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RESEARCH ARTICLE

IMPACTS OF BULK VISCOSITY ON RICCI DARK ENERGY IN A COUPLED MATTER–GEOMETRY *F* (*R, T*) MODIFIED GRAVITY MODEL

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ABSTRACT

In this paper, this paper, the Ricci dark energy (RDE) model associated with bulk viscosity is studied within the framework of f(R, T) gravity in order to describe the accelerated cosmic expansion of the Universe. Assuming that the cosmological evolution of the universe is governed by the RDE model which possesses bulk viscosity, we derive the modified Friedmann equations for a flat universe from f(R, T) gravity. For a specific form of $f(R, T) = (\zeta + \eta\tau T)R$, model and the bulk viscosity coefficient of the form $\xi = \xi_0 + \xi_1 H$, we derive some cosmological parameters, such as the Hubble parameter, the deceleration parameter, the effective equation of state (EoS), the statefinder parameters, and the Om diagnostic parameter. Our results show that for certain values of the free parameters, our model can be considered as an alternative candidate to describe dark energy Pacs numbers: 04.50.Kd, 95.36.+x, 98.80.-k

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INTRODUCTION

The observed accelerated expansion of the Universe is caused by a mysterious energy called dark energy (*DE*), which dominates all other components. Modified gravity is one of the methods to describe this accelerated expansion of the Universe. It is an approach that, while preserving the unquestionably positive results of Einstein's theory, aims to address both the conceptual and experimental aspects. Several other observational data have explained this expansion of the Universe. Examples of such observational data include the cosmic microwave background (*CMB*)(1), Type Ia supernova (*SNIa*) (2), Weak lens (3), Baryon Acoustic Oscillations (*BAO*) (4), and redshift surveys such as 2 *dF* Galaxy Redshift Survey (2*dFGRS*) (5) at low redshift and *DEEP*2redshift have confirmed the accelerated expansion of the Universe. The study of the redshift has shown that our Universe is almost spatially flat, homogeneous, isotropic on a large scale and is developing at an accelerated pace. This accelerated expansion is created from a 'emyst'erieuse energy called dark energy (*DE*), which is the dominant component and is one of the most important challenges in contemporary physics. To describe this dark energy, several models have been proposed: cosmological constant(8), quintessence (9), phantom (10), quintom (11), quintom generalized *DE* (12), tachyon (13), K-petrol and various models of Chaplygin gas (14). Among the above candidates, a new form called 'holographic Dark Energy (HDE)" model 'ele (15)-(18) has built in the context of quantum gravity. It is based on the holographic principle, a principle well known for studying the quantum behavior of black holes. The energy density of *HDE* is defined as follows: $\rho = 3c2Mpl\ 2\ L\text{-}2$, where c is a constant, $M\text{-}2\ pl = 8\pi G$ is Planck's mass and L is supposed to be the size of the Universe. Referring to the holographic principle, the authors of the reference (**?**) introduced a new form of (*DE*) which was inversely proportional to the Ricci scalar namely " Ricci Dark 'edark energy (RDE) ". They have come up with results that show that this model *RDE* r'esolves the problem of causality and the 'eevolution of density perturbations of the power spectra of matter and the anisotropy *CMB* is not much affected by such a modification. The density of such a model is defined in a spatially flat universe by $\rho d = 3\alpha(dotH + 2H2) = (\alpha/2)R$, where we have defined $8\pi G = 1$, R, the Ricci scalar of spacetime, and α, the dimensionless parameter that will be used to determine the behavior of evolution of RDE. Several theoretical models such as $f(R)$, $f(G)$, $f(R, G)$, $f(R, T)$ ..., o'u R, is the Ricci Scalar, T , the trace of the 'eenergy-moment tensor and G, the Gauss-Bonnet invariant have succeeded in describing the cosmic accelerated expansion of the universe. These theories not only correctly the cosmic accelerated expansion of the universe and also respect the principles of Albert Einstein's General Relativity. Cosmological observations in recent years prove that the universe is filled with an imperfect fluid according to which the pressure is negative as has been argued, an effective pressure including the viscosity in bulk can play the role of an agent that drives the current acceleration of the Universe (19). In one of the works of our thesis defended in October 2023, we studied the constraints on some cosmological parameters of observation from the viscous model coupled to the gravity $f(R, T)$. In this work, we have used a specific model of $f(R, T)$ defined as $f(R, T) = R + 2\lambda T$ where λ is an arbitrary constant with a given form of the viscosity coefficient. Dans cet article, nous avons ´etudi´e le mˆeme ph´enom`ene mais avec une autre forme de mod`ele d´efini comme $f(R, T) = (\zeta + \eta\tau T)R$ avec ζ, η et τ des constantes. Pranjal Sarmah *et al*. used this form of $f(R, T)$ in 2024 to study the inflation of the universe. They obtained results consistent with observational data. We found it useful to use this same model as these authors combined with the Ricci dark energy (*RDE*). model associated with viscosity to describe the accelerated cosmological expansion of the Universe.

Preliminary in the modified theory of gravity $f(R, T)$

The total action S of the gravitational field in $f(R, T)$ gravity theory is defined by

$$S = \frac{1}{2\kappa^2}\int_\Omega f(R,T)\sqrt{-g}dx^4 + \int_\Omega \pounds_m \sqrt{-g}dx^4, \tag{1}$$

Using the principle of least action, the variation of this action S by the metric *gmunu* gives us the field equation defined as

$$f_R R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}f(R,T) + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f_R = \kappa^2 T_{\mu\nu} + (T_{\mu\nu} + pg_{\mu\nu})f_T. \tag{2}$$

We can rewrite this field equation in the form:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa^2_{eff}T^{eff}_{\mu\nu}, \tag{3}$$

Where

$$\kappa^2_{eff} = \frac{\kappa^2 + f_T}{f_R}, \tag{4}$$

is the effective gravitational matter depending on the coupling in $f(R, T)$ gravity and

$$T^{eff}_{\mu\nu} = \left[T_{\mu\nu} + \frac{1}{\kappa^2 + f_T}\left(\frac{1}{2}g_{\mu\nu}(f - Rf_R) + f_T p_m g_{\mu\nu} - (g_{\mu\nu} - \nabla_\mu\nabla_\nu)f_R\right)\right], \tag{5}$$

represents the tensor energy impulse of the actual matter era. In *RG* where $f(R, T) = R$, the Eq. (4) and Eq. (5) are reduced 'to $\kappa 2$ *eff* = $\kappa 2$, $T\mu\nu$ *eff* = $T\mu\nu$ and therefore Eq. (3) leads to Einstein's field equations. In addition, we are interested here in the study of the behavior of the Universe for a spatially flat space-time *F LRW* whose line element is expressed by

$$ds^2 = dt^2 - a(t)^2[dx^2 + dy^2 + dz^2], \tag{6}$$

and the Ricci scalar in this context as

$$R = -6(2H^2 + \dot{H}). \tag{7}$$

The Hubble parameter H = *fracdotaa* and the components (0 - 0) and (t - t) of the Eq field. (3) give

$$\rho + \rho_{DE} = \rho_{eff}, \tag{8}$$

$$\bar{p} + p_{DE} = p_{eff}\,. \tag{9}$$

In this way, we have a fluid representation of the matter of the so-called geometric dark energy in the $f(R, T)$ gravity theory, with the energy density and pressure expressed as

$$\rho_{DE} = \frac{1}{f_R}\left[\frac{1}{2}(f - Rf_R) - 3\dot{R}Hf_{RR} + pf_T\right], \tag{10}$$

$$p_{DE} = \frac{1}{f_R}\left[2H\dot{R}f_{RR} + \ddot{R}f_{RR} + \dot{R}^2 f_{RRR} - \frac{1}{2}(f - Rf_R) - pf_T\right] \tag{11}$$

In the expressions above, the dot represents the derivative with respect to cosmic time. Assuming that cosmological evolution is governed by the *f*(*R, T*) theory in which matter and geometry are coupled with the action, we write the *FRW* equations in terms of *H* as:

$$H^2 = \frac{1}{3f_R}\left[(\kappa^2 + f_T)\rho + \frac{1}{2}(f - Rf_R) - 3\dot{R}Hf_{RR} + pf_T\right], \tag{12}$$

$$\dot{H} = -\frac{1}{2f_R}\left[(\kappa^2 + f_T)(\rho + \bar{p}) + \ddot{R}f_{RR} + \dot{R}^2 f_{RRR} - \dot{R}Hf_{RR}\right]. \tag{13}$$

Furthermore, we can calculate the effective equation of state parameter defined as

$$\omega_{eff} = -1 - \frac{2\dot{H}}{3H^2} \tag{14}$$

as

$$\omega_{eff} = -1 + \frac{2}{3f_R H^2}\left[(\kappa^2 + f_T)(\rho + \bar{p}) + \ddot{R}f_{RR} + \dot{R}^2 f_{RRR} - \dot{R}Hf_{RR}\right]. \tag{15}$$

Note that Eq. (ef14) has exactly the same form as in RG with matter. The difference is expressed through additional gravitational terms due to the modification of *RG*. From the conservation law, the effective energy density evolves as

$$\frac{d(\kappa^2_{eff}\rho_{eff})}{dt} + 3H\kappa^2_{eff}(\rho_{eff} + p_{eff}) = 0. \tag{16}$$

From Eq. (12) and Eq. (13), Eq. (16) can be written as follows:

$$18\frac{f_{RR}}{f_R}H(\ddot{H} + 4H\dot{H}) + 3(\dot{H} + H^2) + \frac{\kappa^2 + f_T}{f_R}\rho + \bar{p}\frac{f_T}{f_R} + \frac{f}{2f_R} = 0. \tag{17}$$

Note here that ρ and ¯ p are the energy density and the effective pressure of the matter content in the Univer and u μ, the four-speed vector with normalization condition $u\mu uv = 1$. The effective pressure of the material is given by ¯ $p = pm + pd - 3H\xi$, o`u ξ is the bulk viscosity coefficient, $H = aa^{\cdot}$ is the Hubble parameter and $\rho = \rho m + \rho d$. In its various parameters, ρm and pm represent the energy density and pressure of ordinary matter, while ρd and pd denote respectively the density of energy and the pressure of Ricci dark energy (*RDE*) with $pd = wd.pd$.

Expressions of the modified gravity model *f*(*R, T*) with the viscosity coefficient

We define in this work $f(R, T) = f1(R) + f2(R)f3(T) = (\zeta + \eta\tau T)R$ where $f1(R) = \zeta R$; $f2(R) = \tau R$ and $f3(R) = \eta T$ with ζ; η and τ are constants.

When we substitute this expression into equation, (17) we find

$$3(\dot{H} + H^2) + \frac{1 + \eta\tau T}{\zeta + \eta\tau T}\rho + \frac{\eta\tau T}{\zeta + \eta\tau T}\bar{p} + \frac{\zeta R + \eta\tau RT}{2(\zeta + \eta\tau)} = 0 \tag{18}$$

Considering that the contribution in energy and pressure of ordinary matter is negligible compared to that of the other substances contained in the Universe, $T = \rho = \frac{2\pi}{3}$ and assuming that $pd = wd.\rho d$, o`u wd is the state equation parameter of (*RDE*), we obtain after defining the viscosity coefficient ξ as

$$\xi = \xi_0 + \xi_1\left(\frac{\dot{a}}{a}\right) = \xi_0 + \xi_1 H, \tag{19}$$

where ξ_0 and ξ_1 are constant parameters

$$\dot{H} + \frac{3\zeta}{2\eta\tau\alpha w_d}H^3 + \frac{1+2\alpha\omega_d+\xi_1}{\alpha w_d}H^2 + \frac{\xi_O}{\alpha w_d}H - \frac{2}{9\alpha w_d}H^{-1} - \frac{1}{3\eta\tau\alpha w_d} = 0 \quad (20)$$

The analytical solution of (20) is difficult, and so we move directly to its numerical solution in order to achieve our goal.

Numerical results of some cosmological parameters: The first two figures drawn in this work are the Hubble parameter as a function of cosmic time and the redshift. It is a para meter that measures the rate at which the Universe is expanding at a given moment.

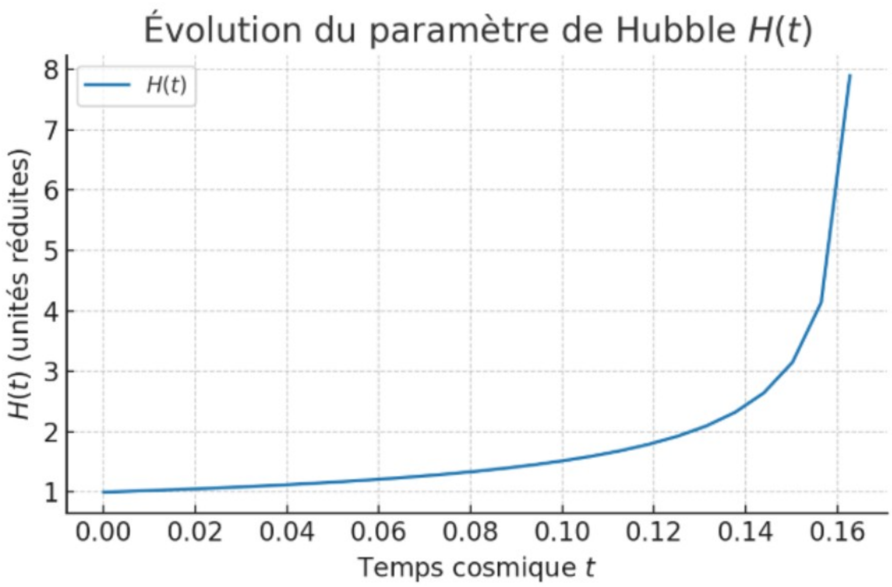

Figure 1. Evolution of the Hubble parameter as a function of cosmic time

We note here that when cosmic time is greater than zero, H increases. This reflects an increasingly rapid cosmic acceleration. Viscosity here acts as a dissipative term equivalent to an additional negative pressure, enhancing the acceleration and leading to potential phantom acceleration. In short, the universe is expanding at an accelerated rate thanks to the dissipative effect of viscosity.

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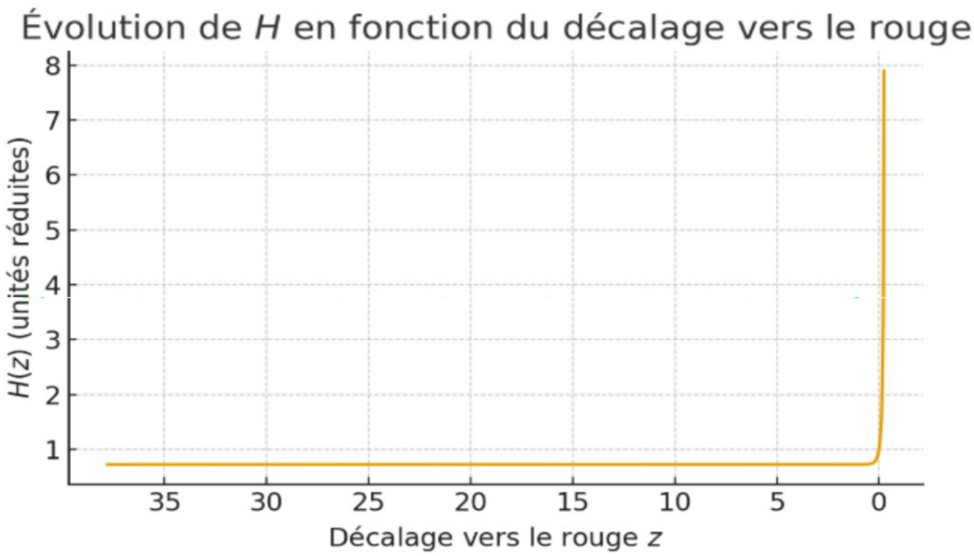

Figure 2. Evolution of H as a function of redshift

We note here that when we decrease z, $H(z)$ decreases but remains greater than in the standard ΛCDM model. We conclude here that the combination of viscosity and matter-geometry coupling intensifies the expansion rate compared to standard cosmology. The following two figures show the behavior of the deceleration parameter q as a function of cosmic time t and as a function of redshift z. It is a fundamental indicator of the dynamics of the Universe's expansion. It allows us to determine whether the Universe is slowing down, accelerating, or transitioning between these two regimes.

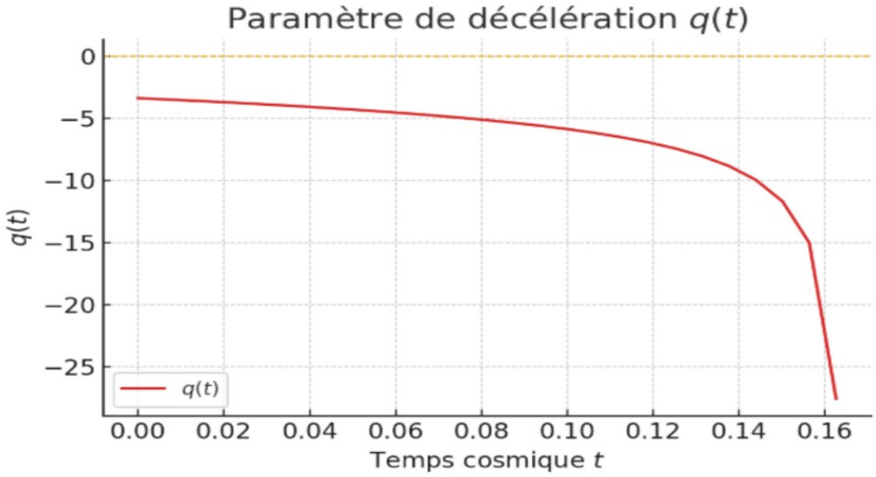

Figure 3. Evolution of the deceleration parameter as a function of cosmic time

We observe in this figure a transition phase from $q > 0$ to $q < 0$, that is, from the decelerated regime (dominated by matter) to the accelerated regime (dominated by dark energy). Conclusion: the model reproduces the observed transition between a matter-dominated universe and a dark energy-dominated universe.

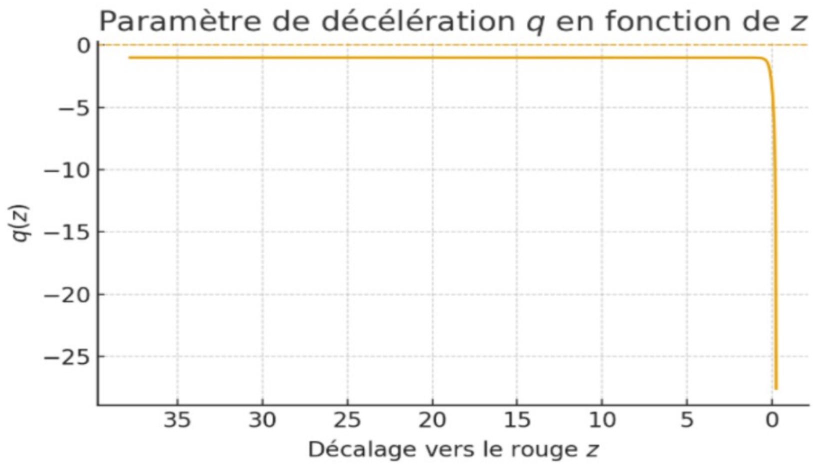

Figure 4. Evolution of the deceleration parameter as a function of redshift

Both phases are observed here: when z is high, $q > 0$ (deceleration in the past); when z is low, $q < 0$ (current acceleration). The point where q=0 gives the transition redshift z between the two phases. This value is generally close toz q = 0.5 - 0.8, consistent with observational data. The two figures below show the evolution of the state parameter as a function of cosmic time and redshift. This is a parameter that links the pressure p of a cosmological component to its energy density ρ. It also informs us about the nature of each form of energy in the universe, its influence on cosmic expansion, and on the dynamic evolution of the scale factor a(t).

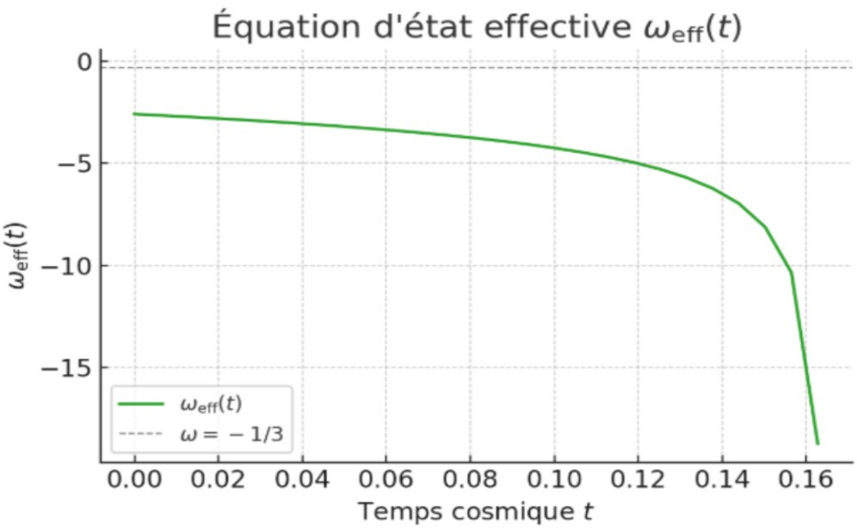

Figure 5. Evolution of the effective state parameter as a function of cosmic time

The curve drops toward or below -1. Now, when *ωeff* < -1, we are in the phase of a phantom-type acceleration model of the universe, which is caused here by viscosity. Conclusion: viscosity stimulates negative pressure, pushing the model beyond vacuum energy.

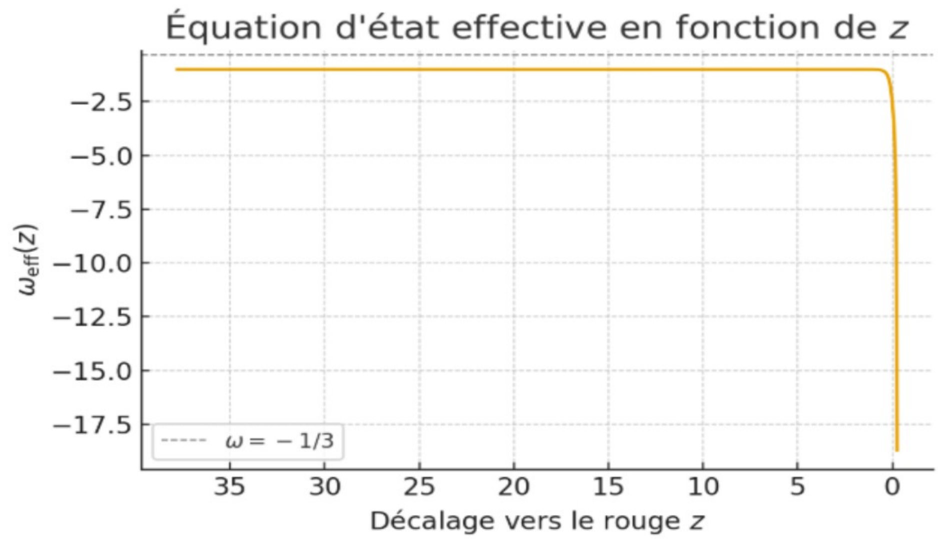

Figure 6. Evolution of the effective state parameter as a function of redshift

In the past (high *z*), *ωeff* is close to 0 (matter domination). At low *z*,*ωeff* becomes less than -1 (phantom acceleration). Clear transition between cosmological epochs: matter →dark energy →phantom

Param`etre de Statefinder: There are an increasing number of models to explain the accelerated cosmic expansion of the Universe, and in order to discriminate between these different contenders, the statefinder parameters as well as future *SNAP* observations can be used to distinguish between the different dark energy models.

These parameters were introduced in 2003 by V. Sahni and al. (20) and are defined as follows: $r = \frac{\dddot{a}}{aH^3}, \ s = \frac{r-1}{3(q-\frac{1}{2})}$ *r* =is the Hubble parameter and *q* is the deceleration parameter. The two parameters *r, s* are dimensionless and geometric since they are derivatives of the scale factor *a*(*t*) alone, although they can be rewritten in terms of dark energy and dark matter parameters. For different models of dark energy, the trajectories in the *r* - *s* plane can exhibit different behaviors. Numerically, we have

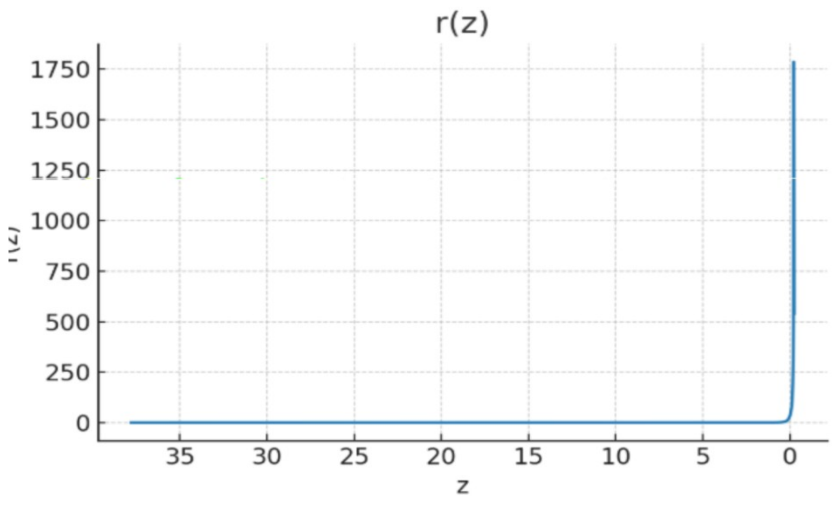

Figure 7. Evolution of r as a function of the red shift0

- *r* is a third-order derivative of the scale factor, sensitive to the dynamics of expansion.
- The variations of *r* with respect to *z* make it possible to distinguish the studied model from Λ*CDM* (where *r* = 1).

The *f*(*R, T*) model chosen in this work, in addition to viscosity, shows notable deviations, indicating a non-standard behavior of dark energy

- *s* evaluates the difference between the actual dynamics and those of the Λ*CDM* model
- The observed variations confirm an evolutionary (not static) scenario for dark energy
- The fixed point(*r* = 1; *s* = 0) corresponds to the standard Λ*CDM* model
- Note that here the trajectory of our model deviates significantly from Λ*CDM*, which means that viscosity and matter-geometry coupling produce a dynamics different from standard cosmology. This illustrates the model's ability to mimic or surpass Λ*CDM* depending on the values of the parameters

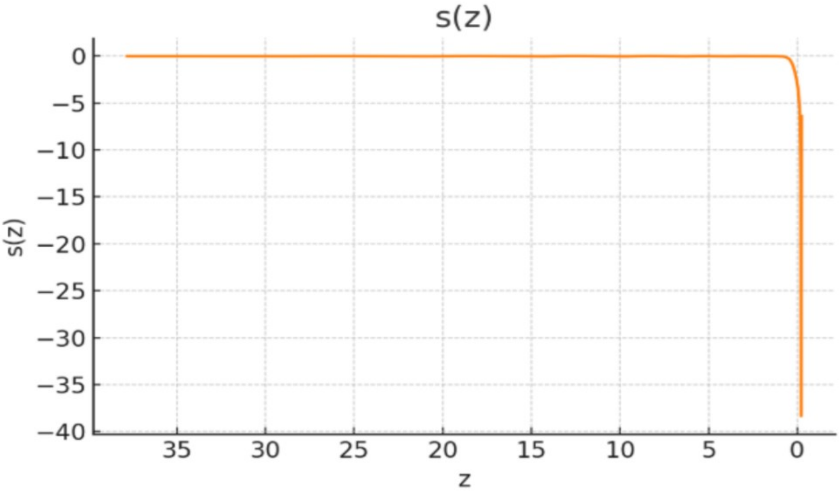

Figure 8. Evolution of s as a function of the redshift

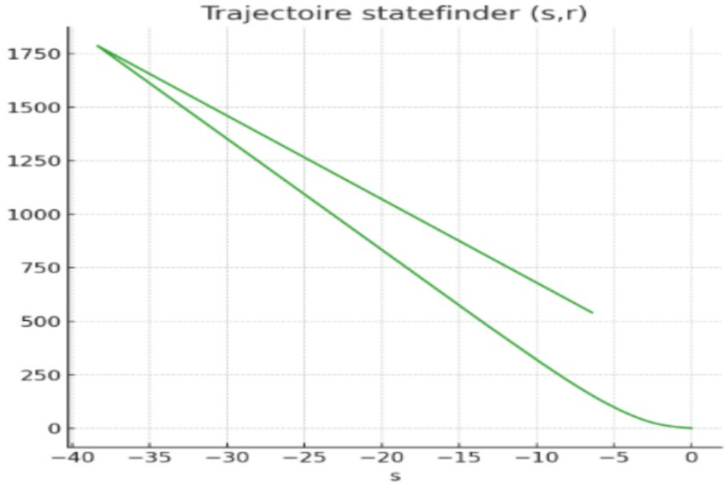

Figure 9. Evolution of r as a function of s

***Om* diagnostics**

Le param`etre *Om* est un autre outil de diagnostic important propos´e par les auteurs (20)-(21). Ce mod`ele distingue efficacement les mod`eles d'´energie sombre qui d´ependent moins de la densit´e de mati`ere $\Omega m0$ du mod`ele standard $\Lambda CDM$. Il est d´eriv´e du param`etre de Hubble et du param`etre cosmologique de d´ecalage vers le rouge et est d´efini par:

$$O_m(z) = \frac{(\frac{H(z)}{H_0})^2 - 1}{(1+z)^3 - 1}. \tag{21}$$

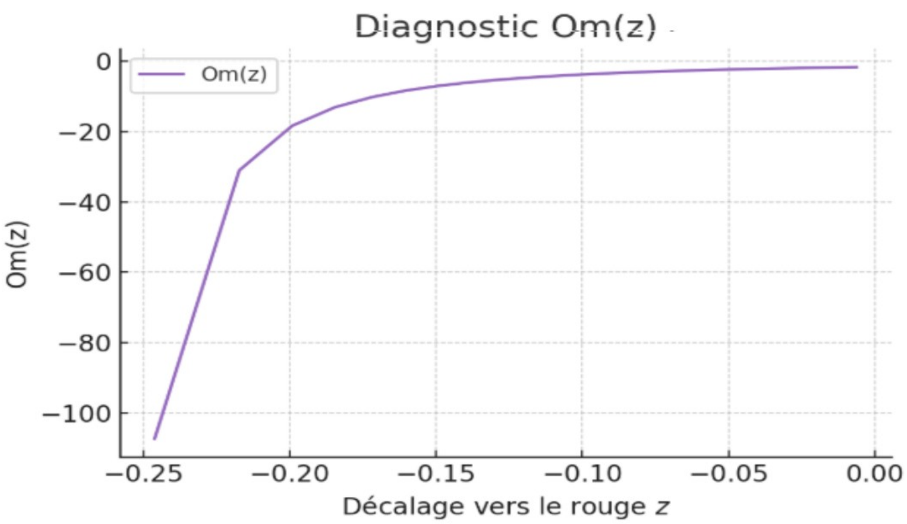

Figure 10. Evolution of the diagnosis as a function of redshift

- We observe that $Om(m)$ varies with z, which confirms that the dark energy equation of state evolves over time.
- Conclusion: the *RDE* and viscosity model in $f(R, T)$ reproduces dynamic dark energy capable of explaining both past expansion and current acceleration.

CONCLUSION

In this work, we have investigated a viscous Ricci dark energy (RDE) model within the framework of modified $f(R, T)$ gravity, considering the specific functional form

$$f(R, T) = (\zeta + \eta\tau T)R. \quad (22)$$

The combined effects of matter–geometry coupling and bulk viscosity provide a consistent and extended theoretical framework to describe the late-time accelerated expansion of the Universe. The numerical analysis of the main cosmological parameters shows that the Hubble parameter confirms a persistent accelerated expansion, while the deceleration parameter clearly exhibits the transition from a decelerated phase in the past to the current accelerated regime. The effective equation of state parameter crosses the phantom divide line $\omega = -1$, indicating the emergence of a phantom-like behavior driven mainly by dissipative bulk viscosity effects. Furthermore, the Statefinder and Om diagnostics reveal significant deviations from the standard ΛCDM model, emphasizing the dynamical nature of dark energy in the present scenario. These results demonstrate that the proposed model is able to reproduce the observed cosmic evolution and offers a viable alternative to the standard cosmological model.
In conclusion, the viscous RDE model in $f(R, T)$ gravity provides a robust framework for describing dynamical dark energy and late-time cosmic acceleration, in good agreement with current observational constraints.

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