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SIMULATION OF DYNAMICAL INTERACTED PROCESS WITH APPLICATION OF SUBCLASSES FUZZY NETS PETRI

*¹Mustafayev V. A., ²Mammadov J. F. and ³Atayev Gh.N.

¹Department of Computer Sciences of Sumgait State University, Azerbaijan

²Department of Information Technology and Programming of Sumgait State University, Azerbaijan

³Department of Computer Sciences of Sumgait State University, Azerbaijan

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ABSTRACT

The modeling of dynamic interacting processes is considered describing the operating of complex objects under uncertainty. Models of dynamic processes are presented in the form of fuzzy algebraic and fuzzy Petri networks. On the example of the flexible manufacturing module stripping card surface is shown that the accepted transition activating rules completely describe the operating process of fuzzy Petri networks.

Key words:

Production model, Fuzzy, Flexible manufacture system, Petri networks, Flexible manufacturing module.

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INTRODUCTION

One of the actual problems of the theory of dynamical systems is the study of interacting processes that operate in the uncertain environment (Pedrycz 1996; Yegorov and Shaykin, 2002; Bodyanskiy *et al.*, 2005). Difficulties associated with the solution of this problem depends on a number of factors, which should also include primarily the following: the presence of a large number of interconnected elements with complex structural and functional relationships between them; operating of the individual elements are not independent and due to their place in the system as a whole; operation of individual elements occurs asynchronously, their interacting rules are described by the complex logical conditions: uncertainty behavior of the individual elements can be both probabilistic and fuzzy character. These challenges require a new approach to modeling the behavior of the systems and processes under uncertainty. The basis for this approach was of Petri networks (PN) construction and the further development of algebraic networks that allow us to describe the interaction of the asynchronous processes with given accuracy. One possible generalization PN is related to the implementation of the additional properties that allow us to describe the uncertainty of system behavior during its operation. There are two approaches to solving this problem.

The first approach is to describe the uncertainty activating transitions in conflict situation. Moreover, the set of admissible sequences of transitions, taken as a complete group of events, and each element of the set is attributed to a certain probability. The disadvantage of this approach is necessity to analyze a large number of elements, the set of admissible sequences of transitions and their sharp increase in network expansion. The second approach involves taking into account the uncertainty of the number of chips in the positions of the network, simulating the condition of the system components. Number of chips in all positions defines the global state of the system. It should be noted that the uncertainty of the availability of chips can be described both by the matter of the probabilistic positions, and fuzzy sets theory. This problem is in the process of modeling of the systems is solved by means of fuzzy and fuzzy algebraic PN. In the paper is considered the modeling of the dynamic interacting processes operating under uncertainty using fuzzy and fuzzy algebraic PN.

Presentation of the dynamic interacting processes by fuzzy models

Feature of fuzzy models (Pedrycz, 1996) is that they should provide a flexible processing strategy for the dissimilar interacting dynamic processes that represent data and knowledge in the essential fuzzy state space of the objects under analysis. Dynamic interaction processes are described by the numerical and linguistic variables. In this regard, fuzzy

*Corresponding author: Mustafayev V. A.

Department of Computer Sciences of Sumgait State University, Azerbaijan.

models are focused on modeling structures, which are characterized by: operating at the level of linguistic terms (fuzzy sets); characteristics of the system may be represented in the same linguistic format; representation and processing of data under uncertainty. Fuzzy models based on the calculation rules with fuzzy sets are clear and efficient means of presenting dynamic interacting processes, mapping data and knowledge in the form of “IF ... THEN ...”. Part of the rule “IF” is called sending and “THEN” - conclusion or action. In general, as a product is taken the expression of the form:

$$(Y); Q; X; A \text{ } \emptyset \text{ } B; s,$$

where Y is a product name; Q is the product using sphere characteristics; X - condition of applicability of the product kernel; $A \text{ } \emptyset \text{ } B$ (A is a condition, B - conclusion or action) is the kernel of the product; s - post condition of the product. In actual designs kernel component A is characterized by the complex structure consisting of certain predicates, logic operations such as the NOT, AND, OR and their derivatives. The product system includes (Osuga, 1986): the rule base; global database; rule interpreter. The rule base is a memory area that contains a knowledge base, i.e. the collection of knowledge represented in the form of rules of the form “IF ... THEN ...”.

Global database is an area of memory containing the actual data that describe the input data and system status. The base data of the different systems have different forms, but they may be described as a group of data comprising a data name, attributes and attribute values. The interpreter is a mechanism for the component output, which forms the conclusion using a base and a database of rules.

Let’s consider the structure of production rules in a clear representation of knowledge:

$$\text{If } A_1 \text{ and } A_2 \text{ and } \dots \text{ and } A_n \text{ Then } \dots \quad \dots(1)$$

This indicates that “if all conditions A_1 to A_n are true, then B is also true” or “when all conditions from A_1 to A_n become true, you should perform the action B ”.

Expression (1) in terms of Boolean logic is as follows:

$$B = \text{TRUE} / (A_1 \text{ and } A_2 \text{ and } \dots \text{ and } A_n) = \text{TRUE} \quad \dots\dots(2)$$

Similarly to (1), (2) can be shown the validity of the relevant decisions of the rules for products containing operations NOT, OR, and their derivatives. Expressions (1), (2) in a fuzzy representation are determined as follows:

$$\text{IF } \overline{A_1} \text{ is } \sim_{\overline{A_1}}(x) \text{ and } \overline{A_2} \text{ is } \sim_{\overline{A_2}}(x) \text{ and.. and } \overline{A_n} \text{ is } \sim_{\overline{A_n}}(x) \\ \text{then } \overline{B} \text{ is } \sim_{\overline{B}}(x).$$

$$\overline{B} = \text{TRUE}[(\overline{A_1} \text{ and } \overline{A_2} \text{ and } \dots \text{ and } \overline{A_n}) = \text{TRUE}] \text{ and} \\ \text{and } [\sim_{\overline{A_1}}(x_1) \geq \sim_{\overline{A_1}}(x_1)^*] \text{ and } [\sim_{\overline{A_2}}(x_1) \geq \sim_{\overline{A_2}}(x_1)^*] \text{ and} \\ \dots \text{ and } [\sim_{\overline{A_n}}(x_1) \geq \sim_{\overline{A_n}}(x_1)^*] \text{ and } [\sim_{\overline{B}}(x_1) \geq \sim_{\overline{B}}(x_1)^*], \text{ where}$$

$\overline{A_1}, \overline{A_2}, \dots, \overline{A_n}$ - fuzzy condition, \overline{B} – fuzzy conclusion and action, $\sim_{\overline{A_1}}(x_1)^*, \sim_{\overline{A_2}}(x_1)^*, \dots, \sim_{\overline{A_n}}(x_1)^*, \sim_{\overline{B}}(x_1)^*$ - admissible values of the corresponding membership functions. Membership function of the set of processes that determine the conditions and actions of the domain are mapped on the set of positions and transitions of fuzzy or fuzzy algebraic PN.

Considering the above, the fuzzy production model of the dynamic interacting parallel processes can be represented in the form of fuzzy PN (Cao T., Sanderson A., 1993): $N = (P, T, I, O, \sim)$, where $P = \{p_i\}$ ($i=1, \dots, n$; n – positions number) – fuzzy set of positions; $T = \{t_j\}$ ($j=1, \dots, m$; m – transitions number) – fuzzy set of transitions; $I: P \times T \rightarrow \tilde{E}(0, 1, \dots)$; $O: T \times P \rightarrow \tilde{E}(0, 1, \dots)$ – respectively the input and output functions of incidence; the map $\sim: P \rightarrow \tilde{E}[0,1]$ assigns to each position p_i - vector of the distribution membership function degrees of the chips to the position $\sim(p_i)$.

Let’s consider the representation of interacting processes in cases where:

- The process is carried out with one or more input and one or more output conditions:
 $5 t_j \in T, [\sim_{\overline{A_1}}(p_i: [1]] \text{ and } [O(p_i: [1]]$
- Some condition of the process has one or more inputs and one or more output processes:
 $5 p_i \in P, [\sim_{\overline{A_1}}(t_j: [1]] \text{ and } [O(t_j: [1]]$
- The process is carried out with more than one input and more than one output conditions:
 $5 t_j \in T, [\sim_{\overline{A_1}}(p_i: [0 1]] \text{ and } [O(p_i: [0 1]]$
- A condition for implementation of the process has more than one input and more than one output process:
 $5 p_i \in P, [\sim_{\overline{A_1}}(t_j: [0 1]] \text{ and } [O(t_j: [0 1]]$.

Algorithm operating fuzzy petri nets

Solving the practical problems it is convenient to use the matrix representation of the structure of fuzzy PN. The elements of the input matrix f_{ij} , output positions h_{ij} and incidence d_{ij} are determined as follows (Yegorov and Shaykin, 2002):

$$f_{ij} = \begin{cases} 1, & \text{if } p_i \in I(t_j); \\ 0, & \text{if } p_i \notin I(t_j); \end{cases} \\ h_{ij} = \begin{cases} 1, & \text{if } p_i \in O(t_j); \\ 0, & \text{if } p_i \notin O(t_j); \end{cases}$$

$$d_{ij} = \begin{cases} -1, & \text{if } p_i \in I(t_j) \text{ and } p_i \notin O(t_j); \\ 1, & \text{if } p_i \notin I(t_j) \text{ and } p_i \in O(t_j); \\ 0, & \text{if } p_i \notin I(t_j) \text{ and } p_i \notin O(t_j). \end{cases}$$

Activating the transitions and changes of the states of the fuzzy PN are defined by the rule (Akhmedov and Mustafayev, 2010):

- if the vector of the distribution of the membership degree of each input position $p_i \in P$ has a non zero component,

with the number equal to or greater than number of arcs, connecting this position with the transition $t_j \in T$, then transition t_j is activated;

- after activating the transition occurs in the process of redistribution of chips in the positions;
- the number of chips in the positions determines the state of the network.

Transition t_j at marking \sim is allowed under the conditions:

- select all $f_{ij} \hat{=} 0$ by $i=1, n$;
- for each fixed i must hold $\exists \sim_{ik} \hat{=} 0, k = \overline{f_{ij}, k_i}$, where k_i is a length of the distribution vector of membership degrees of the i^{th} position.
- after activating the transition t_j dynamics of the network state, the process of redistribution of chips in the positions and the new marking are defined by the following algorithm:

1. Form the vector of the distribution of the membership degrees of each input position after activating the transition t_j .

1.1. Calculate the zero component of the vector of the membership degree distribution

$$\sim_{i0} = \bigvee_{r=0} f_{ij} \sim_{ir},$$

where \vee is the maximum logic operation.

1.2. Determine the remaining components of the membership distribution degree

$$\sim_{is} = \sim_{is} + f_{ij}, S = 1, 2, \dots, k_i - f_{ij}. 1.3.$$

Activating the transition t_j the dimension of the membership distribution vector of the n^{th} input position is reduced by the number of input arcs:

$$k_i = k_i - f_{ij}.$$

2. Form the vector of the membership distribution degree for each output position after activating the transition. This vector indeed is the vector of the diagonal convolution of the Gram matrix source output vector and an intermediate vector $r = (r_0, r_1, \dots, r_{h_{jz}})$.

2.1. Select all $h_{jz} \hat{=} 0$, by $z = \overline{1, n}$.

2.2. Calculate the last component of the vector r for all fixed i at $f_{ij} \hat{=} 0$:

$$r_{h_{jz}} = \bigwedge_i \bigvee_{r=f_{ij}}^{k_i-1} \sim_{ir},$$

where \wedge is the minimum logic operation.

2.3. Calculate the zero component:

$$r_0 = 1 - r_{h_{jz}}.$$

2.4. Determine the remaining components: $r_i = 0, i \in \overline{1, h_{jz}} > 1$.

3. Form the Gram matrix of the vectors \sim_z and r^T (\sim_z denotes the transpose):

$$G(\sim_z, r^T) = \begin{pmatrix} \sim_{z0} \\ \sim_{z1} \\ \dots \\ \sim_{zk_z} \end{pmatrix} \times \begin{pmatrix} r_0 \\ r_1 \\ \dots \\ r_{h_{jz}} \end{pmatrix} =$$

$$\begin{vmatrix} \sim_{z0} \wedge r_0 & \sim_{z0} \wedge r_1 & \dots & \sim_{z0} \wedge r_{h_{jz}} \\ \sim_{z1} \wedge r_0 & \sim_{z1} \wedge r_1 & \dots & \sim_{z1} \wedge r_{h_{jz}} \\ \dots & \dots & \dots & \dots \\ \sim_{zk_z} \wedge r_0 & \sim_{zk_z} \wedge r_1 & \dots & \sim_{zk_z} \wedge r_{h_{jz}} \end{vmatrix}$$

where $g_{\ell k} = \sim_{z\ell} \wedge r_k, \ell = \overline{0, k_z}, k = \overline{0, h_{jz}}$.

4. Calculate of the vector of the diagonal convolution of the Gram matrix $C(G(\sim_z, r^T))$:

$$C(G(\sim_z, r^T)) = \begin{vmatrix} (\sim_{z0} \wedge r_0) \\ (\sim_{z1} \wedge r_0) \vee (\sim_{z0} \wedge r_1) \\ \dots \\ (\sim_{zk_z} \wedge r_0) \vee (\sim_{zk_{z-1}} \wedge r_1) \vee \dots \vee (\sim_{z-h_{jz}} \wedge r_{h_{jz}}) \\ \dots \\ (\sim_{zk_z} \wedge r_{h_{jz}}) \end{vmatrix}$$

Elements of this vector are calculated as follows:

$$C_\ell = \bigvee_{k+i=\ell} (\sim_{zk} \wedge r_i) \quad \text{by}$$

$$k = \overline{0, k_z}, i = \overline{1, h_{jz}};$$

The dimension of the membership degree distribution is increased by the number

$$h_{jz}: k_z = k_z + h_{jz}.$$

5. Form the vector of the membership degree distribution $\tilde{z} = (\tilde{z}_0, \tilde{z}_1, \dots, \tilde{z}_k)$ by output position p_z :

$$\tilde{z}_k = c_k, \quad k = \overline{0, k_z}.$$

Transition activating rules and change of the states of algebraic petri networks

Let $A = \{a, b, c, d, \dots\}$ is a finite alphabet; \overline{A} – free monoid over the alphabet A , the set of the elements of which is the set of all finite sequences of the elements of A called words, and operation $\bar{\cdot}$ – the concatenation of the words. Identity element ε of the free monoid \overline{A} is an empty word, i.e. empty sequence of the symbols from the alphabet A . Monoids represented as

$$\overline{A} = (\overline{A}, \bar{\cdot}, \varepsilon),$$

where \overline{A} – a set of words;

$\bar{\cdot}$ – concatenation operation;

ε – the identity element.

Finite nonempty set $X \subseteq \overline{A}^*$ is called a Code, if all of the words \overline{A}^* admit a unique decomposition into words X , in other words \overline{A}^* is a free monoid.

If \overline{A}^* – a free monoid, it means validity of the of the implication:

$$x_1 x_2 \dots x_m = y_1 y_2 \dots y_n \Rightarrow m = n, \\ x_1 = y_1, x_2 = y_2, \dots, x_m = y_m, \quad x_i, y_i \in X.$$

Algebraic network is PN (Leskin A.A., Maltsev P.A., Spiridonov A.M., 1996)

$$N = (P \cup F, T, A, V, \mu_0),$$

where $P = \{p_1, p_2, \dots, p_n\}$, $F = \{f_1, f_2, \dots, f_m\}$, $T = \{t_1, t_2, \dots, t_r\}$ – finite sets of the types of positions, f and transitions, respectively; $V : [(P \cup F) \times T] \cup [T \times (P \cup F)] \rightarrow \overline{A}^*$ is a mapping, indicating arcs, connecting the positions with transitions and transitions with positions; $\mu_0 : F \cup P \rightarrow \overline{A}^*$ – initial marking of the positions. Existence of the arcs connecting position and transitions is defined as follows: if $V(a, b) = \varepsilon$ (ε – empty word), then arc between a and b is absent, and $a \in P \cup F$, $b \in T$ or $a \in T$, $b \in P \cup F$. If $V(a, b) = R$, $R \in \overline{A}^*$, then there is an arc from a to b , marked with the word R , $a \in P \cup F$, $b \in T$ or $a \in T$, $b \in P \cup F$.

Let's denote each element $a \in P \cup F \cup T$:

$$G^-(a) = \{b \in P \cup F \cup T \mid V(a, b) \neq \varepsilon\} \text{ – set of inputs } ; \\ G^+(a) = \{b \in P \cup F \cup T \mid V(b, a) \neq \varepsilon\} \text{ – set of outputs } .$$

Let's $a \in \overline{A}^*$, $a = a_1, a_2, \dots, a_n$, $a_i \in \overline{A}$, $i = \overline{1, n}$. Word \tilde{a} is specular with respect to the a :

$$\tilde{a} = a_n, a_{n-1}, \dots, a_2, a_1.$$

Global state of the network is determined by the type of words in positions p and f . Activation of some transition t words at positions $G^-(t) \cup G^+(t)$ are modified. Transition activating rules provide algebraic dynamics PN and clarify conditions of change of the network status, which is determined by the maps:

$$\mu : P \cup F \rightarrow \overline{A}^*; \quad \mu : \in (\overline{A}^*)^{n+m},$$

where $n + m = \text{card}(P \cup F)$. The word $\mu(a)$, $a \in P \cup F$ marks the position a . Network marking is represented by the vector of dimension $[1 \times (n + m)]$:

$$\mu = (\mu(f_1), \mu(f_2), \dots, \mu(f_m), \mu(p_1), \dots, \mu(p_n)),$$

where the first m -places are the marking the positions of type f and in the field $(m + 1)$ to $(m + n)$ – type marking positions p . Initial marking of the positions is described similarly:

$$\mu_0 = (\mu_0(f_1), \dots, \mu_0(f_m), \dots, \mu_0(p_1), \dots, \mu_0(p_n)).$$

Transition t is allowed for marking μ , if for all positions of types f , such that $f_i \in G^+(t)$, $V(f_i, t)$ there exists the left multiplier $\mu(f_i)$ and for all positions of type p , such that $p_i \in G^-(t)$, $V(p_i, t)$ there exists is the left multiplier $\tilde{\mu}(p_i)$. The case when transition t is allowed for marking μ is denoted as $\mu(t >)$. Activating the transition t allowed for marking μ leads to the new marking μ' , $\mu(t > \mu')$, if

$$\forall f_i \in F, \quad \mu'(f_i) = g(V(f_i, t), \mu(f_i) \circ V(t, f_i)), \\ \forall p_j \in P, \quad \mu'(p_j) = d(V(p_j, t), \mu(p_j) \circ V(t, p_j)),$$

where $g(a, b) = b$ and $d(a, b) = b$.

Thus, in the words of the positions type of f having arcs with transition t marked by the words $V(f_i; t)$ the left multiplier $V(f_i; t)$ is absorbed in the words $\mu(f_i)$, and there is an addition to the right hand side as a result of the concatenation of the factor $V(t; f_i)$. In turn, in the words of the positions type p having arcs with transition t marked by the words $V(p_j; t)$ right factor $V(p_j; t)$ is absorbed and there is an addition adding the right hand side as a result of the concatenation of the factor $V(t; p_j)$. The sequence $t \in T^*$, where T^* is the set of the finite words in the alphabet T is called admissible for marking μ and gives marking μ' , $\mu(t > \mu')$, if the following conditions (Kotov V.E., 1984) hold:

- a) $t = \varepsilon \Rightarrow \mu = \mu'$;
- b) $t = t' t_i, t' \in T^*, t_i \in T$.

Then there exists a marking μ'' , such that $\mu(t' > \mu'' \quad \mu''(t_i > \mu')$. In turn, marking μ' is called reachable marking μ , if there exists a finite sequence $t \in T^*$, such that $\mu(t > \mu'$.

Algorithm to operating of the fuzzy algebraic petri networks

Fuzzy algebraic network is a PN

$$N = (P \cup F, T, A, V, \sim_0^R),$$

where $\sim_0^R : P \cup F \rightarrow A^* \times [0, 1]^\ell$ is the initial marking of the positions of words from A^* , $\ell = \text{card}(x^f(a))$, $a \in P \cup F$.

Initial marking for each positions of fuzzy algebraic PN is the tuple

$$\sim_0^R(a) = (x_1, x_2, \dots, x_k; R(x_1), R(x_2), \dots, R(x_k)). \quad \dots\dots\dots(3)$$

The rules of permission for the transitions and their operation in fuzzy algebraic PN are similar to the rules of the simple algebraic PN and are determined by the first element of the tuple (3). Activating the transition t permitted for marking $\sim^R(a)$ leads to the new marking $\sim_1^R(a)$, moreover the first element of the tuple is computed similarly to algebraic PN. The second element of the tuple is calculated by the formula

$$R(V(t, a)) = \min [R(V(y, t)) \mid y \in G^+(t)], \forall a \in P \cup F \quad \dots\dots\dots(4)$$

In this case, the activating transition t to the word $\sim^R(a)$ are added the right multiplier $V(t; a)$ and the corresponding degree of the membership function of the network distribution $R(V(t; a))$. The above process of operating of the network is uniquely defined only if the output code for each position consists of the elements of the input code. Therefore it is necessary to define the rules for computing the degree of the distribution function membership $R(V(t; a))$ in the presence of the code $X^f(a)$ with the elements not belonging to $X^b(a)$. It is known that $X^f(a)$ can be obtained from $X^b(a)$ only by the join operations and the dismemberment of its elements.

Consider options for obtaining Code $X^f(a)$ consistently

– Code $X^f(a)$ obtained from the Code $X^b(a)$ only by a join operation, i.e.

$$\forall x^b \in \overline{X^b(a)}, \quad x^b = x_{i_1}^b, x_{i_2}^b, \dots, x_{i_k}^b, \quad x_{i_j}^f \in X^f(a), j=1, k$$

in this case it is possible to assume that

$$R(x_{i_j}^f) = R(x^b), \quad \forall j = \overline{1, k};$$

– Code $X^f(a)$ obtained from the Code $X^b(a)$ only by a join operation, i.e.

$$\forall x^f \in X^f(a), \quad x^f = x_{i_1}^b, x_{i_2}^b, \dots, x_{i_n}^b, \quad x_{i_j}^b \in X^b(a), j = \overline{1, n},$$

in this case it is obvious that

$$R(x^f) = \min [V(x_{ij}^b) \mid x_{ij}^b \in X^b]. \quad \dots\dots\dots(5)$$

For given values of probabilities for each sub-word of $\mu^s(a)$, $\forall a \in P \cup F$, and the expression (4), (5) allow us to determine the values of $R(V(t, b))$, $\forall b \in P \cup F$, after activating any transition $t \in T$.

Considering the above, the algorithm of operating of fuzzy algebraic PN is as follows:

1. Constructing of the input incidence matrix of the transitions sets with the dimension $(m + n) \times r$:

$$g_{ji}^- = \begin{cases} s, & \text{if there is an arc from the } j^{\text{th}} \text{ position} \\ & \text{to } i^{\text{th}} \text{ transition;} \\ v, & \text{otherwise,} \end{cases}$$

where $i = \overline{1, r}$; $j = \overline{1, m+n}$. For $j = \overline{1, m}$ the arcs are denoted from the positions type of f and $j = \overline{m+1, m+n}$ are denoted the arcs from the positions type of p .

2. Constructing of the output incidence matrix of the transitions sets with the dimension $r \times (m + n)$:

$$g_{ij}^+ = \begin{cases} s, & \text{if there is an arc from the } i^{\text{th}} \text{ position} \\ & \text{to } j^{\text{th}} \text{ transition;} \\ v, & \text{otherwise} \end{cases}$$

where $i = \overline{1, r}$; $j = \overline{1, m+n}$. For $j = \overline{1, m}$ the arcs are denoted from the positions type of f and $j = \overline{m+1, m+n}$ are denoted the arcs from the positions type of p .

3. Constructing the input matrix of the distribution function membership degree of the set of transitions with the dimension $(m + n) \times r$:

$$W_{ji}^- = \begin{cases} W(s), & \text{if there is an arc from the } j^{\text{th}} \text{ position} \\ & \text{to } i^{\text{th}} \text{ transition;} \\ 0, & \text{otherwise} \end{cases} \quad \text{where}$$

$$i = \overline{1, r}; j = \overline{1, m+n}; W(s) \in [0, 1].$$

4. Constructing the matrix of the initial marking with the dimension $1 \times (m + n)$:

$$\mu_j = \begin{cases} s, & \text{if the position is marked with the word } s; \\ v, & \text{if the position is not marked} \end{cases}$$

where $j = \overline{1, m+n}$. The elements $\mu_j, j = \overline{1, m}$ determine marking of the positions type of f , and the elements $\mu_j, j = \overline{m+1, m+n}$, determine marking of the positions type of \bar{f} .

5. Constructing the matrix of the distribution function membership degree for the initial marking:

$$e_j = \begin{cases} W(\sim_j), & \text{if } j^{\text{th}} \text{ position is marked;} \\ 0, & \text{if } j^{\text{th}} \text{ position is not marked,} \end{cases}$$

where $j = \overline{1, m+n}$; $W(\mu_j) \in [0, 1]$.

6. Search of the allowed transition. For each transition $t_i, i = \overline{1, r}$, activating condition is checked:

6.1. All input positions are defined from the input matrix positions t_i . For all $\bar{g}_{ji} \neq \varepsilon, j = \overline{1, m}$, the condition is checked whether \bar{g}_{ji} is a factor for μ_j : the length of these elements $n_1 = \text{card}(\bar{g}_{ji})$, are calculated and the word $p = \text{copy}(\mu_j, 1, n_1)$ is chosen and the same symbols from the first position of the marking element μ_j . If $p \neq \bar{g}_{ji}$, then index i is increased by unit $i = i + 1$ i.e. and proceeds to step 6.1;

6.2. For all $\bar{g}_{ji} \neq \varepsilon, j = \overline{m+1, m+n}$, specular word is constructed: $\tilde{\mu}_j = \varepsilon \circ \text{copy}(\mu_j, n_1, 1)$, is accepted and substitution of the symbols is carried out by the formula $\tilde{\mu}_j = \tilde{\mu}_j \circ \text{copy}(\mu_j, k, 1)$ by $k = \overline{n_1, 1}$;

6.3. Condition is checked whether $\bar{g}_{ji}, j = \overline{m+1, m+n}$, is the left multiplier for the specular word: $p = \text{copy}(\mu_j, 1, n_1)$ is chosen from $n_1 = \text{card}(\bar{g}_{ji})$ number of symbols from the first position of the specular word. If $p \neq \bar{g}_{ji}$, then the index i is increased by unit, i.e. $i = i + 1$.

7. If $i > r$ a message about deadlock is sent.

8. Go to step 6.1.

9. Calculating the new marking matrix:

$$\mu'_j = \begin{cases} \text{copy}(\sim_j, n_1 + 1, m_1 - n_1) \circ \bar{g}_{ji}^+ & \text{at } j = \overline{1, m} \\ \text{copy}(\sim_j, 1, m_1 - n_1) \circ \bar{g}_{ji}^+ & \text{at } j = \overline{m+1, m+n} \end{cases}$$

where $m_1 = \text{card}(\mu_j)$; $n_1 = \text{card}(\bar{g}_{ji})$.

10. The new marking is accepted as a current one:

$$\mu_j = \mu'_j, i = \overline{1, m+n}.$$

11. Forming of the matrix of distribution function membership degree for the obtained new marking:

11.1. Calculating the elements of the output matrix of distribution function membership degree for the transitions set:

$$W^+(i, k) = \min [W^-(j, i)^\ell] \quad \text{for all } W^-(j, i) \neq 0,$$

where $i = \overline{1, r}$; $k = \overline{1, m+n}$;

$$11.2. \mathbf{e}_j = \begin{cases} W^+(j, k), & \text{if } \mu_j \neq \varepsilon; \\ 0, & \text{if } \mu_j = \varepsilon, \end{cases}$$

where $j = \overline{1, m+n}$; $\ell = \text{card}(\mu_n)$.

12. Go to step 6. Process continues until the desired marking is obtained. After analyzing the basic properties - boundedness (finiteness of the states of the separate system element), security (number of states is not greater than one), conceivability (inability destruction or emergence of additional resources), liveness (absence of the deadlock states in the operating of the system), reachability (sequence of transitions that transform system from one state to another) -fuzzy algebraic PN, can the behavior of the modeled system (Zaitsev, 2004).

Operation model of flexible manufacturing module for stripping cards surface

Let us consider the operation of the Flexible Manufacturing System (FMS) of stripping card surface. The composition of FMS includes: stripping machine (SM) to perform the technological process of stripping card surface; input and output drives for card storing, before and after stripping respectively; vision device (VD) for determining the quality of operation of stripping card surface; OL- loading the operation defect drives, corrigible defect and stripped cards. The module works as follows: if there are cards in the input drive ID pusher is turned on and the card is fed into the working area of the peeling device, where the stripping process is carried out; initial and final state of the pusher is fixed; after the operation, the peeled cards loaded in the output drives. If there are cards on the output drive, VD performs quality control of the operation of stripping card surface; after quality control of operation, the peeled cards loaded in the appropriate drive; depending on the monitoring results the values of the membership function are defined (0; 0.5) – admissibility interval; (0.5; 0.8) – corrigibility interval; (0.8; 1) – incorrigibility interval.

Fuzzy operating model of the active elements of FMS of stripping cards surface are presented in the form of fuzzy PN and set of positions and transitions are defined. The set of positions p_1 - presence of the cards in the input drive SM; p_2 - absent of the cards in the input drive SM; p_3 - SM performs stripping operation; p_4 - presence of the cards in the output drive SM; p_5 - absent of the cards in the output drive SM; p_6 - VD is in the initial state; p_7 - VD performs quality control of

operation stripping card surface ; p_8 - VD is in the final state; p_9 - on the output drive SM in the card surface is not detected; p_{10} - on the output drive SM corrigible defect is found in the card surface; p_{11} - in the output drive SM incorrigible defect is found in the card surface; p_{12} - OL is in the initial state; p_{13} - drive of the peeled cards is loaded; p_{14} - drive of the cards with corrigible defects is loaded; p_{15} - drive of cards with incorrigible defects is loaded.

The set of transitions: t_1 - the cards is sent for stripping operations; t_2 - loading the input drive SM ; t_3 - SM completed stripping operation; t_4 - switch of VD; t_5 - VD completed the quality control of operations of stripping cards surface; t_6 - turning off VD; t_7 - switch of OL; t_8 - turning off OL.

The initial marking \sim_0 is presented by the vectors:

$$\begin{aligned} \sim(0,1) &= (1.000 \ 0.000), \\ \sim(0,9) &= (0.200 \ 0.600 \ 0.100 \ 0.100), \\ \sim(0,2) &= (1.000 \ 0.000), \\ \sim(0,10) &= (0.100 \ 0.500 \ 0.400 \ 0.000), \\ \sim(0,3) &= (1.000 \ 0.000), \\ \sim(0,11) &= (0.100 \ 0.400 \ 0.500 \ 0.000), \\ \sim(0,4) &= (1.000 \ 0.000), \\ \sim(0,12) &= (0.000 \ 0.000 \ 0.000 \ 1.000), \\ \sim(0,5) &= (1.000 \ 0.000), \\ \sim(0,13) &= (0.000 \ 0.400 \ 0.600 \ 0.000), \\ \sim(0,6) &= (1.000 \ 0.000), \\ \sim(0,14) &= (0.000 \ 0.200 \ 0.000 \ 0.800), \\ \sim(0,7) &= (0.200 \ 0.300 \ 0.500), \\ \sim(0,15) &= (0.000 \ 0.300 \ 0.500 \ 0.200), \\ \sim(0,8) &= (0.000 \ 0.400 \ 0.600), \end{aligned}$$

On the base of the developed algorithm the elements of the Gram matrix and the diagonal vectors of the reconciliation fuzzy PN are calculated. As a result of the computer experiment the sequence of the operated transitions are obtained: $\uparrow = (t_1 t_2 t_3 t_4 t_5 t_6 t_7 t_8)$ taking into account the initial marking \sim_0 .

Conclusion

Developed algorithms to operating of the fuzzy and fuzzy algebraic PN, provide a convenient conversion of external data into the internal format used in the simulation, an effective

form of representation of the structure, dynamics model state, space of reached states and a sequence of the operating of the transitions as a set of vectors and matrices, simplifying and accelerating the modeling process, automatic detection of deadlocks. The software was developed in the system DELPHI 7.0 based on the above algorithms. Resources of modern computers allow us to solve the problem with sufficiently large size matrices that completely satisfies the requirements for modeling of complex real objects, operating under uncertainty conditions.

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