\}, \{b, c\}\}$ and $\sigma^c = \{\emptyset, X, \{b, c, d\}, \{c, d\}, \{d\}\}$. Also $\hat{g}^*sC(X, \tau) = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{c, d\} \{b, d\}, \{b, c\}, \{d\}, \{c\}, \{b\}\}$.

Define f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = b, f(b) = d, f(c) = a, f(d) = c. We have $f^{-1}(\{b, c, d\}) = \{a, b, d\}$, $f^{-1}(\{c, d\}) = \{b, d\}$, $f^{-1}(\{d\}) = \{b\}$. Thus $f^{-1}(U)$ is $\hat{g}^*$s - closed in X for each set U in Y. Therefore f is $\hat{g}^*$s -continuous.

Proposition 3.5. The following are equivalent for f: $(X, \tau) \rightarrow (Y, \sigma)$.

(i) f is $\hat{g}^*$s -continuous.
(ii) $f^{-1}(U)$ is $\hat{g}^*$s-open for each open set U in Y.

**Proof:** (i) ⇒ (ii)

Suppose that f is $\hat{g}^*$s -continuous. Let U be open in Y. Then $U^c$ is closed in Y. Since f is $\hat{g}^*$s –continuous, we have $f^{-1}(U^c)$ is $\hat{g}^*$s - closed in X. But $f^{-1}(U^c) = (f^{-1}(U))^c$. Therefore $f^{-1}(U)$ is $\hat{g}^*$s-open in X.

(ii) ⇒ (i)

Suppose that $f^{-1}(U)$ is $\hat{g}^*$s-open for each open set U in Y. Let V be closed in Y. Then $V^c$ is open in Y. By assumption $(f^{-1}(V))^c$ is $\hat{g}^*$s - open in X and hence $f^{-1}(V)$ is $\hat{g}^*$s - closed in X. Thus f is $\hat{g}^*$s -continuous.

Proposition 3.6 Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be a function.

(i) f is $\hat{g}^*$s -continuous.
(ii) For each x in X and for each open set V containing f(x), there is a $\hat{g}^*$s –open set U containing x such that f(U) ⊆ V.
(iii) $f(\hat{g}^*\mathrm{scl}(A)) \subseteq \mathrm{cl}(f(A))$ for each subset A of X.
(iv) $\hat{g}^*\mathrm{scl}(f^{-1}(B)) \subseteq f^{-1}(\mathrm{cl}(f(B)))$ for each subset B of Y.

Then (i) ⇒ (ii) ⇒ (iii) ⇒ (iv)

**Proof:** (i) ⇒ (ii)

Let $x \in X$ and V be an open set containing f(x). Then, by (i), $f^{-1}(V)$ is $\hat{g}^*$s –open set of X containing x. If U = $f^{-1}(V)$, then f(U) = $f(f^{-1}(V)) \subseteq V$.

(ii) ⇒ (iii)

Let A be a subset of a space X and $f(x) \notin \mathrm{cl}(f(A))$. Then there exists open set V of Y containing f(x) such that $V \cap f(A) = \emptyset$. Now, by (ii), there is a $\hat{g}^*$s –open set U containing x such that $f(x) \in f(U) \subseteq V$. Hence $f(U) \cap f(A) = \emptyset$. i.e., $f(U \cap A) = \emptyset$. i.e., $U \cap A = \emptyset$. Therefore $x \notin \hat{g}^*\mathrm{scl}(A)$.Therefore $f(x) \notin f(\hat{g}^*\mathrm{scl}(A))$.Therefore $f(\hat{g}^*\mathrm{scl}(A)) \subseteq \mathrm{cl}(f(A))$.

(iii)⇒ (iv)

Let B be a subset of Y such that A = $f^{-1}(B)$. By (iii), $f(\hat{g}^*\mathrm{scl}(A)) \subseteq \mathrm{cl}(f(A))$ for each subset A of X. Therefore, $f(\hat{g}^*\mathrm{scl}(f^{-1}(B))) \subseteq \mathrm{cl}(f(f^{-1}(B)))$. i.e., $f\left(\hat{g}^*\mathrm{scl}\left(f^{-1}(B)\right)\right) \subseteq \mathrm{cl}(B)$. i.e., $\hat{g}^*\mathrm{scl}(f^{-1}(B)) \subseteq f^{-1}(\mathrm{cl}(B))$.

Lemma 3.7 (Pious Missier and Anto): A subset A of a topological space (x , $\tau$) is $\hat{g}^*$s -open iff F ⊆ sint(A) whenever F ⊆ A and F is $\hat{g}$-closed.

Proposition 3.8: Let B be a $\hat{g}^*$s -open (or $\hat{g}^*$s –closed) subset of (Y,$\sigma$) satisfying sint(B) = int(B). Then $f^{-1}(B)$ is $\hat{g}^*$s -open (or $\hat{g}^*$s –closed) in (X , $\tau$) if f: (X, $\tau$) → (Y,$\sigma$) is $\hat{g}^*$s –continuous and if image of a $\hat{g}$-closed set in X under f is $\hat{g}$-closed set Y.

**Proof:** Let B be a $\hat{g}^*$s –open set in Y. Let F ⊆ $f^{-1}(B)$ where F is a $\hat{g}$-closed set in X. Then f(F) ⊆ B holds. By our assumption, f(F) is $\hat{g}$-closed set in Y and B be a $\hat{g}^*$s –open set in Y. Therefore, by Lemma 3.7, f(F) ⊆ sint(B) holds. Again, by our assumption, f(F) ⊆ int(B) and hence F ⊆ $f^{-1}$(int(B)) holds. Since f is $\hat{g}^*$s –continuous and int(B) is open in Y, $f^{-1}$(int(B)) is $\hat{g}^*$s –open in X. So, by Lemma 3.7,

F ⊆ sint($f^{-1}$(int(B))) holds. i.e., F ⊆ sint($f^{-1}$(int(B))) ⊆ sint($f^{-1}$(B)) holds. Therefore $f^{-1}$(B) is $\hat{g}^*$s –open. By taking complements, we can show that if B is $\hat{g}^*$s –closed in Y, then $f^{-1}$(B) is $\hat{g}^*$s –closed in X.

Proposition 3.9: The following are equivalent for f: (X, $\tau$) → (Y,$\sigma$).

(i) f is $\hat{g}^*$s-irresolute.
(ii) $f^{-1}$(U) is $\hat{g}^*$s –open for each $\hat{g}^*$s –open set U in Y.

**Proof:** (i) ⇒ (ii)

Suppose that f is $\hat{g}^*$s-irresolute. Let U be $\hat{g}^*$s –open in Y. Then $U^c$ is $\hat{g}^*$s –closed in Y. Since f is $\hat{g}^*$s –irresolute, we have $f^{-1}(U^c)$ is $\hat{g}^*$s –closed in X. But $f^{-1}(U^c) = (f^{-1}(U))^c$.Therefore $f^{-1}$(U) is $\hat{g}^*$s –open in X.

(iii)⇒(i)

Suppose that $f^{-1}$(U) is $\hat{g}^*$s –open for each $\hat{g}^*$s –open set U in Y. Let V be $\hat{g}^*$s –closed in Y. Then $V^c$ is $\hat{g}^*$s –open in Y. Therefore $f^{-1}(V^c)$ is $\hat{g}^*$s –open in X. Therefore $f^{-1}$(V) is $\hat{g}^*$s – closed in X. Therefore f is $\hat{g}^*$s-irresolute.

Proposition 3.10: If a function f: (X, $\tau$) → (Y,$\sigma$) is $\hat{g}^*$s-irresolute, then f is $\hat{g}^*$s-continuous.

**Proof:** Let V be a closed set of Y. But every closed set is $\hat{g}^*$s-closed. Therefore V is a $\hat{g}^*$s-closed set of Y. Since f is $\hat{g}^*$s-irresolute, $f^{-1}$(V) is $\hat{g}^*$s-closed in X. Therefore, by Definition 3.2, f is $\hat{g}^*$s-continuous.

Remark 3.11: The converse of Proposition 3.10 need not be true as seen from the following example.

Example 3.12 Let $(X, \tau)$ and $(Y,\sigma)$ be two topological spaces where X = Y = { a, b, c, d } with $\tau = \{ \emptyset, X, \{a\}, \{ a, b, c \}\}$ and $\sigma = \{\emptyset, X, \{a\}, \{a, b\}, \{ a, b, c \}\}$.Then $\tau^c = \{ \emptyset, X, \{b, c, d\}, \{ d \}\}$ and $\sigma^c = \{\emptyset, X, \{b, c, d\}, \{c, d\}, \{ d \}\}$. Also $\hat{g}^*sC(X, \tau) = \{ \emptyset, X, \{b, c, d\}, \{a, c, d\}, \{ a, b, d \}, \{c, d\}, \{b, d \}, \{b, c\}, \{a, d\}, \{d\}, \{c\}, \{b\} \}$and $\hat{g}^*sC(X, \tau) = \{ \emptyset, X, \{b, c, d\}, \{a, c, d\}, \{ a, b, d \}, \{c, d\}, \{b, d \}, \{b, c\}, \{a, d\}, \{d\}, \{c\}, \{b\} \}$. Define f: $(X, \tau) \rightarrow (Y,\sigma)$ by f(a) = a, f(b) = c, f(c) = d, f(d) = b. We have $f^{-1}(\{b, c, d\}) = \{b, c, d\}$, $f^{-1}(\{ c, d\}) = \{ b, c\}$, $f^{-1}(\{d\}) = \{c\}$. Thus $f^{-1}(U)$ is $\hat{g}^*$s - closed in X for each set U in Y. Therefore f is $\hat{g}^*$s -continuous. But $f^{-1}(\{a, c, d\}) = \{a, b, c\}$ is not $\hat{g}^*$s - closed in X, whereas {a,c,d} is $\hat{g}^*$s - closed in Y. Therefore f: $(X, \tau) \rightarrow (Y,\sigma)$ is not $\hat{g}^*$s-irresolute.

Proposition3.13 Let Y be a $T_b$ space. A function f**:** $(X, \tau) \rightarrow (Y,\sigma)$ is not $\hat{g}^*$s-irresolute if it is $\hat{g}^*$s –continuous.

**Proof:** Let f: $(X, \tau) \rightarrow (Y,\sigma)$ be $\hat{g}^*$s-continuous. Let A be a $\hat{g}^*$s– closed in Y. But every $\hat{g}^*$s– closed is gs-closed.Therefore A is gs-closed in Y. Since Y is a $T_b$ space, A is closed. Since f is $\hat{g}^*$s–continuous,

$f^{-1}(A)$ is $\hat{g}^*$s– closed in X. Hence f is $\hat{g}^*$s-irresolute.

Proposition3.14 Let Y be a $T_d$ space and a $T_{\frac{1}{2}}$ space. A function f**:** $(X, \tau) \rightarrow (Y,\sigma)$ is $\hat{g}^*$s-irresolute if it is $\hat{g}^*$s–continuous.

**Proof:** Let f**:** $(X, \tau) \rightarrow (Y,\sigma)$ be $\hat{g}^*$s– continuous. Let A be a $\hat{g}^*$s– closed in Y. But every $\hat{g}^*$s– closed is gs-closed. Therefore A is gs-closed in Y. Since Y is a $T_d$ space, A is g-closed in Y. Since Y is a $T_{\frac{1}{2}}$ space, A is closed in Y.Since f is $\hat{g}^*$s –continuous, $f^{-1}(A)$ is $\hat{g}^*$s – closed in X. Hence f is $\hat{g}^*$s-irresolute.

Proposition3.15 Let f**:** $(X, \tau) \rightarrow (Y,\sigma)$ and g**:** $(Y,\tau) \rightarrow (Z, \sigma)$ be two functions. Let Y be a $T_{\frac{1}{2}}$ space, g a g-continuous function and f a $\hat{g}^*$s –continuous function. Then $g \circ f$ is $\hat{g}^*$s–continuous.

**Proof:** Let U be closed in Z. Since g is g-continuous, $g^{-1}(U)$ is g-closed in Y. But Y is a $T_{\frac{1}{2}}$ space. Therefore, $g^{-1}(U)$ is closed in Y. Since f is $\hat{g}^*$s –continuous, $f^{-1}(g^{-1}(U))$ is $\hat{g}^*$s– closed, Therefore $g \circ f$ is $\hat{g}^*$s–continuous.

Proposition3.15 Let f**:** $(X, \tau) \rightarrow (Y,\sigma)$ and g**:** $(Y,\sigma) \rightarrow (Z, \mu)$ be two functions. Let Y be a $T_{\frac{1}{2}}$ space, g a g-continuous function and f a $\hat{g}^*$s –continuous function. Then $g \circ f$ is $\hat{g}^*$s –continuous.

**Proof:** Let U be closed in Z. Since g is g-continuous, $g^{-1}(U)$ is g-closed in Y. But Y is a $T_{\frac{1}{2}}$ space. Therefore , $g^{-1}(U)$ is closed in Y. Since f is $\hat{g}^*$s –continuous, $f^{-1}(g^{-1}(U))$ is $\hat{g}^*$s– closed, Therefore $g \circ f$ is $\hat{g}^*$s –continuous.

Proposition 3.16 Let f**:** $(X, \tau) \rightarrow (Y,\sigma)$ be $\hat{g}^*$s-irresolute and X is $T_b$. Then f is continuous.

**Proof:** Let V be closed subset of Y. Then V is semi closed and hence $\hat{g}^*$s–closed in Y. Since f is $\hat{g}^*$s-irresolute, $f^{-1}(V)$ is $\hat{g}^*$s–closed in X. But every $\hat{g}^*$s –closed is gs-closed. Therefore $f^{-1}(V)$ is gs-closed in X. But X is $T_b$. Therefore $f^{-1}(V)$ is closed in X. Therefore f is continuous.

Proposition 3.17: Let f: $(X, \tau) \rightarrow (Y,\sigma)$ be $\hat{g}^*$s-irresolute and X is $T_b$. Then f is irresolute.

**Proof:** Let V be a semi closed subset of Y. Then V is $\hat{g}^*$s–closed in Y. Since f is $\hat{g}^*$s-irresolute, $f^{-1}(V)$ is $\hat{g}^*$s –closed in X. But every $\hat{g}^*$s –closed is gs-closed. Therefore $f^{-1}(V)$ is gs-closed in X. But X is $T_b$. Therefore $f^{-1}(V)$ is closed in X. Therefore f is irresolute

Lemma 3.18: If a function f: $(X, \tau) \rightarrow (Y,\sigma)$ is surjective and if image of a $\hat{g}$-closed set is $\hat{g}$-closed under f, then for every subset S of T and each $\hat{g}$-open set U of X containing $f^{-1}(S)$, there exists $\hat{g}$-open set V of Y such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.

**Proof:** Let $S \subseteq Y$ and U be a $\hat{g}$-open set in X, containing $f^{-1}(S)$. Put V = Y – f (X – U). Then V is $\hat{g}$-open in Y containing S.

$\Rightarrow f^{-1}(V) = f^{-1}(Y - f(X - U))$
$\Rightarrow f^{-1}(V) = X - f^{-1}(f(X - U))$
$\Rightarrow f^{1}(V) \subseteq X - (X - U)$
$\Rightarrow f^{-1}(V) \subseteq U$

Proposition 3.19: Let f: $(X, \tau) \rightarrow (Y,\sigma)$ be surjective and if image of a $\hat{g}$-closed set is $\hat{g}$-closed under f. Then for every $\hat{g}^*$s -closed set B in Y, $f^{1}(B)$ is $\hat{g}^*$s -closed in X.

**Proof:** Let B be a $\hat{g}^*$s -closed set in Y. Suppose that $f^{-1}(B) \subseteq U$ where U is $\hat{g}$-open set of X. By assumption and by 3.18, there is $\hat{g}$-open set V in Y such that $B \subseteq V$ and $f^{-1}(V) \subseteq U$. Since B is $\hat{g}^*$s -closed in Y and $B \subseteq V$ and $scl(B) \subseteq V$. Hence $f^{-1}(scl(B)) \subseteq f^{-1}(V) \subseteq U$. By Lemma 2.12, $scl(f^{-1}(B)) \subseteq U$. Therefore $f^{-1}(B)$ is $\hat{g}^*$s -closed in X.

Proposition 3.20: Let f: $(X, \tau) \rightarrow (Y,\sigma)$ be a function such that f is pre semi closed and $\hat{g}$- irresolute. Then for every $\hat{g}^*$s -closed set A in X, f(A) is $\hat{g}^*$s -closed set in Y.

Proof: Let A be a $\hat{g}^*$s -closed set in X. Suppose that $f(A) \subseteq U$ where U is $\hat{g}$-open in Y. Then $A \subseteq f^{-1}(U)$ and $f^{-1}(U)$ is $\hat{g}$-open in Y. Since A is $\hat{g}^*$s-closed set in X, $scl(A) \subseteq f^{-1}(U)$ and hence $f(scl(A)) \subseteq U$. But $scl(f(A)) \subseteq scl(f(scl(A)))$. Since f is pre semi closed, $scl(f(A)) \subseteq f(scl(A))$. Therefore $scl(f(A)) \subseteq U$.Hence f(A) is $\hat{g}^*$s-closed set in Y.

Proposition 3.21: If a function f: $(X, \tau) \rightarrow (Y,\sigma)$ is $\hat{g}^*$s-irresolute, then for every subset A of X, Then $f(\hat{g}^*scl(A)) \subseteq scl(f(A))$

**Proof:** Let $A \subseteq X$. We know that every semi closed set is $\hat{g}^*$s -closed set in Y.Therefore, we have scl(f(A)) is $\hat{g}^*$s -closed in Y. Since f is $\hat{g}^*$s -irresolute, then $f^{-1}(scl(f(A)))$ is $\hat{g}^*$s -closed in X.. Also $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(scl(f(A)))$. Since $f^{-1}(scl(f(A)))$ is $\hat{g}^*$s -closed, we have $\hat{g}^*$s $scl(A) \subseteq f^{-1}(scl(f(A)))$. Therefore$f(\hat{g}^*s\, cl(A)) \subseteq f(f^{-1}(scl(f(A)))) \subseteq scl\big(f(A)\big)$.

Proposition 3.22: If a function f: $(X, \tau) \rightarrow (Y,\sigma)$ is bijective, $\hat{g}^*$s -continuous, scl(A) = cl(A) for all subsets A in Y and if image of a $\hat{g}$-open set is $\hat{g}$-open under f, then f is $\hat{g}^*$s-irresolute.

**Proof:** Let V be a $\hat{g}^*$s -closed set of Y. Let $f^{-1}(V) \subseteq U$ where U is $\hat{g}$-open in X. Then $f(f^{-1}(V)) \subseteq f(U)$. Since f is surjective, $V \subseteq f(U)$. Since f(U) is $\hat{g}$-open and since V is $\hat{g}^*$s-closed in Y, we have $scl(V) \subseteq f(U)$. By our assumption, $cl(V) \subseteq f(U)$. Since f is injective, $f^{-1}( cl(V)) \subseteq U$. Since f is $\hat{g}^*$s-continuous and since cl(V) is closed in Y, $f^{-1}( cl(V))$ is $\hat{g}^*$s-closed in X. Therefore $f^{-1}(V)$ is $\hat{g}^*$s-closed in X and hence f is $\hat{g}^*$s-irresolute.

Definition 3.23 A map f: $X \rightarrow Y$ is called a $\hat{g}^*$s-closed map if f(U) is $\hat{g}^*$s-closed in Y for every closed set U of X.

Definition 3.24 A map f: $X \rightarrow Y$ is called a $\hat{g}^*$s-open map if f(U) is $\hat{g}^*$s-open in Y for every open set U of X.

**Proposition 3.25** If f : $X \rightarrow Y$ is $\hat{g}$-irresolute and $\hat{g}^*$s-closed and A is a $\hat{g}^*$s-closed subset of X, then f(A) is $\hat{g}^*$s-closed in Y.

**Proof:** Let $f(A) \subseteq U$ and U is $\hat{g}$-open in Y. Then $f^{-1}(f(A)) \subseteq f^{-1}(U)$. i.e., $A \subseteq f^{-1}(U)$. Since f is $\hat{g}$-irresolute, $f^{-1}(U)$ is $\hat{g}$-open in X. Since A is $\hat{g}^*$s-closed, $cl(A) \subseteq f^{1}(U)$. So, $f(cl(A)) \subseteq f(f^{1}(U))$. i.e., $f(cl(A)) \subseteq U$. Since f is $\hat{g}^*$s-closed and cl(A) is closed in X, f(cl(A)) is $\hat{g}^*$s-closed in Y. Therefore $scl(f(cl(A))) \subseteq U$. Since $f(A) \subseteq f(cl(A))$, we have $scl(f(A)) \subseteq scl(f(cl(A))) \subseteq U$. Therefore f(A) is $\hat{g}^*$s-closed in Y.

**Proposition 3.26:** If f**:** $X \rightarrow Y$ is $\hat{g}^*$-closed and g**:** $Y \rightarrow Z$ is $\hat{g}$-iresolute and $\hat{g}^*$s-closed, then $g \circ f$ is $\hat{g}^*$s –closed.

**Proof:** Let F be a closed set of X. Since f is $\hat{g}^*$-closed, f(F) is $\hat{g}^*$-closed in Y. Since g is $\hat{g}$-iresolute and $\hat{g}^*$s –closed and f(F) is $\hat{g}^*$-closed in Y, by Proposition 3.25, g(f(F)) is $\hat{g}^*$s-closed in Z. Hence $g \circ f$ **:** $X \rightarrow Z$ is $\hat{g}^*$s-closed.

**Proposition 3.27:** If f**:** $X \rightarrow Y$ is closed and g**:** $Y \rightarrow Z$ is $\hat{g}^*$s-closed, then $g \circ f$ is $\hat{g}^*$s –closed.

**Proof:** Let F be a closed set of X. Since f is closed, f(F) is closed in Y. Since g is $\hat{g}^*$s –closed, g(f(F)) is $\hat{g}^*$s-closed in Z. Hence $g \circ f$ **:** $X \rightarrow Z$ is $\hat{g}^*$s-closed.

**Proposition 3.28:** Let f**:** $X \rightarrow Y$ and g: $Y \rightarrow Z$ be two maps such that $g \circ f$: $X \rightarrow Z$ is $\hat{g}^*$s-open map, if f is continuous and surjective.

**Proof:** Let A be open in Y. Since f is continuous, $f^{-1}(A)$ is open in X. Since $f^{-1}(A)$ is open in X, $g \circ f(f^{-1}(A))$ is $\hat{g}^*$s-open in Z. i.e., g(A) is $\hat{g}^*$s-open in Z. Therefore, g is a $\hat{g}^*$s-open map.

**Proposition:3.29** For any bijection f**:** $X \rightarrow Y$, the following are equivalent:

(i) $f^{-1}$**:** $Y \rightarrow X$ is $\hat{g}^*$s-continuous
(ii) f is $\hat{g}^*$s-open
(iii)f is $\hat{g}^*$s-closed

**Proof**

(i)$\Rightarrow$ (ii)

Let F be open in X. Then X – F is closed in X. Since $f^{-1}$ is $\hat{g}^*$s-continuous, $(f^{-1})^{-1}(X - F) = f(X - F) = Y - f(F)$ is $\hat{g}^*$s-closed in Y. Then f(F) is $\hat{g}^*$s-open in Y. Hence f is $\hat{g}^*$s-open.

(ii) ⇒ (iii)

Let F be closed in X. Then X – F is open in X. Since f is $\hat{g}^*$s-open, $f(X - F) = Y - f(F)$ is $\hat{g}^*$s-open in Y. Then f(F) is $\hat{g}^*$s-closed in Y. Hence f is $\hat{g}^*$s-closed.

(iii)⇒ (i)

Let V be closed in X. Since $f: X \to Y$ is $\hat{g}^*$s-closed. f(V) is $\hat{g}^*$s-closed in Y. i.e., $(f^{-1})^{-1}(V)$ is $\hat{g}^*$s-closed in Y. Therefore $f^{-1}$ is $\hat{g}^*$s-continuous.

## REFERENCES

Balachandran, K., Maki, H. and Sundaram, P. 1991. On generalized continuous maps in topological spaces, Mem. Fac. Kochi Univ. Ser. A. Math., 12, 5-13.
Crorrly, S.G. and Hilderbrand, S.K.1972. Semi topological *properties, Fund. Math.,*74, 233-254
Devi, R. Maki, H. and Balachandran, K. 1993. Semi generalized closed maps and generalized semi closed maps, Mem. Fac. Sci. Kochi Univ. *Ser. A. Math.,* 14, 41-54.
Dunham, W. and Levine, N. 1982. A new closure operator for non-$T_1$ Topologies, Kyungpook *Math. J.,* 22,55-60
Dunham, W. and Levine, N. Further results on generalized closed sets in topological spaces, *Kyungpook Math. J.,* 20(1980), 169-175.
Levine, N.1963. Semi open sets and semi continuity in topological spaces, *Amer. Math. Monthly* 70, 36-41
Levine, N.1970. Generalised closed sets in topology, Rend. *Circ., Mat.* Palermo 19(2), 89 -96.
Pious, S. Missier and Anto, M. $\hat{g}^*$s-closed sets in topological spaces
Veerakumar, M.K.R.S. 2001. $\hat{g}$-locally closed sets and $\hat{g}$LC-functions, *Indian J.of Mathematics,* 43(2) 231-247.
Veerakumar, M.K.R.S. 2006. Between $g^*$-closed sets and g-closed sets, *Antartica J. Math.,* 3(1), 43-65.

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