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RESEARCH ARTICLE

THE RELATION BETWEEN THE DERIVATIVE OF THE FUNCTION AND THE INTEGRAL OF THE INVERSE FUNCTION TO THE DERIVATIVE FUNCTION

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ABSTRACT

In the first what is the calculus? Before defining the calculus we can divide it into two categories; the first is differentiation, but the second is the integration.
The differentiation: that category relates with the rate of change and the slope of the graphs, whereas we use the limits in finding the derivative of any curve:

$$\boxed{\begin{aligned}\frac{df}{dx} &= \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}\\ &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &= \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}.\end{aligned}}$$

-The integration: that category relates with the area under and between the graphs, whereas appeared many theorems like Riemann integration, Monte Carlo integration and the fundamental theorem of calculus.
-The differentiation is connected with integration by The fundamental theorem of calculus that tells us that:

$$\boxed{\int_a^b f(x)dx = F(b) - F(a)}$$

-The differential calculus and integral calculus is well-connected by many thing and from the new things that the derivative of a function can be found by knowing the integral of the inverse function to the derivative function, and that can be understood from the following equation:

$$\boxed{\frac{\mathcal{F}(x) - \left(\mathcal{F}(0) - \int_{f(0)}^{f(x)} f^{-1}(y)\,dy\right)}{x} = \mathcal{F}'(t)}$$

-The previous equation told us that:
1-we can find the derivative of any curve without taking the limits.
2-That equation is an good evidence for the strong connection between the differentiation and integration.

INTRODUCTION

The limits is very important in finding the derivative of any function:

$$\begin{aligned}\frac{df}{dx} &= \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}\\ &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &= \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}.\end{aligned}$$

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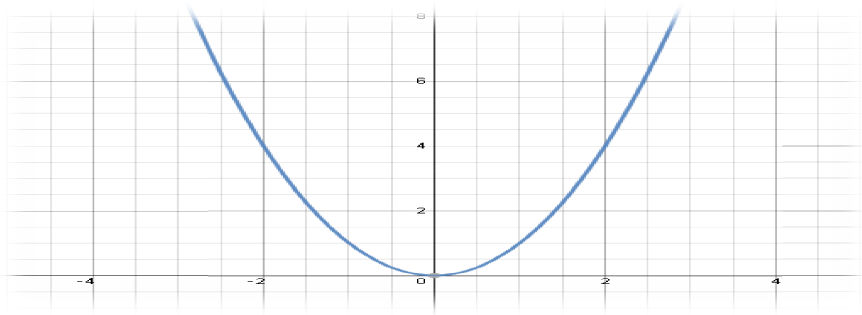
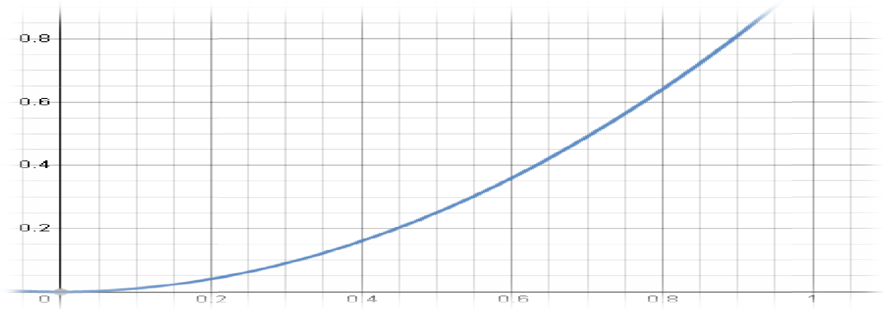
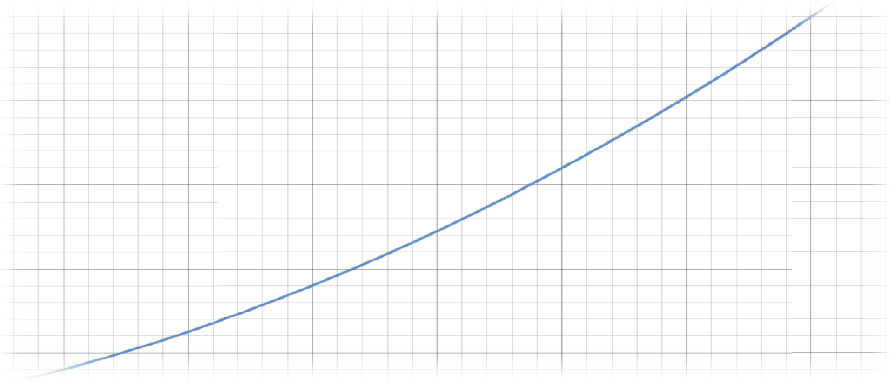
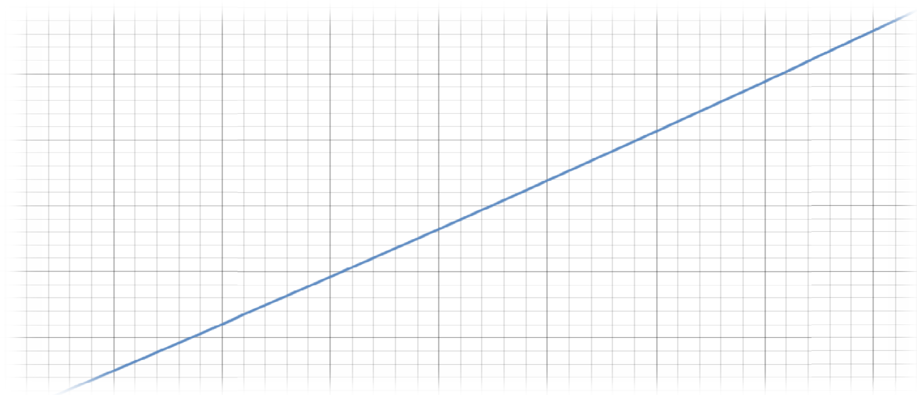
whereas the limits make any curve as a line at some point and the following figures explain that:









Finding the antiderivative is a inverse process of differentiation and it represents the main connection between the differentiation and integration for example:

$d(x^2)/dx = 2x$ but the integral of $2x$ is x^2

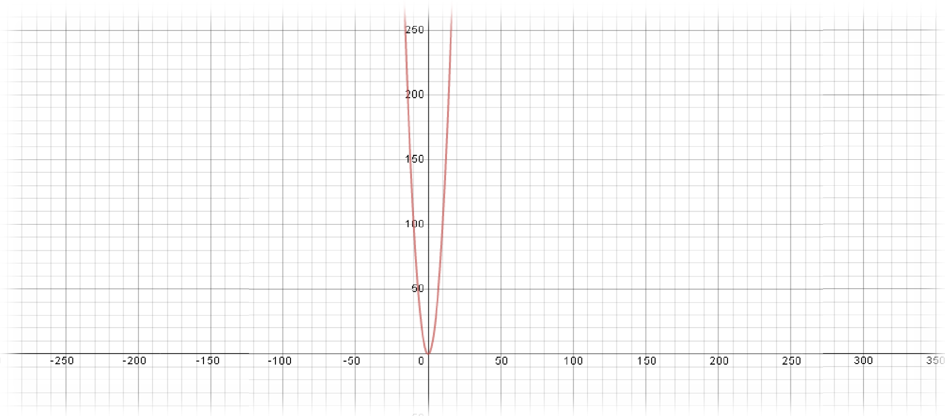
According to knowing that the derivative at some point is the slope of the tangent line to the curve at that point, we can find the derivative by specifying two points which are $(x,F(x))$ and

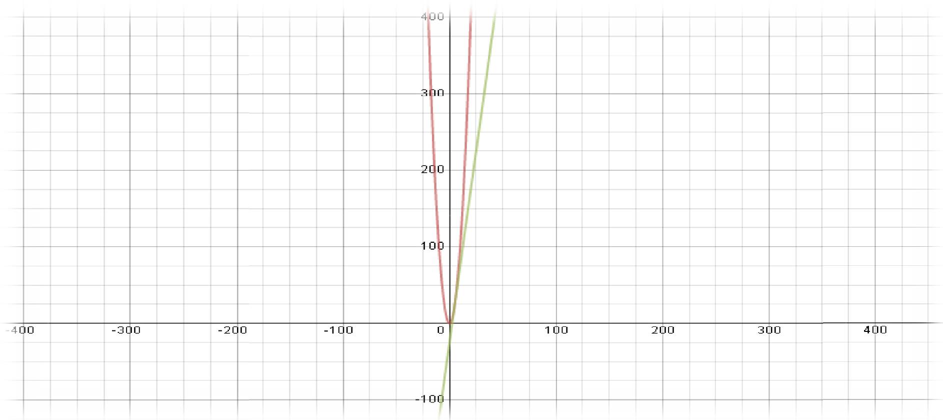
$$\boxed{\left(0 , \mathcal{F}(0) - \int_{f(0)}^{f(x)} f^{-1}(y)\, dy \right)}$$

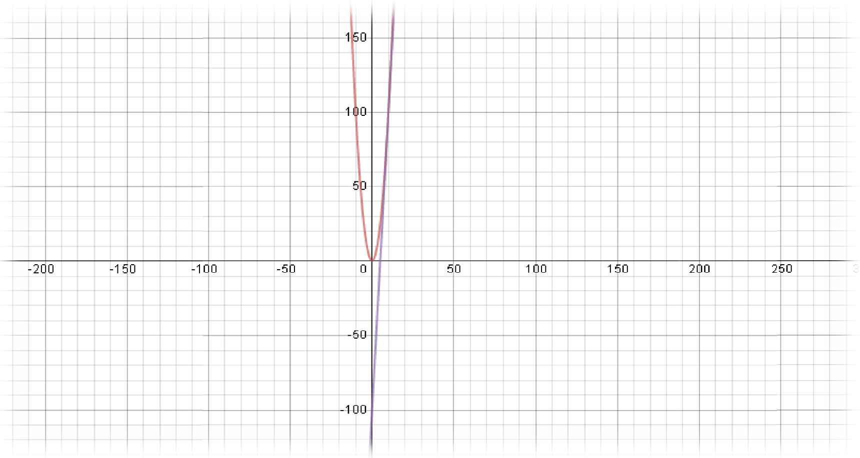
The proof

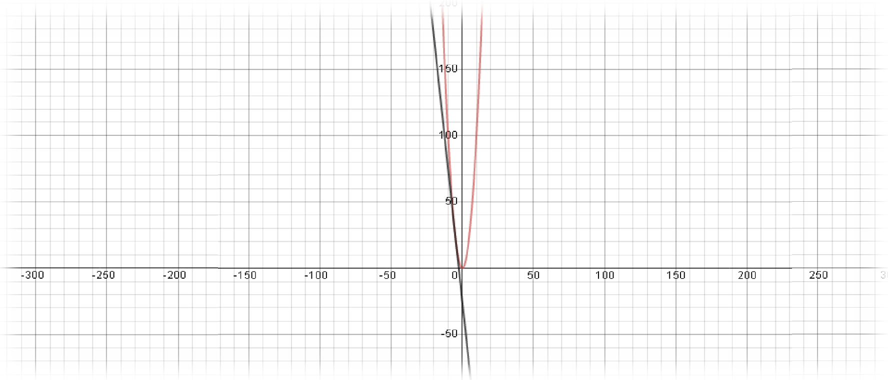
$$\frac{\mathcal{F}(x) - S(x)}{x - 0} = \mathcal{F}'(x)$$

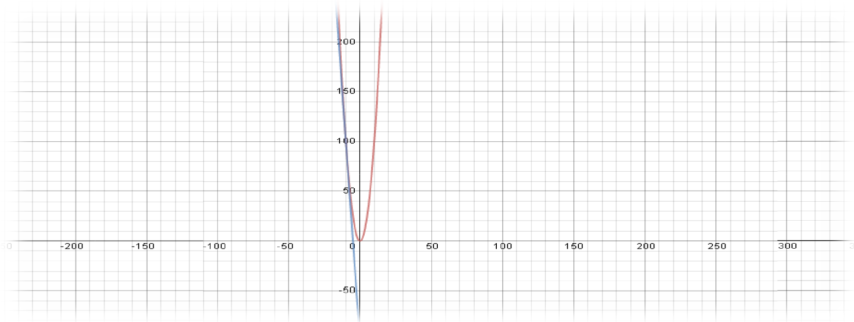
That is the slope between two points; the first is on the curve and the second lie on the y-axis, whereas the x-value is zero but the y-value follows specific function. when we find the slope between these two points, we will find it equal to the derivative of the function at that point if we specified the y-value of the second point correctly. we can find the y-value correctly from the graph of the function as the following:







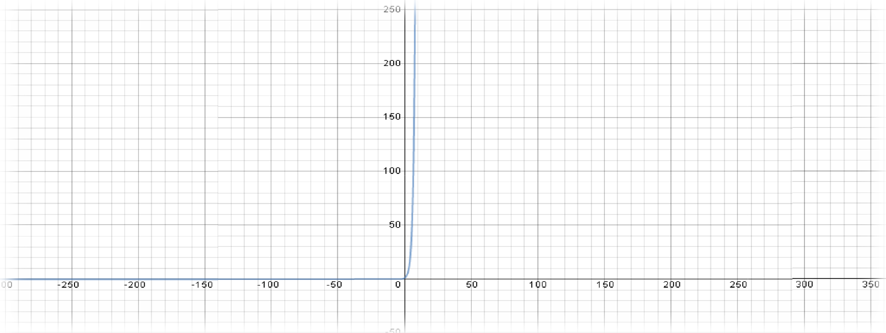


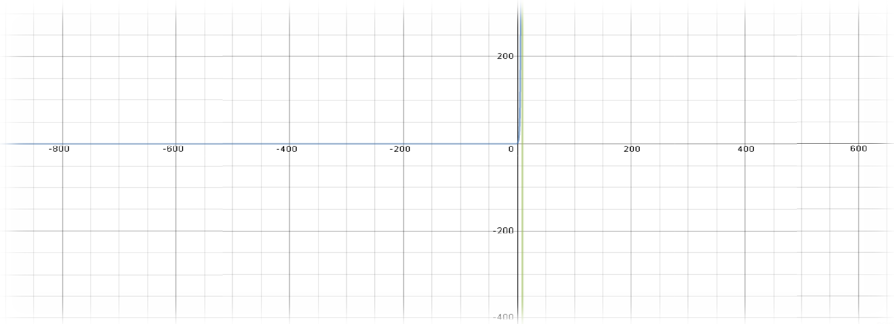


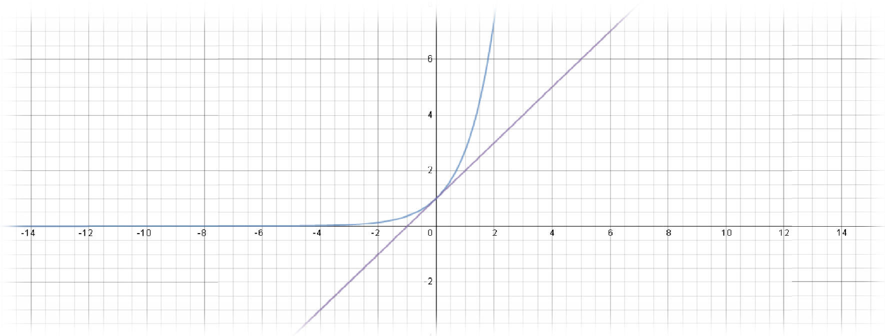
from the previous graphs, we can conclude the next equation for the polynomials:

$$\frac{\mathcal{F}(x) - (- \mathcal{A}\ (n - 1)\ x^{n})}{x - 0} = \mathcal{F}'(x)$$

-Another type of functions:







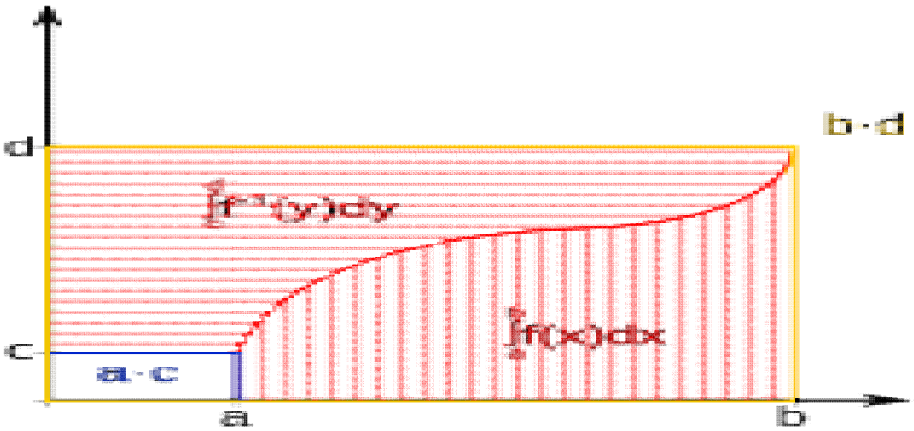
-The following one for the exponentials:

$$\frac{\mathcal{F}(x) - \mathcal{A}^{x}\,(1 - x - \ln(\mathcal{A}))}{x - 0} = \mathcal{F}'(x)$$

-the following proof is the second part of the theorem that relates the function from which we can find the y-value of the second point and the integration of the inverse of the derivative:

-The following equation relates the relation between the integral of a function and the integral of its inverse:

$$\boxed{\int_c^d f^{-1}(y)\,dy + \int_a^b f(x)\,dx = bd - ac.}$$



that graph represent the interpretation for the rule above.

We can change the previous equation into:

$$\int_{f(a)}^{f(b)} f^{-1}(y)\,dy + \int_a^b f(x)\,dx = f(b)\cdot b - f(a)\cdot a$$

$$\int_{f(a)}^{f(b)} f^{-1}(y)\,dy + \int_a^b f(x)\,dx = f(b)\cdot b$$

$$\int_{f(0)}^{f(x)} f^{-1}(y)\,dy + \int_0^x f(x)\,dx = f(x)\cdot x$$

$$f(x) = \mathcal{F}'(x)$$

$$\int_{f(0)}^{f(x)} f^{-1}(y)\,dy + \mathcal{F}(x) - \mathcal{F}(0) = f(x)\cdot x$$

$$\frac{\left(\int_{f(0)}^{f(x)} f^{-1}(y)\,dy - \mathcal{F}(0)\right) + \mathcal{F}(x)}{x} = f(x)$$

$$\frac{\mathcal{F}(x) - \left(\mathcal{F}(0) - \int_{f(0)}^{f(x)} f^{-1}(y)\,dy\right)}{x - 0} = \mathcal{F}'(x)$$

that is the slope between two points ;
the first is
$(x , \mathcal{F}(x))$
,but the second is

$$\left(0 , \mathcal{F}(0) - \int_{f(0)}^{f(x)} f^{-1}(y)\,dy\right)$$

From the previous result we can conclude two things:
1-We can find the derivative of any curve without the needing to take the limits.
2-the differential calculus and integral calculus is more connected and the the last equation is an good evidence for that.

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