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RESEARCH ARTICLE

FUZZY CONTROL CHART ANALYSIS OF MEAN WAITING TIME IN M / M / S QUEUING MODEL

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ABSTRACT

Queuing problems are most common features not only in our daily-life situations such as bank counters, post offices, ticket booking centers, public transportation systems, but also in more technical environments such as in manufacturing, computer networking and telecommunications. The main observable performance characteristics for any queueing system are length and waiting time. Control chart is a graphical technique used to monitor changes in process over time and signifies to evolve methods to control. To deal with practical situations the control chart with fuzzy parameters are much more realistic than that of assumed classical crisp parameters. In this paper for M/M/s queueing model control limits are established for the mean waiting time of customers both in queue and in the system under fuzzy environment. Numerical illustrations are given to highlight its applications.

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INTRODUCTION

In general, queues form when the demand for service exceeds its supply. Waiting time in the facility depends on the number of customers in the queue, the number of servers and the length of service time. Providing system with too much of service capacity involves loss to service providers whereas not providing system with enough service capacity results in excessive waiting time involving loss of customers. This warrants the optimal system to accommodate reduction in waiting time along with staffing and service facilities. Queueing model with multiple servers has been discussed by Gross and Harris (1998) and several other authors. Shewhart developed control chart technique based on one or several quality related characteristics of product or service to identify whether a production process or service facility meets required quality standards. Montgomery (2005) proposed a number of applications of Shewhart control charts in analyzing quality in manufacturing industries. Fazel Zarandi (2006) developed fuzzy control charts for quality characteristics of variable and attribute nature. In analysis of a textile industry, Hamid Reza Feili (2010) compared fuzzy control charts with probability control charts.

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Siamak Noori (2008) applied fuzzy control chart in earned value analysis. The applications of control charts in evaluating queueing models with many servers are discussed by Shore (2000) for random queue length of M/M/s queueing model in terms of the first three moments, Shore (2006) for G/G/S queueing system using skewness, Khaparde and Dhabe (2010) for random queue length of M/M/1 queueing model using method of weighted variance and Poongodi and Muthulakshmi (2013) constructed for mean waiting time in the system of M/M/s queuing model by assuming the inter-arrival times and service times under non-fuzzy environment. However, in many practical situations, the arrival pattern and service pattern are typically described by linguistic values such as fast, slow or moderate rather than with complete probability distributions. The queueing models are more suitable for real life applications by considering system parameters as fuzzy numbers.

Chen (2006) provided a method to conserve the fuzziness of input information when some information of bulk service queueing system is ambiguous. Also Chen (2006) developed a non-linear programming approach to derive the membership function of steady state performance measures in bulk arrival queueing system with varying batch sizes.

In this paper, based on Zadeh's extension principle by employing α- cut approach, the membership functions of the parameters of the control chart relating to the mean waiting time of customers in the queue and in the system of M/M/s queueing model are formulated by assuming the arrival rate and service rate as fuzzy numbers.

MATERIALS AND METHODS

M /M /s Queuing Model Description

M/M/s queuing system has s servers arranged in parallel. The customers arrive in a Poisson fashion with mean arrival rate λ and the service time at each server has an independent and identically distributed exponential distribution with rate μ. A customer on arrival may go to any of the free counters. The system has infinite capacity with First Come First Served (FCFS) queue discipline.

Parameters of the control chart

For M/M/s queuing model let W_q be the waiting time of customer in queue and W_s be the waiting time in the system. Then the mean waiting time in queue is derived as

$$E(W_q) = \frac{(\rho s)^s}{s!(s\mu)\ (1-\rho)^2} P_o$$

and the mean waiting time in the system is

$$E(W_s) = \frac{(\rho s)^s}{s!(s\mu)\ (1-\rho)^2} P_o + \frac{1}{\mu}$$

where $P_o = \left[\sum_{n=0}^{s-1} \frac{1}{n!}(\rho s)^n + \frac{(\rho s)^s}{s!(1-\rho)}\right]^{-1}$ and $\rho = \lambda/(s\mu)$

The variance of waiting time in queue is derived as,

$$Var(W_q) = \frac{2(\rho s)^s}{s!\ (s\mu)^2\ (1-\rho)^3} P_o - (E(W_q))^2$$

and the variance of waiting time in system is

$$Var(W_s) = \frac{1}{\mu^2}\left[2\left(1 + \frac{(\rho s)^s(\mu\ (s+1) - \lambda)}{\mu s!\, s^2\ (1-\rho)^3} P_o\right) - \left(1 + \frac{(\rho s)^s}{s!s\ (1-\rho)^2} P_o\right)^2\right]$$

The parameters of the control chart for the mean waiting of customers in queue are

$$UCL = E(W_q) + 3\sqrt{Var(W_q)}$$

$$CL = E(W_q)$$

and $LCL = E(W_q) - 3\sqrt{Var(W_q)}$

The parameters of the control chart for the mean waiting time of customers in the system may be calculated using $E(W_s)$ and $Var(W_s)$.

Model with fuzzy parameters

The membership functions of the parameters of the control chart for the mean waiting time in queue and in the system of M /M /s queueing systemare constructed by assuming the arrival rate $\tilde{\lambda}$ and the service rate $\tilde{\mu}$ as fuzzy numbers. Then these fuzzy numbers are represented as

$$\tilde{\lambda} = \left\{ (x,\ \varphi_{\tilde{\lambda}}(x))\ /\ x \in X \right\}$$
$$\tilde{\mu} = \left\{ (y,\ \varphi_{\tilde{\mu}}(y))\ /\ y \in Y \right\}$$

where $\varphi_{\tilde{\lambda}}(x)$ and $\varphi_{\tilde{\mu}}(y)$ denote the membership functions and X and Y are the support of the fuzzy numbers $\tilde{\lambda}$ and $\tilde{\mu}$ respectively.

Let P (x, y) and $\tilde{P}(\tilde{\lambda}, \tilde{\mu})$ denote the parameters of the control chart relating to mean waiting time of customers in the crisp and fuzzy environments respectively, where P stands for the control chart parameters CL, UCL and LCL. As $\tilde{\lambda}$ and $\tilde{\mu}$ are fuzzy numbers $\tilde{P}(\tilde{\lambda}, \tilde{\mu})$ will also be fuzzy. Using Zadeh's extension principle (1978) the membership function of the control chart parameters of system performance measures $\tilde{P}(\tilde{\lambda}, \tilde{\mu})$ is defined as

$$\varphi_{\tilde{P}(\tilde{\lambda}, \tilde{\mu})}(z) = \sup_{x\in X,\, y\in Y} \min \left\{\varphi_{\tilde{\lambda}}(x),\ \varphi_{\tilde{\mu}}(y)\ /\ z = P(x,y)\right\} \quad (1)$$

Now we consider the derivation of the control chart parameters for mean waiting time of customers bothin queue and in the system of M /M /s queueing model.

Control chart parameters for the meanwaiting time of customers in queue

The parameters of the fuzzy control chart for mean waiting time of customers in queue are

$$CL(x, y) = \frac{(x)^s}{(s-1)!(y)^{s-1}\ (sy-x)^2} P_o,$$

$$Var(x, y) = \frac{2(x)^s}{(s-1)!(y)^{s-1}(sy-x)^3} P_o - (CL(x,y))^2$$

$$UCL(x,y) = CL(x,y) + 3\sqrt{Var(x,y)}$$

and $LCL(x,y) = CL(x,y) - 3\sqrt{Var(x,y)}$

where $P_o = \left[\sum_{n=0}^{s-1} \frac{(x/y)^n}{n!} + \frac{(x)^s}{(y)^{s-1}(s-1)!(sy-x)}\right]^{-1}$

A mathematical programming approach is developed for deriving the desiredmembership functions for CL, UCL and LCL on the basis of α- cuts.

The α- cut approach based on the extension principle

The α- cuts of $\tilde{\lambda}$ and $\tilde{\mu}$ as crisp intervals are

$$\lambda_\alpha = \left\{ x \in X / \varphi_{\tilde{\lambda}}(x)\ \geq \alpha \right\} \quad (2)$$

$$\mu_\alpha = \{ y \in Y / \varphi_{\tilde{\mu}}(y) \geq \alpha \} \quad (3)$$

These crisp sets may be expressed as

$$\lambda_\alpha = \left[x_\alpha^L, x_\alpha^U\right] = \left[\min_{x\in X}\{x \in X / \varphi_{\tilde{\lambda}}(x) \geq \alpha\}, \max_{x\in X}\{x \in X / \varphi_{\tilde{\lambda}}(x) \geq \alpha\}\right] \quad (4)$$

$$\mu_\alpha = \left[y_\alpha^L, y_\alpha^U\right] = \left[\min_{y\in Y}\{y \in Y / \varphi_{\tilde{\mu}}(y) \geq \alpha\}, \max_{y\in Y}\{y \in Y / \varphi_{\tilde{\mu}}(y) \geq \alpha\}\right] \quad (5)$$

These intervals provide information on the arrival rate and the service rate with possibilityα.

The bounds of these intervals in (4) and (5) are functions of α and may be obtained as

$$x_\alpha^L = \min \varphi_{\tilde{\lambda}}^{-1}(\alpha), \quad x_\alpha^U = \max \varphi_{\tilde{\lambda}}^{-1}(\alpha)$$

and $y_\alpha^L = \min \varphi_{\tilde{\mu}}^{-1}(\alpha)$, $y_\alpha^U = \max \varphi_{\tilde{\mu}}^{-1}(\alpha)$

The membership function defined in (1) is also parameterized byα. Thereforeα-cuts may be used to construct the membership function.

Construction of membership function

Consider the membership function relating to CL of the control chart for the mean waiting time of customers in queue as $\varphi_{\tilde{CL}}(z)$. As given in (1), $\varphi_{\tilde{CL}}(z)$ is the minimum of $\varphi_{\tilde{\lambda}}(x)$ and $\varphi_{\tilde{\mu}}(y)$

To deal with the value of the membership function, we need at least one of the following two cases to hold such that z = CL(x, y) and $\varphi_{\tilde{CL}}(z) = \alpha$ (i) : $\varphi_{\tilde{\lambda}}(x) = \alpha, \quad \varphi_{\tilde{\mu}}(y) \geq \alpha$

and (ii) : $\varphi_{\tilde{\lambda}}(x) \geq \alpha, \quad \varphi_{\tilde{\mu}}(y) = \alpha$ (6)

From the definitions of λ_α and μ_α in equations (2) and (3), x ϵ λ_α and y ϵ μ_α may be replaced by

x ϵ [x_α^L, x_α^U] and y ϵ [y_α^L, y_α^U] respectively. The following are the formulated parametric non-linear programs (NLPs) for finding the lower and upper bounds of the α-cut of $\varphi_{\tilde{CL}}(z)$ corresponding to the two cases stated in (6).

Case (i):

$$(CL)_\alpha^{L_1} = \min \frac{(x)^s}{(s-1)!(y)^{s-1}\ (sy-x)^2} P_o$$

subject to $x_\alpha^L \leq x \leq x_\alpha^U$ and $y \in \mu_\alpha$

$$(CL)_\alpha^{U_1} = \max \frac{(x)^s}{(s-1)!(y)^{s-1}\ (sy-x)^2} P_o$$

subject to $x_\alpha^L \leq x \leq x_\alpha^U$ and $y \in \mu_\alpha$

Case (ii):

$$(CL)_\alpha^{L_2} = \min \frac{(x)^s}{(s-1)!(y)^{s-1}\ (sy-x)^2} P_o$$

subject to $x \in \lambda_\alpha$ and $y_\alpha^L \leq y \leq y_\alpha^U$

$$(CL)_\alpha^{U_2} = \max \frac{(x)^s}{(s-1)!(y)^{s-1}\ (sy-x)^2} P_o$$

subject to $x \in \lambda_\alpha$ and $y_\alpha^L \leq y \leq y_\alpha^U$

It is sufficient to find the left shapefunction L(z) and the right shapefunction R(z) of $\varphi_{\tilde{CL}}(z)$ which in turnare used to find the lower bound $(CL)_\alpha^L$ and the upper bound $(CL)_\alpha^U$ of α- cuts of $(\tilde{CL})$.

These may be rewritten as

$$(CL)_\alpha^L = \min_{x\in X, y\in Y} \frac{(x)^s}{(s-1)!(y)^{s-1}(sy-x)^2} P_o$$

subject to $x_\alpha^L \leq x \leq x_\alpha^U$ and $y_\alpha^L \leq y \leq y_\alpha^U$ (7)

$$(CL)_\alpha^U = \max_{x\in X, y\in Y} \frac{(x)^s}{(s-1)!(y)^{s-1}(sy-x)^2} P_o$$

subject to $x_\alpha^L \leq x \leq x_\alpha^U$ and $y_\alpha^L \leq y \leq y_\alpha^U$ (8)

At least one of x and y must be on the boundary of the constraints in equations (7) and (8) to satisfy $\varphi_{\tilde{CL}}(z) = \alpha$. This pair of mathematical programs falls into the category of parametric non- linear programs which facilitates the systematic study of how the optimal solutions change when x_α^L ,x_α^U ,y_α^L and y_α^U as α ranges over the interval [0,1]. The interval $\left[(CL)_\alpha^L, (CL)_\alpha^U\right]$ is a crisp interval representing the α-cuts of $(\tilde{CL})$. Again based on the extension principle and the convexity property of fuzzy numbers we obtain

$$\left[(CL)_{\alpha_1}^L, (CL)_{\alpha_1}^U\right] \subseteq \left[(CL)_{\alpha_2}^L, (CL)_{\alpha_2}^U\right] \quad \text{for } 0 < \alpha_2 < \alpha_1 < 1$$

In other words, as α increases, the values of $(CL)_\alpha^L$ increase and $(CL)_\alpha^U$ decrease.

The α-cuts give the possible range for the performance measure. At α = 0, the range for the support of the performance measure is calculated and at α = 1, the most possible range for the performance measure is calculated.

If both the lower bound $(CL)_\alpha^L$ and the upper bound $(CL)_\alpha^U$ of the α- cuts of $(\tilde{CL})$ are invertible with respect to α, then a left shape function L(z) and a right shape function R(z) may be obtained as $L(z) = \left[(CL)_\alpha^L\right]^{-1}$ and $R(z) = \left[(CL)_\alpha^U\right]^{-1}$. Then the membership function $\varphi_{\widetilde{CL}}(z)$ can be expressed as

$$\varphi_{\widetilde{CL}}(z) = \begin{cases} L(z), & (CL)_{\alpha=0}^L \le z \le (CL)_{\alpha=1}^L \\ 1, & (CL)_{\alpha=1}^L \le z \le (CL)_{\alpha=1}^U \\ R(z), & (CL)_{\alpha=1}^U \le z \le (CL)_{\alpha=0}^U \end{cases}$$

Yager ranking index method based on area compensation is used to defuzzify $(\widetilde{CL})$ of the mean waiting time of customers in queue into a crisp one. TheYager ranking index is

$$\widetilde{Y}(\widetilde{CL}) = \frac{1}{2}\int_0^1 \left[(CL)_\alpha^L + (CL)_\alpha^U\right] d\alpha$$

By following the similar procedure, the membership functions $\varphi_{\widetilde{UCL}}(z)$ and $\varphi_{\widetilde{LCL}}(z)$ and the Yager ranking indices relating to the parameters of the control chart namely UCL and LCL may be derived.

Control chart parameters for mean waiting time of customers in the system

The parameters of the fuzzy control chart for mean waiting time of customers in the system are

$$CL(x, y) = \frac{(x)^s}{(s-1)!(y)^{s-1}(sy - x)^2} P_0 + \frac{1}{y},$$

$$Var(x, y) = \frac{2}{y^2}\left(1 + \frac{x^s (y(s+1) - x)}{y^{s-2}(s-1)!(sy-x)^3} P_o\right) - (CL(x,y))^2$$

$$UCL(x,y) = CL(x,y) + 3\sqrt{Var(x,y)}$$

and $LCL(x,y) = CL(x,y) - 3\sqrt{Var(x,y)}$

The membership functions and the Yager ranking indices of the parameters of fuzzy control chart relating to the mean waiting time of customers in the system may be derived by adopting the procedure described to mean waiting time of customers in queue presented in the previous section.

Numerical analysis

Three counters are being run on the frontier of a country to check the passports and the necessary documents of the tourists. The arrival pattern of the tourists for verification is assumed to be Poisson with rate λ and the service time of the tourists is assumed to have an exponential distribution with parameter μ.

The arrival rate and service rate are assumed as triangular fuzzy numbers represented by

$\widetilde{\lambda} = [5, 7, 9]$ and $\widetilde{\mu} = [4, 5, 6]$.

To avoid congestion in waiting room the authorities of the office wish to analyze the mean waiting time oftourists both in the queue and in the system using fuzzy control charts. The upper and lower bounds of $\widetilde{\lambda}$ and $\widetilde{\mu}$ are obtained as

$[x_\alpha^L, x_\alpha^U] = [2\alpha + 5, 9 - 2\alpha]$, $[y_\alpha^L, y_\alpha^U] = [\alpha + 4, 6 - \alpha]$
and s = 3.

Fuzzy control chart for mean waiting time of tourists in the queue

Using the software MATLAB the upper and lower bounds of the parameters of fuzzy control chart for mean waiting time of tourists in the queue is derived as

$(CL)_\alpha^L = (2\alpha + 5)^3/((5\alpha - 13)(\alpha - 6)(2\alpha^2 - 24\alpha + 361))$

$(CL)_\alpha^U = -(2\alpha - 9)^3/((5\alpha + 3)(\alpha + 4)(2\alpha^2 + 16\alpha + 321))$

$(var)_\alpha^L = -((2\alpha + 5)^3(12\alpha^3 - 12\alpha^2 + 1160\alpha - 4207))/((5\alpha - 13)^2(\alpha - 6)^2(2\alpha^2 - 24\alpha + 361)^2)$

$(UCL)_\alpha^L = (CL)_\alpha^L + 3\sqrt{(var)_\alpha^L}$

$(var)_\alpha^U = -((2\alpha - 9)^3(12\alpha^3 - 60\alpha^2 + 1256\alpha + 1839))/((5\alpha + 3)^2(\alpha + 4)^2(2\alpha^2 + 16\alpha + 321)^2)$

$(UCL)_\alpha^U = (CL)_\alpha^U + 3\sqrt{(var)_\alpha^U}$

$(LCL)_\alpha^L = (CL)_\alpha^L - 3\sqrt{(var)_\alpha^L}$

and

$(LCL)_\alpha^U = (CL)_\alpha^U - 3\sqrt{(var)_\alpha^U}$

For various values of $\alpha \in [0, 1]$, the values of the upper and lower bounds of the parameters of fuzzy control chart are calculated and presented in Table 1.

At $\alpha = 0$ the range of CL is [0.0044, 0.1893] indicating that CL of mean waiting time of tourists in the queue will never exceed 0.1893 or fall below 0.0044. The UCL will never exceed 1.0910 or fall below 0.0817.
[
At $\alpha = 1$ the values of CL and UCL are 0.0253 and 0.2515 respectively. Theupper andlower bounds of LCL are By using Yager ranking index method the expected CL and UCL of the mean waiting time of tourists in queue are

$$\widetilde{Y}(\widetilde{CL}) = \frac{1}{2}\int_0^1 \left[(CL)_\alpha^L + (CL)_\alpha^U\right] d\alpha = 0.0441$$

$$\widetilde{Y}(\widetilde{UCL}) = \frac{1}{2}\int_0^1 \left[(UCL)_\alpha^L + (UCL)_\alpha^U\right] d\alpha = 0.3424$$

With the help of MATLAB, the inverse functions L(z) and R(z) of $(CL)_\alpha^L$, $(CL)_\alpha^U$, $(UCL)_\alpha^L$ and $(UCL)_\alpha^U$ are obtained. The membership functions $\varphi_{\widetilde{CL}}(z)$ and $\varphi_{\widetilde{UCL}}(z)$ can be expressed as

$$\varphi_{\widetilde{CL}}(z) = \begin{cases} L(z), & 0.0044 \le z \le 0.0253 \\ U(z), & 0.0253 \le z \le 0.1893 \end{cases}$$

and

$$\varphi_{\widetilde{UCL}}(z) = \begin{cases} L(z), & 0.0817 \le z \le 0.2515 \\ U(z), & 0.2515 \le z \le 1.0910 \end{cases}$$

Table 1. α- cuts of arrival and service rates and the parameters of fuzzy control chart for mean waiting time of tourists in the queue

α	x_α^L	x_α^U	y_α^L	y_α^U	$(CL)_\alpha^L$	$(CL)_\alpha^U$	$(UCL)_\alpha^L$	$(UCL)_\alpha^U$	$(LCL)_\alpha^L$	$(LCL)_\alpha^U$
0.0	5.0	9.0	4.0	6.0	0.0044	0.1893	0.0817	1.0910	-0.0728	-0.7125
0.1	5.2	8.8	4.1	5.9	0.0053	0.1472	0.0913	0.8969	-0.0807	-0.6025
0.2	5.4	8.6	4.2	5.8	0.0064	0.1168	0.1021	0.7513	-0.0894	-0.5178
0.3	5.6	8.4	4.3	5.7	0.0076	0.0940	0.1140	0.6384	-0.0989	-0.4505
0.4	5.8	8.2	4.4	5.6	0.0090	0.0765	0.1274	0.5484	-0.1093	-0.3954
0.5	6.0	8.0	4.5	5.5	0.0107	0.0628	0.1423	0.4751	-0.1209	-0.3495
0.6	6.2	7.8	4.6	5.4	0.0127	0.0519	0.1591	0.4145	-0.1336	-0.3107
0.7	6.4	7.6	4.7	5.3	0.0151	0.0431	0.1780	0.3636	-0.1478	-0.2773
0.8	6.6	7.4	4.8	5.2	0.0179	0.0360	0.1994	0.3204	-0.1635	-0.2484
0.9	6.8	7.2	4.9	5.1	0.0213	0.0301	0.2237	0.2834	-0.1811	-0.2231
1.0	7.0	7.0	5.0	5.0	0.0253	0.0253	0.2515	0.2515	-0.2009	-0.2009

The shape of membership functions $\varphi_{\widetilde{CL}}(z)$ and $\varphi_{\widetilde{UCL}}(z)$ α-cuts of CL and UCL relating to mean waiting time oftourists in the queue are displayed in Figs.1 and 2 respectively.

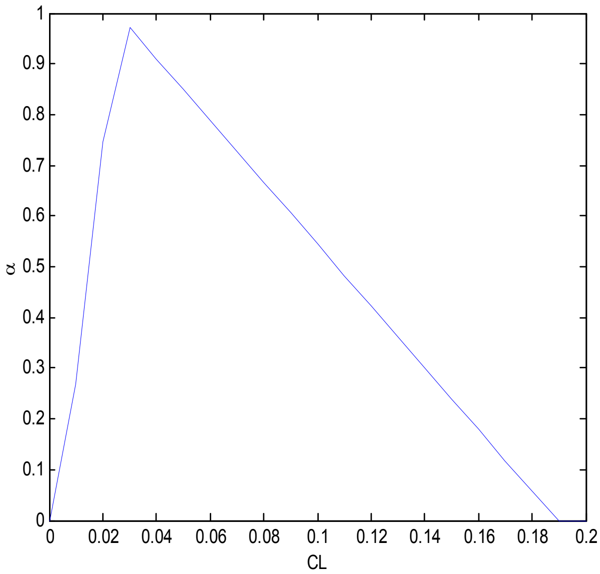

Fig.1. The Membership function of CL of the mean waiting time of tourists in the queue

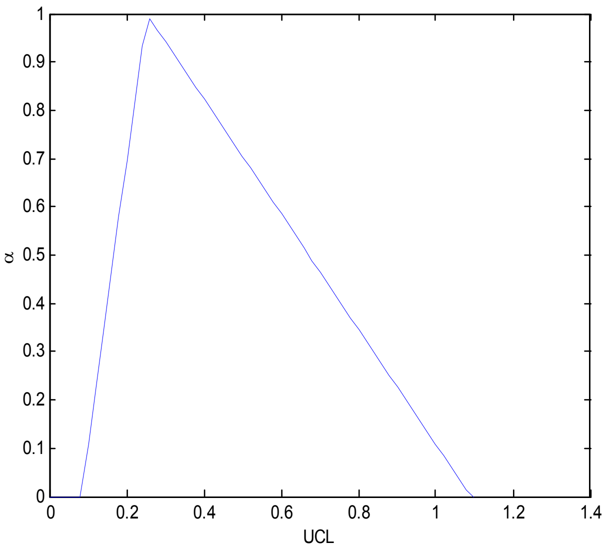

Fig. 2. The Membership function of UCL of the mean waiting time of tourists in the queue

The upper and lower bounds of the parameters of fuzzy control chart for mean waiting time of tourists in the system are

$$(CL)_\alpha^L = 34/(2\alpha^2 - 24\alpha + 361) - 1/(5\alpha - 13)$$

$$(CL)_\alpha^U = 1/(5\alpha + 3) + 34/(2\alpha^2 + 16\alpha + 321)$$

$$(var)_\alpha^L = (4\alpha^5 - 3520\alpha^4 + 32176\alpha^3 - 554612\alpha^2 + 2604293\alpha - 3758354)/((5\alpha - 13)^2(\alpha - 6)(2\alpha^2 - 24\alpha + 361)^2)$$

$$(UCL)_\alpha^L = (CL)_\alpha^L + 3\sqrt{(var)_\alpha^L}$$

$$(var)_\alpha^U = (4\alpha^5 + 3480\alpha^4 + 4176\alpha^3 + 445716\alpha^2 + 659637\alpha + 567000)/((5\alpha + 3)^2(\alpha + 4)(2\alpha^2 + 16\alpha + 321)^2)$$

$$(UCL)_\alpha^U = (CL)_\alpha^U + 3\sqrt{(var)_\alpha^U}$$

$$(LCL)_\alpha^L = (CL)_\alpha^L - 3\sqrt{(var)_\alpha^L}$$

and

$$(LCL)_\alpha^U = (CL)_\alpha^U - 3\sqrt{(var)_\alpha^U}$$

For various values of α ϵ [0,1] the values of the upper and lower bounds of the parameters of fuzzy control chart are calculated and presented in Table 2.

At α = 0 the range of the CL is [0.1711, 0.4393] indicating that CL of mean waiting time of tourists in the system will never exceed 0.4393 or fall below 0.1711. The UCL will never exceed 1.6121 or fall below 0.6770.

At α = 1 the values of CL and UCL are 0.2253 and 0.8665 respectively.

By using Yager ranking index method the expected CL and UCL of the meanwaiting time of tourists in the system are

$$\tilde{Y}(\widetilde{CL}) = \frac{1}{2}\int_0^1 \left[(CL)_\alpha^L + (CL)_\alpha^U\right] d\alpha = 0.2468$$

$$\tilde{Y}(\widetilde{UCL}) = \frac{1}{2}\int_0^1 \left[(UCL)_\alpha^L + (UCL)_\alpha^U\right] d\alpha = 0.9397$$

With the help of MATLAB, the inverse functions L(z) and R(z) of $(CL)_\alpha^L$, $(CL)_\alpha^U$, $(UCL)_\alpha^L$ and $(UCL)_\alpha^U$ are obtained.

Table 2. α- cuts of arrival and service rates and the parameters of fuzzy control chart for mean waiting time of tourists in the system

α	x_α^L	x_α^U	y_α^L	y_α^U	$(CL)_\alpha^L$	$(CL)_\alpha^U$	$(UCL)_\alpha^L$	$(UCL)_\alpha^U$	$(LCL)_\alpha^L$	$(LCL)_\alpha^U$
0.0	5.0	9.0	4.0	6.0	0.1711	0.4393	0.6770	1.6121	-0.3348	-0.7336
0.1	5.2	8.8	4.1	5.9	0.1748	0.3911	0.6905	1.4387	-0.3409	-0.6565
0.2	5.4	8.6	4.2	5.8	0.1788	0.3548	0.7048	1.3103	-0.3473	-0.6006
0.3	5.6	8.4	4.3	5.7	0.1830	0.3265	0.7200	1.2115	-0.3540	-0.5584
0.4	5.8	8.2	4.4	5.6	0.1876	0.3037	0.7362	1.1329	-0.3611	-0.5254
0.5	6.0	8.0	4.5	5.5	0.1925	0.2850	0.7536	1.0689	-0.3686	-0.4989
0.6	6.2	7.8	4.6	5.4	0.1979	0.2693	0.7724	1.0155	-0.3766	-0.4769
0.7	6.4	7.6	4.7	5.3	0.2038	0.2559	0.7928	0.9701	-0.3852	-0.4583
0.8	6.6	7.4	4.8	5.2	0.2102	0.2443	0.8150	0.9310	-0.3946	-0.4423
0.9	6.8	7.2	4.9	5.1	0.2173	0.2342	0.8394	0.8968	-0.4047	-0.4284
1.0	7.0	7.0	5.0	5.0	0.2253	0.2253	0.8665	0.8665	-0.4159	-0.4159

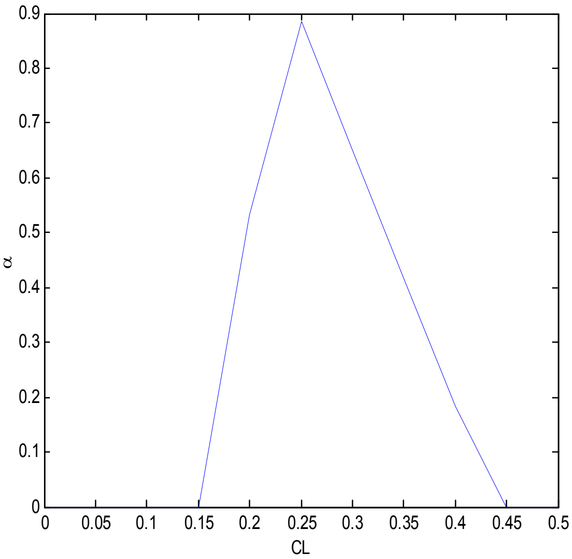

Fig.3 The Membership function of CL of the mean waiting time of tourists in the system

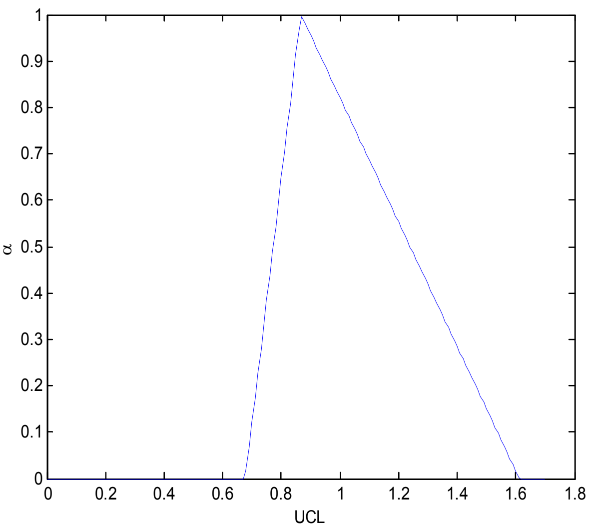

Fig. 4 The Membership function of UCL of the mean waiting time of tourists in the system

The corresponding membership functions $\varphi_{\tilde{CL}}(z)$ and $\varphi_{\tilde{UCL}}(z)$ may be expressed as

$$\varphi_{\tilde{CL}}(z)=\begin{cases} L(z), & 0.1711\le z\le 0.2253 \\ U(z), & 0.2253\le z\le 0.4393 \end{cases}$$

and

$$\varphi_{\tilde{UCL}}(z)=\begin{cases} L(z), & 0.6770\le z\le 0.8665 \\ U(z), & 0.8665\le z\le 1.6121 \end{cases}$$

The shapes of the membership functions $\varphi_{\tilde{CL}}(z)$ and $\varphi_{\tilde{UCL}}(z)$ are displayed in Figs.3 and 4 respectively.

Conclusion

In this paper, fuzzy control chart for waiting timeis constructed to analyze the queueing model by employing a pair of parametric non-linear programming to findα- cuts of the membership functions. By this analysis the system designers and the decision makers may improve the existing systems. This approach maintains the fuzziness of input information to represent the real time system.

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