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RESEARCH ARTICLE

INVESTIGATION OF FORCED OSCILLATIONS VISCOELASTIC SHELLS

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ABSTRACT

The paper proposes a new method for solving integral-differential equation of forced oscillations of linear viscoelastic shells built on the basis of the operational calculus for arbitrary hereditary functions at a low viscosity. The solution is built as a series, the first member of which is the solution of this problem, obtained by averaging method and it is shown that at low impacts of amplitude true fluctuations remain finite. Fundamental results that at low frequencies the effect of subsequent terms slightly and they increase with increasing frequency. It is shown that in the particular case of certain values influence the amplitude of the oscillation frequency of the second term is 20-25% of the amplitude of the first term and the amplitude of all the members of the series over time fit exponentially, and the phases are shifted.

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INTRODUCTION

In the design of modern buildings of different type and values for various materials are used in the form of a cylindrical shell. Therefore, only on the basis of the dynamic viscoelasticity of oscillations can be fully ascertaining the optimum size and operational conditions of work con-struction with the rheological properties of the material. Among the tasks of dynamic viscoelasticity should highlight the problem of vibrations of viscoelastic systems and non-stationary wave problems. In solving the problem of oscillations of viscoelastic systems (Larionov, 1969; Matyash, 1971 and Ilyasov, 2007) has been applied well-known method of averaging, which was further developed in the works (Brilla, 1997; Abdou, 2002 and Kim and Youn, 2001). It should be noted that in addressing problems of non-stationary dynamic analytical view of the relaxation of the nuclei is not specified. This solutions are built with the help of some approximate methods, which are the final decision to the solution of integral-differential equations of free and forced vibrations of viscoelastic systems implemented by various numerical methods (Ilyasov, 2007; Gurtin Morton and Hrusa William, 1988; Kim and Youn, 2001 and Park and Schapery, 1997). A particularly complex and important is to make a deep analysis and build a more accurate solutions of integro-differential equation of oscillations of viscoelastic systems and use them to explore the effect of different initial and boundary conditions, and rheological properties of heterogeneous materials, etc. on the wave field (Eshmatov and Khodjaev, 2007; Brilla, 1997 and Kurbanov and Babajanova, 2014). To study these issues the focus of this work.

MATERIALS AND METHODS

For the analytical solution of the problem using mathematical methods of dynamic theory of linear and nonlinear viscoelastic, the theory of partial differential equations of hyperbolic type, the theory of integral equations of the second type of Voltaire, operational calculus and the method of separation of variables. Using these methods, the generalized mathematical model and the development of methods for solving the problems of the free movement of hereditary solids, with which it will be possible to describe the behavior of viscoelastic systems with arbitrary rheology by intense external loading. This paper investigates the forced oscillations of a viscoelastic shell for any hereditary nuclei at low viscosity by using the integral Laplace transform. As is known, the equation of forced oscillations of viscoelastic shells obtained from the equation of oscillations of elastic shells (2,4,12).

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$$L_k[u_1,u_2,u_3]+\rho h F_k(\alpha_1,\alpha_2,t)=\rho h\frac{\partial^2 u_k}{\partial t^2}$$

replacement u_1, u_2 and u_3 by $\{J(u_1), J(u_2), J(u_3)\}$. Thus the equation of forced oscillations of viscoelastic shells will be in the form:

$$L_k[J(u_1),J(u_2),J(u_3)]+\rho h F_k(\alpha_1,\alpha_2,t)=\rho h\frac{\partial^2 u_k}{\partial t^2} \quad (k=1,2,3) \quad (1)$$

where the operator $J(z)$ is given by equation

$$J(z)=z-\int_0^t \Gamma(t-\tau)z(\tau)d\tau \quad (2)$$

$F(\alpha_1,\alpha_2,t)-$ the external force,
$\rho-$ the material density,
$h-$ the shell thickness,
u_k-transformation.

Note that the equation (1) describing the oscillations of a viscoelastic shell should be connected the initial and boundary conditions.

The boundary conditions can be defined in different ways and are used to determine the eigenvalues and eigenfunctions, which is not difficult, and the initial conditions take the form:

$$u_k(\alpha_1,\alpha_2,t)=T_0 \quad at \quad t=0$$
$$\frac{\partial u_k(\alpha_1,\alpha_2,t)}{\partial t}=T_0' \quad at \quad t=0 \quad (3)$$

In many practical problems investigation of oscillations of viscoelastic systems reduces to the solution of integral-differential equations depending only on time, which are obtained from the equations (1) or by separation of variables or Bubnov-Galerkin method.

Therefore, introducing the solution of equation (1) in the form of a series

$$u_k(\alpha_1,\alpha_2,t)=\sum_{m=1}^{\infty}u_k^{(m)}(\alpha_1,\alpha_1)T_m(t)$$

from this equation for determining the $T_m(t)$ we obtain:

$$\frac{d^2T_m}{dt^2}=-P_m^2T_m(t)+\varepsilon P_m^2T_m(t)\int_0^t\omega(t-\tau)T_m(\tau)d\tau+F_m(t) \quad (4)$$

Therefore that solution of the problem is mathematically reduced to solving integral-differential equation (4) with the following initial conditions:

$$T(t)=T_0 \quad \text{at} \quad t=0$$

$$T'(t)=T_0' \quad \text{at} \quad t=0 \quad (5)$$

Using the integral Laplace transform in time t to the equation (4) with regard to the conditions (5) and dropping indexes for simply of record we obtain (6,7,12).

$$\overline{T}(s)=\frac{sT_0+T_0'}{s^2+p^2-\varepsilon p^2\overline{\omega}(s)}+\frac{\overline{F}(s)}{s^2+p^2-\varepsilon^2\overline{\omega}(p)} \quad (6)$$

where $s-$the parameter of the Laplace transform. The over bar denotes the Laplace transform of similar functions.

Let's introduce the following notation:

$$\overline{\psi}(s)=\frac{sT_0+T_0'}{s^2+p^2-\varepsilon p^2\overline{\omega}(s)}$$
$$\overline{\varphi}(s)=\frac{\overline{F}(s)}{s^2+p^2-\varepsilon p^2\overline{\omega}(s)}$$

Let us find the originals of these functions.

It is known that (4,8,10) at small values of the time parameter S is large enough, because the materials in question with instant elasticity image of relaxation kernel $\overline{\omega}(s)$ with increasing S approaches to zero. At other times we use the inequality

$$0\le\varepsilon\int_0^t\omega(\tau)d\tau<<1; \qquad \varepsilon\Gamma(t)\ge 0$$

established by A.A. Ilyuschin (2,6) and is valid for all values of time t.

The validity of this inequality is obtained from the physical and mechanical nature of medium y hard composites materials viscous resistance are small compared with the main elastic. Then the value $|\varepsilon\overline{\omega}(s)|$ in the specified intervals will be small. This shows that inequality

$$\left|\frac{\varepsilon p^2\overline{\omega}(s)}{s^2+p^2}\right|<1$$

will be true for any time t (Abdou, 2002 and Park and Schapery, 1997).

With these assumptions the first term on the right-hand side of equation (6) can be expanded in a series:

$$\overline{\psi}_k^{(s)}=\frac{s\,T_0+T_0'}{s^2+p^2}\sum_{k=0}^{\infty}\left(\frac{\varepsilon\,p^2\,\overline{\omega}(s)}{s^2+p^2}\right)^k \quad (7)$$

For calculate the number of the original present in the form (10,14):

$$\overline{\psi}_k^{(s)} = \frac{s T_0 + T_0'}{\overline{A}(s) - \varepsilon p^2 \overline{B}(s)} \tag{8}$$

where

$$\overline{A}(s) = \left(s + \frac{1}{2}\varepsilon\omega_s p\right)^2 + p^2\left(1 - \frac{1}{2}\varepsilon\omega_c\right)^2$$

$$\overline{B}(s) = \overline{\omega}(s) + \omega_s \frac{s}{p} + \omega_c + \frac{\varepsilon}{4}\left(\omega_s^2 + \omega_c^2\right)$$

$$\omega_s = \int_0^\infty \omega(\tau)\sin p\tau\, d\tau; \quad \omega_c = \int_0^\infty \omega(\tau)\cos p\tau\, d\tau$$

Note that if the denominator of the equation (8) neglect the term $\varepsilon p^2 \overline{B}(s)$, we obtain the solution of this problem the image obtained by the averaging method (Larionov, 1969; Matyash, 1971 and Badalov *et al*., 1987).

If the variable t is large enough then the value $\left|\varepsilon p^2 \overline{B}(s)\right|$ is small

Therefore, for all values of parameter s and consequently time t can be shown validity of the inequality

$$\left|\frac{\varepsilon p^2 \overline{B}(s)}{\overline{A}(s)}\right| < 1$$

Then the function $\overline{\psi}(s)$ we can represented as:

$$\overline{\psi}_k(s) = \frac{s T_0 + T_0'}{\overline{A}(s)}\left[1 + \varepsilon p^2 \frac{\overline{B}(s)}{\overline{A}(s)} + \varepsilon^2 p^4 \frac{\overline{B}^2(s)}{\overline{A}^2(s)} + \ldots\right] \tag{9}$$

Original first term of this series is defined as

$$\psi_1(t) = e^{\exp\left(-\frac{1}{2}\varepsilon\omega_s pt\right)}\left[T_0 \cos p\left(1 - \frac{1}{2}\varepsilon\omega_c\right)t + \frac{T_0' - \frac{1}{2}\varepsilon\omega_s p}{p\left(1 - \frac{1}{2}\varepsilon\omega_c\right)}\sin p\left(1 - \frac{1}{2}\varepsilon\omega_c\right)t\right] \tag{10}$$

This formula is the solution, the first obtained by the averaging method for integro-differential equation of free oscillations of viscoelastic systems in (Matyash, 1971; Abdou and Salama, 2004 and Kurbanov and Babajanova, 2014).

Following the approach of the series (9) is determined by the convolution of functions (Govindjee and Reese, 1997 and Kurbanov and Babajanova, 2014).

$$\psi_2(t) = \varepsilon p^2 \int_0^t g(t-\tau)\psi_1(\tau)d\tau$$

where

$$g(t) = \omega(t) * e^{\exp\left(-\frac{1}{2}\varepsilon\omega_s pt\right)} \frac{\sin p\left(1 - \frac{1}{2}\varepsilon\omega_c\right)}{p\left(1 - \frac{1}{2}\varepsilon\omega_c\right)} +$$

$$+ \frac{\omega_s}{p} e^{\exp\left(-\frac{1}{2}\varepsilon p\omega_s t\right)}\left[\cos p\left(1 - \frac{1}{2}\varepsilon\omega_c\right)t + \frac{d - \frac{\varepsilon}{2}\omega_s p}{p\left(1 - \frac{1}{2}\varepsilon\omega_c\right)}\sin p\left(1 - \frac{1}{2}\varepsilon\omega_c\right)t\right]$$

$$d = \frac{\omega_c}{\omega_s}p + \frac{\varepsilon p}{4\omega_s}\left(\omega_s^2 + \omega_c^2\right)$$

Analogously we can find the next approximation of a function $\psi_k(t)$.

Now consider the function

$$\overline{\varphi}(s) = \frac{\overline{F}(s)}{s^2 + p^2 - \varepsilon p^2 \overline{\omega}(p)}$$

Let's expand the denominator in the following order:

$$\frac{1}{s^2 + p^2 - \varepsilon p^2\overline{\omega}(p)} = \frac{1}{s^2 + p^2} + \frac{\varepsilon p^2 \overline{\omega}(s)}{\left(s^2 + p^2\right)^2} + \ldots + \frac{\varepsilon^m p^{2m}\overline{\omega}^m(s)}{\left(s^2 + p^2\right)^{m+1}} +$$

$$+ \frac{\varepsilon^{m+1} p^{2(m+1)}\overline{\omega}^{m+1}(s)}{\left(s^2 + p^2\right)^{m+1}} \frac{1}{\left(s + \frac{1}{2}\varepsilon\omega_s p\right)^2 + p^2\left(1 - \frac{1}{2}\varepsilon\omega_c\right)^2}$$

Hence, we find:

$$\frac{1}{s^2 + p^2} \doteq \frac{1}{p}\sin pt$$

$$\frac{\overline{\omega}(s)}{s^2 + p^2} \doteq \frac{1}{p}\int_0^t \sin p(t-\tau)\omega(\tau)d\tau = g_0(t)$$

$$\frac{p^2\,\overline{\omega}(s)}{\left(s^2 + p^2\right)^2} \doteq p\int_0^t \sin p(t-\tau) g_0(\tau)d\tau = g_1(t)\omega$$

....

$$\frac{p^{2m}\overline{\omega}^m(s)}{\left(s^2 + p^2\right)^m} \doteq \lambda\int_0^t g_0(t-\tau)g_{m-1}(\tau)d\tau = g_m(t)$$

$$\frac{p^{2m}\overline{\omega}^m(s)}{\left(s^2 + p^2\right)^{m+1}} \cdot \frac{1}{\left(s^2 + \frac{1}{2}\varepsilon\omega_s p\right)^2 + p^2\left(1 - \frac{1}{2}\varepsilon\omega_c\right)^2} \doteq \frac{1}{1 - \frac{1}{2}\varepsilon\omega_c}\int_0^t g_m(t-\tau)\Omega(\tau)d\tau$$

where

$$\Omega(t) = \int_0^t \omega(t-\tau)e^{\exp\left(-\frac{1}{2}\varepsilon\omega_s pt\right)} \sin\left(1 - \frac{1}{2}\varepsilon\omega_c\right)p\tau\, d\tau$$

Then for $\overline{\varphi}(s)$ given:

$$\varphi_k(t) = \frac{1}{p}\int_0^t \sin p(t-\tau)F(\tau)d\tau + \varepsilon\int_0^t g_1(t-\tau)F(\tau)d\tau +$$

$$+ \ldots + \varepsilon^m \int_0^t g_m(t-\tau)F(\tau)d\tau + \frac{\varepsilon^{m+1} p}{1 - \frac{1}{2}\varepsilon\omega_c}\int_0^t F(t-\tau)\int_0^t g_m(\tau - t)\Omega(z)d\tau\, d\tau$$

Therefore, to a first approximation solutions assigned the problem we obtain:

$$T_1(t) = \psi_1(t) + \varphi_1(t) = e^{\exp(-\frac{1}{2}\varepsilon\omega_s pt)} \times$$

$$\times\left[T_0 \cos p\left(1-\frac{1}{2}\varepsilon\omega_c\right)t + \frac{T_0' - \frac{1}{2}\varepsilon\omega_c T_0 p}{p\left(1-\frac{1}{2}\varepsilon\omega_c\right)} \times \sin p\left(1-\frac{1}{2}\varepsilon\omega_c\right)t\right] +$$

$$+\frac{1}{p\left(1-\frac{1}{2}\varepsilon\omega_c\right)}\int_0^t F(\tau) e^{\exp(-\frac{1}{2}\varepsilon\omega_s pt)} \sin p\left(1-\frac{1}{2}\varepsilon\omega_c\right)(t-\tau)d\tau \qquad (11)$$

To investigate the obtained solutions assume that $F(t) = a\sin\alpha t$ then for $T_1(t)$ we obtain:

$$T_1(t) = e^{\exp(-\frac{1}{2}\varepsilon\omega_s pt)} M \sin\left[\lambda\left(1-\frac{1}{2}\varepsilon\omega_c\right)t - \beta\right] +$$

$$+\frac{a\sqrt{d_1^2 + d_2^2}}{\lambda\left(1-\frac{1}{2}\varepsilon\omega_c\right)} \sin(\alpha t - \theta) \qquad (12)$$

Where

$$\mu = \sqrt{T_0^2 + \left[\frac{T_0' - \frac{1}{2}\varepsilon\omega_c T_0 p}{\lambda\left(1-\frac{1}{2}\varepsilon\omega_c\right)}\right]^2}$$

$$\beta = \arcsin\frac{T_0}{\mu}$$

$$d_1 = \frac{1}{(\alpha+m)^2 - \left(\frac{1}{2}\varepsilon\omega_s p\right)^2} \cdot \frac{(\alpha_1 + \alpha_2)}{(\alpha-m)^2 - \left(\frac{1}{2}\varepsilon\omega_s pt\right)^2}$$

$$d_2 = \frac{1}{(\alpha-m)^2 - \left(\frac{1}{2}\varepsilon\omega_s p\right)^2} \cdot \frac{(\alpha_1 + \alpha_2)}{(\alpha+m)^2 + \left(\frac{1}{2}\varepsilon\omega_s pt\right)^2}$$

$$\alpha_1 = \frac{1}{2}\varepsilon\omega_s p e^{-\frac{1}{2}\varepsilon\omega_s pt} \sin(\alpha+m);$$

$$\alpha_2 = (\alpha+m)\left[1 - e^{-\frac{1}{2}\varepsilon\omega_s pt}\cos(\alpha+m)t\right]$$

$$\alpha_3 = \frac{1}{2}\varepsilon\omega_s p e^{-\frac{1}{2}\varepsilon\omega_s pt} \sin(\alpha-m)t;$$

$$\alpha_4 = (\alpha-m)\left[1 - e^{-\frac{1}{2}\varepsilon\omega_s pt}\cos(\alpha-m)t\right];$$

$$\alpha_5 = (\alpha+m)e^{-\frac{1}{2}\varepsilon\omega_s pt} \sin(\alpha+m)t;$$

$$\alpha_6 = \frac{1}{2}\varepsilon\omega_s p\left[1 - e^{-\frac{1}{2}\varepsilon\omega_s pt}\cos(\alpha+m)t\right];$$

$$\alpha_7 = (\alpha-m)e^{-\frac{1}{2}\varepsilon\omega_s pt} \sin(\alpha-m)t;$$

$$\alpha_8 = \frac{1}{2}\varepsilon\omega_s p\left[1 - e^{-\frac{1}{2}\varepsilon\omega_s pt}\cos(\alpha-m)t\right];$$

$$m = p\left(1 - \frac{1}{2}\varepsilon\omega_c\right);$$

$$\theta = \arccos\frac{d_1}{\sqrt{d_1^2 + d_2^2}}$$

Thus, if the external impact is small, the true amplitude oscillations remain finite and there is amplitude damping of free oscillations of by the exponential law and a frequency shift.

DISCUSSION

The purpose of work is to construct a generalized mathematical model and the development of new methods of solving integral-differential equation fluctuations viscoelastic systems with arbitrary rheology. With the help of the integral Laplace transform solved integral-differential equation of forced vibrations of linear viscoelastic systems for arbitrary kernel. The solution in the form of a series shown that at low frequencies the effect of the subsequent terms of the series to address a little with increasing frequency, they are increasing.
Analysis of the decisions shows that the inclusion of the following terms of the series improves the accuracy of the solution, and the amplitudes of all members of the series over time decreases exponentially, and the phases are shifted. The results can be used directly in the solution of complex applications in engineering calculations on the strength and durability of the operational reliability of the viscoelastic elements technology.

Conclusions

Solved the problem of forced vibrations of linear viscoelastic shells for any kernel at low viscosity material. The solution is built as a series, and shows that the first term of this series is a solution to this problem, obtained by averaging, and taking into account subsequent amendments give the number of the members of these decisions. The resulting solution is investigated for specific actions and obtained that at small amplitudes of the true impacts fluctuations remain finite and there is the damping of the amplitude of free oscillations exponentially, and the phases are shifted.

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