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RESEARCH ARTICLE

UNSTEADY MAGNETOHYDRODYNAMIC FLOW DUE TO AN EXPONENTIALLY DECAY LINE SOURCE BETWEEN TWO PARALLEL DISKS

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ABSTRACT

An unsteady MHD flow of homogeneous fluid due to an exponentially decay line source placed between two axially symmetric infinite parallel disks is considered under the action of a transverse magnetic field. Considering axial symmetry within the system the expression for radial velocity, pressure and skin friction at the disks are obtained for small values for Reynolds number and at a large distance from the source. The influence of decay parameter, applied field and Reduced Reynolds number over the physical parameter of the problem are analyzed graphically followed by conclusions.

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INTRODUCTION

The study of 'Magneto-hydrodynamics viscous flow' has been to a great extent in the recent years. This is due to the fact that it has many practical applications viz. natural, industrial, astrophysical and geophysical and many other. Free and forced flow in fluid medium in various geometrical set up can be modeled or approximated to various flow and transport phenomena in many natural circumstances such as geothermal extraction, storage of nuclear waste material, ground water flows; industrial situations such as industrial and agricultural water distribution, oil recovery processes, thermal insulation processes, pollutant dispersion in aquifers cooling of electronic components, cooling of nuclear reactor, food processing, casting and welding of manufacturing processes, dispersion of chemical contaminants in various processes, in fibrous insulation and even for obtaining approximate solutions for flow through turbo-machinery etc. In a system of viscous flow, the inertia forces acting are proportional to the square of the velocity, whereas the viscous forces are only proportional to its first power. Therefore, when the viscous forces are considerably greater than the inertia forces, the Navier-stokes equations are limited under some approximations. Such flow where Reynolds number is very small ($R<1$) also called creeping motion. Such kind of flow occurs not only in nature but also in many areas of science and technology. One such study is related to the effects of MHD free convection flow, which plays an important role in agriculture, engineering and petroleum industries.

The oldest known problem on creeping motion was studied by Stokes (Stokes, 1851) who investigated the case of parallel flow past a sphere. Later Prandtl, (1935) discussed the motion in details. Stroke's problem on a sliding surface with respect to the direction of motion was carried out by Reynolds (Reynolds, 1886). He also considered the problem of motion between two parallel flat walls with a pressure gradient. He showed that the inertia forces can be neglected with respect to the viscous forces if the reduced Reynolds number (R^*) <1. The occurrence of high pressure that slows down the viscous motion is a peculiar property of the fluid motion. Such flows generally encounter in case of sphere falling in air, lubrication etc. In reality heat is evolved through friction and the temperature of the fluid (e.g. lubricating oil) is increased. Since the viscosity of oil decreases rapidly with the increasing temperature, the viscous thrust also decreases.

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Several authors /researchers have carried out various problems on magneto hydrodynamics viscous flow where Reynolds number is small enough (R<1). The incompressible radial flow between two parallel stationary disks using the integral approach and the assumption of a parabolic velocity profile was discussed by Livesey (Liversey, 1881). Savage (Savage, 1964) has obtained the solution by expanding velocity components and pressure terms of the downstream coordinate by omitting the no slip condition on the disk. Garandet *et al*. (1992) analyzed the equations of the MHD, which were used to model the effect of a transverse magnetic field on the buoyancy driven convection in a two-dimensional cavity. Elkouh (1975) has given an analysis for a system in which the flow rate was sinusoidal about a zero mean value. His solutions were valid for small values of reduced Reynolds number and all values of frequency Reynolds number. Kim and Lee (2002) set up an experiment using a circular cylinder, where electrodes and magnets were installed in an alternative sequence in the axial direction of the cylinder to generate magnetic force in the circumferential direction. Duwairi *et al*. (2006) discussed MHD-conjugate mixed convection heat transfer over a vertical hollow cylinder embedded in a porous medium.

Raptis and Singh (1983) studied the effect of a uniform transverse magnetic field on the free convection flow of an electrically conducting fluid past an infinite vertical plate for impulsive and uniformly accelerated motions of the plate. Shehadeh and Duwairi (2009) studied the MHD natural convection heat transfer problem with Joule and viscous heating effects inside a porous medium in which two sides of the rectangular enclosure were adiabatic and the other two were isothermal. Gourla and Mehta (1994) studied for laminar flow due to an exponential source between two parallel stationary infinite disks. They obtained solutions for the motion of liquid in form of an infinite series expanded in terms of reduced Reynolds number. They discussed the significance of convective inertia over the viscous motion and concluded that the effects of non linear-inertia is significant over the motion for small values of decay factor of the source; while for the higher values of the decay factor the effect is less significant.

In this paper, we have studied laminar flow of a homogenous viscous incompressible fluid due to an exponentially decay line source positioned between two parallel stationary disks in presence of uniform magnetic field. The source line has flux (Q) that decays exponentially with time. Assuming creeping motion between the parallel infinite disks, solutions are obtained for small values of reduced Reynolds number (R*) and large value of r (the distance from the source line) which give the effects of linear and non-linear convective inertia on the flow and the pressure under the action of a uniform magnetic field applied transversely to the direction of flow. Considering cylindrical coordinate system, the distribution of radial velocities at different R* are shown graphically for different values of decay and magnetic field parameter. The results obtained are meant for simultaneous effect of inertia and magnetic field on fluid motion, skin friction and the pressure whose variations with respect to the magnetic field and decay factor of the source are presented and analyzed graphically.

Formulation of the Problem

An unsteady flow of electrically conducting, incompressible, viscous and homogeneous fluid between two axially symmetric infinite parallel disks is considered. In a cylindrical polar coordinate system, disks are placed at z = ± h and a line source of the fluid decaying exponentially is situated on the z-axis at r= 0. The source of fluid flow varies exponentially, given as

$$Q(t) = Q_0 e^{-nt} \quad \ldots\ldots (1)$$

Where, the initial strength of the line source is defined as

$$Q_0 = \int_{-h}^{+h} 2\pi r u dz \quad \ldots\ldots (2)$$

u and v are components of fluid velocities along radial and z-direction ; n is the decay factor of the source

Assumptions

- The flow is free and forced convective flow under Bossinesq's approximation.
- The fluid flowing has infinite conductivity so that viscous dissipation and the Joule heat are negligible.
- A transverse magnetic field B_0 is applied perpendicular to the axis of the disks.
- The value of Magnetic Reynolds Number is so small that the effect of induced magnetic field is negligible.
- The level of foreign masses are very low such that the Soret and Dufour are negligible
- The velocity field $\overline{V}$ and magnetic field $\overline{B}$ are assumed as $\overline{V}$ = [u ,0, v] , $\overline{B}$ = [B_0, 0, 0]
- Considering axial symmetry, and using Curl E = 0 and E_θ= 0 everywhere, the magnetic body force (omitting electrical body part) given as $f_r = -\sigma B_o^2 u$
- Hall Effect, Polarization effect, Buoyancy effect and the effects of heat radiation are negligible.

Governing Equations

Under above condition and assumptions, the basic equations of combined free and forced convective flow under usual Bossinesq's approximation in cylindrical coordinate are as follows.

$(\frac{\partial u}{\partial r}) + (\frac{u}{r}) + (\frac{\partial v}{\partial z}) = 0$ ………… (3)

$\rho(\frac{\partial u}{\partial t} + u(\frac{\partial u}{\partial r}) + v(\frac{\partial u}{\partial z})] = -(\frac{\partial p}{\partial r}) + \mu(\frac{\partial^2 u}{\partial r^2}) + \frac{1}{r}(\frac{\partial u}{\partial r}) - \frac{u}{r^2} + (\frac{\partial^2 u}{\partial z^2}) - \sigma B_o^2 u$ ………… (4)

$\rho(\frac{\partial v}{\partial t} + u(\frac{\partial v}{\partial r}) + v(\frac{\partial v}{\partial z})] = -(\frac{\partial p}{\partial z}) + \mu(\frac{\partial^2 v}{\partial r^2}) + \frac{1}{r}(\frac{\partial v}{\partial r}) - \frac{u}{r^2} + (\frac{\partial^2 v}{\partial z^2})$ ………… (5)

The boundary conditions of the problem are

u = 0,v = 0 , at z = ± h , ………… (6)

$Q_0 e^{-nt} = \int_{-h}^{+h} 2\pi r u dz$ ………… (7)

Introducing non-dimensional quantities

$r' = \frac{r}{h}$; $z' = \frac{z}{h}$, ; $u' = (\frac{hu}{\upsilon})$, $v' = (\frac{hv}{\upsilon})$,
$p' = (\frac{1}{\rho}(\frac{h}{\upsilon})^2 p)$, $n' = (\frac{h^2}{\upsilon})n$, $t' = (\frac{\upsilon}{h^2})t$ ………… (8)

Substituting (8) and removing primes, the equations (3) to (5) are

$(\frac{\partial u}{\partial r}) + (\frac{u}{r}) + (\frac{\partial v}{\partial z}) = 0$ ………… (9)

$(\frac{\partial u}{\partial t} + u(\frac{\partial u}{\partial r}) + v(\frac{\partial u}{\partial z})] - (\frac{\partial^2 u}{\partial r^2}) - \frac{1}{r}(\frac{\partial u}{\partial r}) + \frac{u}{r^2} - (\frac{\partial^2 u}{\partial z^2}) + M^2 u + (\frac{\partial p}{\partial r}) = 0$ ………… (10)

$(\frac{\partial v}{\partial t} + u(\frac{\partial v}{\partial r}) + v(\frac{\partial v}{\partial z})] - (\frac{\partial^2 v}{\partial r^2}) - \frac{1}{r}(\frac{\partial v}{\partial r}) - (\frac{\partial^2 v}{\partial z^2}) + (\frac{\partial p}{\partial z}) = 0$ ………… (11)

Where,μ = Coefficient of viscosity of the fluid; ρ = fluid density, $\upsilon = \frac{\mu}{\rho}$, Kinematic viscosity; p = fluid pressure, $\sigma = electrical\ conductivity$,

M = $(\sigma\frac{B_o^2 h^2}{\rho\upsilon})$, Hartmann Number ($Magnetic\ field\ factor$)

Using (8) and then removing primes, the non-dimensional boundary conditions (6)and (7) are,

u = 0,v = 0 , at z = ±1, ………… (12)

$\int_{-h}^{+h} 2\pi r u dz = 2(\frac{R}{r})\, e^{-nt}$ ………… (13)

Where, $\frac{Q_o}{4\pi\upsilon h}$=R, Reynold number, the controlling factor of the fluid flow ………… (14)

Solutions

In order to solve the (9) - (11) under the boundary conditions (12) - (13), we consider the series expansions in non-dimensional form as given below

$u = (\frac{R}{r})(f_o'(z,t) + (R^*) f_1'(z,t) + (R^*)^2 f_2'(z,t) + \text{- - - - - - - -})$ ………… (15)

$v = (2(R^*)^2 f_1(z,t) + 4(R^*)^3 f_2(z,t) + \text{- - - - - - - -})$ ………… (16)

$p = K(z,t) + R(K_o \log(r) + R^* K_1(z,t) + \text{- - - - - - -})$ ………… (17)

where,

$R^{*}= (\frac{R}{r^2})$, reduced Reynold number, the controlling force of the flow.

Here, prime denotes partial differentiation with respect to z-axis

The expansion (15) - (17) which satisfy the equation of continuity, valid for small values of R^* and large values of r i.e. at a large distance from the source line.

Using above expansions in equations (9) - (11) and comparing coefficients of various powers of r, we have the relations as follows.

$$\frac{\partial^3 f_0}{\partial z^3} + \frac{\partial^2 f_0}{\partial z \partial t} - M^2 \frac{\partial f_0}{\partial z} = K_0 (z, t) \quad (18)$$

$$\frac{\partial K_0}{\partial z} = 0 \quad (19)$$

$$\frac{\partial K}{\partial z} = 0 \quad (20)$$

$$\frac{\partial^3 f_1}{\partial z^3} \frac{\partial^2 f_1}{\partial z \partial t} - M^2 \frac{\partial f_1}{\partial z} = -(2 K_1 (z, t) + (\frac{\partial f_0}{\partial z})^2) \quad (21)$$

$$\frac{\partial K_1}{\partial z} = 0 \quad (22)$$

The boundary conditions in terms of $f_0(z, t)$,$f_1(z, t)$ - - - - & K (z, t) , $K_1(z, t)$, $K_2(z, t)$, - - - - - are as follows.

$$f_0(1, t) - f_0(-1, t) = 2e^{-nt\prime} \quad (21)$$
$$f_1(1, t) - f_1(-1, t) = 0 \quad (22)$$
$$f_2(1, t) - f_2(-1, t) = 0 \quad (23)$$

For streamline flow,

$$f_0(1, t) = e^{-nt} \quad and \quad f_0(-1, t) = -e^{-nt} \quad (21)$$

$$f_1(1, t) = f_1(-1, t) = 0 \quad (22)$$

Combining the above conditions we may write

$$f_i(z, t) = \alpha_I(z) e^{-(i+1)nt} \quad (23)$$

where, i = 0, 1, 2, - - - - - - -

Similarly, from relations (19) - (21)

$$K_o(z, t) = K_o(t) \quad (24)$$
$$K(z, t) = K(t) \quad (25)$$

$$K_i(z, t) = K_i(t) \quad (26)$$

Combining relations (9) - (11), we may write

$$K_i(z, t) = \beta_i e^{-(i+1)t} \quad (27)$$

$where, i = 0, 1, 2, - - - - - - -$

Equations (18) - (22), under the relations (23) - (27), transform as follows

$$\alpha_o''' + A\, \alpha_o' - \beta_o = 0 \quad (28)$$

$$\alpha_1''' + B\, \alpha_1' + (2 \beta_1 + (\alpha_o')^2) = 0 \quad (29)$$

Where, A = (n$-M^2$) , B = (2 n $-M^2$)

The corresponding boundary conditions (12) & (13) are now

At z= ± 1 ,$\alpha_o(\pm 1) = \pm 1, \alpha_o'(\pm 1) = 1, \alpha_1'(\pm 1) = 0$ ... (30)

Solutions of the equations (28) & (29), under boundary conditions (30), are as follows.

$$\alpha_o = C_1 \ \mathrm{Sin}\sqrt{A}z \ + \frac{\beta_o}{A} \quad \ldots\ldots (31)$$

$$\alpha_1 = C_2 \mathrm{Sin}\sqrt{B}z \ -((2\,\beta_1 + C_3)\ \frac{z}{B} + C_6 \ \mathrm{Sin}\ (2\sqrt{A}z) + C_7 \ \mathrm{Sin}\ (\sqrt{A}z) \quad \ldots\ldots (32)$$

Where, $A = (n - M^2)$, $B = (2n - M^2)$

$$C_1 = \frac{1}{Sin\sqrt{A} - \sqrt{A}Cos\sqrt{A}};\ \beta_0 = \frac{A\sqrt{A}Cos\sqrt{A}}{\sqrt{A}Cos\sqrt{A} - Sin\sqrt{A}};$$

$$C_3 = \left(\frac{AC_1}{2} + \frac{B_o^2}{A^2}\right);\ C_2 = \frac{1}{Sin\sqrt{B}}\left(\frac{C_3 + 2\,\beta_1}{B} + C_8\ \right)$$

$$C_8 = C_6 + \mathrm{Sin}\ 2\sqrt{A} + \ C_7\ \mathrm{Sin}\ \sqrt{A}\ ,\ \ C_9 = 2\sqrt{A}\ C_6 \mathrm{Cos} 2\sqrt{A} + \sqrt{A}\ C_7 \mathrm{Cos}\ \sqrt{A}$$

$$C_{10} = \frac{\sqrt{B}Cos\sqrt{B}}{Sin\sqrt{B}};\quad \beta_1 = \frac{B}{2(C_{10} - 1)}\left((1 - C_{13})\ \frac{C_3}{B} + \ C_9 - C_8\ \right) \quad \ldots\ldots (33)$$

Radial velocity, Pressure and Skin friction

The quantities of our physical interest are Radial velocity, Pressure and Skin friction at the plates ($z \pm 1$). The non-dimensional velocity, pressure along r-axis, and viscous drag per unit lengh that the plates acting along z-axis i.e. the Skin friction at the plates are represented as follows.

$$U_r = (\ f_o'(z , t) \ + \ (R^*)\ \ f_1'\ (z , t) + (R^*)^2 f_2'\ (z , t) + \text{- - - - - - - -}) \quad \ldots\ldots (34)$$

$$P_r = \ (K_o \log (r) + R^* K_1 \ \ (t) + \ (R^*)^2 K_2 (t) + \text{- - - - - -}) \quad \ldots\ldots (35)$$

$$\tau_z = -\left(\frac{du}{dy}\right)_{z=\pm 1}$$
$$= \ -(f_o''(z , t) \ + \ (R^*)\ \ f_1''\ (z , t) + (R^*)^2 f_2''\ (z , t) + \text{- - - - - - - -}) \quad \ldots\ldots (36)$$

RESULTS AND DISCUSSION

The aim of this study is to learn the nature of variation of radial velocity (Ur), Pressure (Pr) and the Skin friction (τ_z) along z-axis at the disks under the action of magnetic field parameter (M), decay parameter (n) and reduced Reynolds number (R*).

Since the expansions (15)-(17) are valid for smaller values of R* and larger values of r, we consider R* = 0.1 and R*=0.001 which correspondence to R=10 & r=10 and R=0.1 & r=10 respectively as higher and lower value of R*.Numerical values are obtained for Ur, Pr and τ_z using the relations (34) - (36). Graphs are plotted to show distribution of radial velocity Ur along z-axis, its variation with n& M, variation of skin friction at the disks ($\tau_{z=\pm 1}$), and variation of Pressure (Pr) with n & M for the values of higher and smaller values of R* (= 0.1 & 0.001).The plots incurred the following observations.

Radial Velocity Distribution

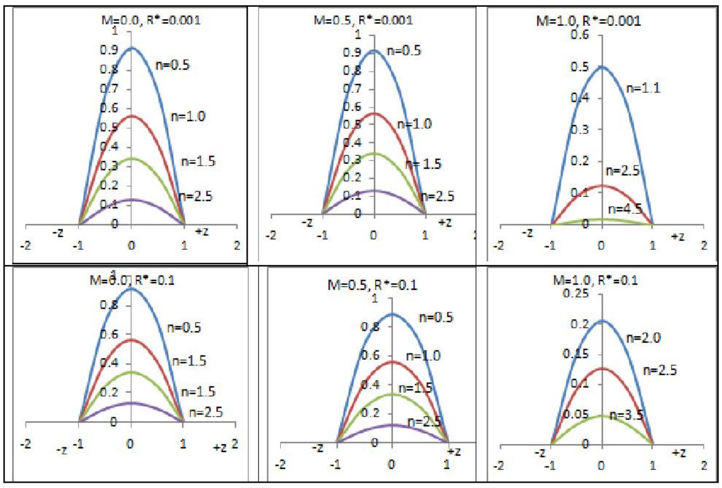

Fig. 1. Radial velocity distribution for M= 0.0, 0.5, 1.0; n=0.5, 1.0, 1.5, 2.5 at R*= 0.001

Fig 1. Show distribution of radial velocity at R* = 0.001 & 0.1 for various values of n and M. In figures, Ur decreases with the increase of n both in presence and absence of the field i.e. M=0.0, M=0.5& M=1.0. Magnitude of Ur decreases with the increase of M; e.g. at z=0.0, Ur= 0.55898 at n=1.0 & M=0.5 while Ur=0.49997 at n=1.0 & M=1.0. With the rise of R*, 0.001 to 0.1, Ur increases for all values of n & M. At a certain higher value of n, Ur vanishes

Radial Velocity Variation

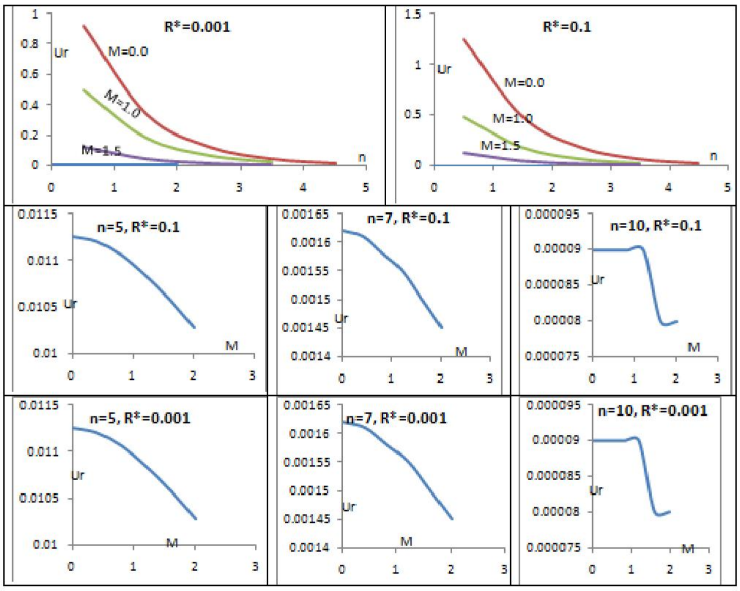

Fig. 2. Variation of radial velocity with n & M at R*=0.001 & 0.1

Fig.2 show the variation of Ur with n & M at R* =0.001 & 0.1. Ur varies inversely with both n and M; for n, the variation is exponential while for M, Ur falls directly, for all values of R*. The rate of decrease is less when field is higher. For a certain higher value of M, Ur vanishes which implies strong effect of magnetic field on the motion. At constant M & n, the value of Ur is higher when R* is smaller; e.g. at z=0.0, n=3.5 & M=1.5, Ur =0.4549 at R*=0.001 while Ur= 0.04543 at R*=0.1

Skin friction variation (at the disks)

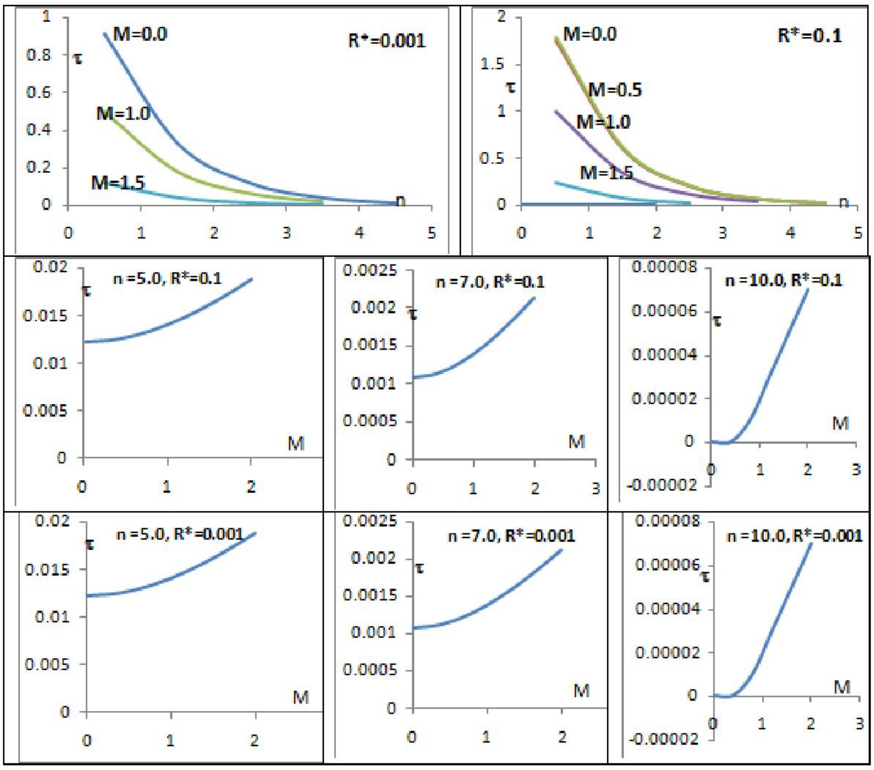

Fig. 3. Variation of skin-friction with n and M, at R*= 0.001 & 0.1

Fig3. Represent variation of skin friction with n and M. With the rise of n, skin friction at the disk $(\tau)_{z=\pm 1.}$decays exponentially with the increase of n while it increases with the rise of M for all values of R*. The rate of variation of τ_z with n is higher when fieldis smaller ; while the rate of variation of τ_z with M is higher when n is higher. For higher values of n($\geq$ 10), $(\tau)_{z=\pm 1.}$ vanishes when M $\cong 0$.

Pressurevariation

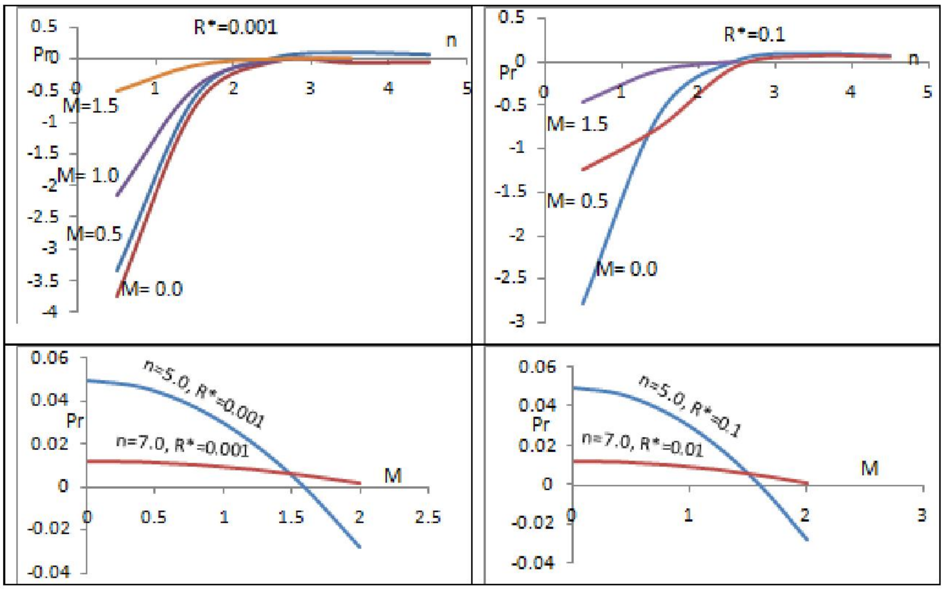

Fig. 4. Variation of Pressure with n ; for M= 0.0, 0.5, at R*= 0.1 & 0.001

Fig.4 show variation of pressure (Pr) with n and M. Pr rises with n for all values of M and R* while decreases with the rise of M for all values of n and R*. For higher value of n ($\geq$ 3), Pr is almost constant. The rate of rise of Pr is higher when field is higher. Beyond a certain value M, the direction of Prreverses. The rate of decrease of Pr with M is higher when n is less.

Conclusion

- Radial velocity varies inversely with both decay factor and field factor; their nature of variations are not same.
- Radial velocity decays exponentially with decay factor while falls directly with the field factor. The rate of decrease of radial velocity less when field is higher.
- The field has strong effect on radial velocity; for a certain higher value of the field, radial velocity becomes minimum.
- Skin friction at the disk. decaysexponentially with the rise of decay factor while increases with the of the field. The rate of variation of skin friction with decay factor ishigherwhenfieldisless. The rate of variation of skin friction with field is higher when decay factor is higher.
- Pressure increases with the increase of decay factor while decreases with the increase the field. The rate of increase of pressure is higher when field is higher. Beyond a certain value of decay factor, pressure remains unchanged. While pressure varying with fields, beyond a certain value the field, pressure reverses direction. This implies strong effect of magnetic field on pressure.

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