



RESEARCH ARTICLE

MONITORING LINEAR PROFILES USING MULTIVARIATE CONTROL  
CHART IN PRESENCE OF OUTLIERS

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ABSTRACT

The method ordinary least square is widely used for parameter estimation in regression, if the outliers are present in data then then OLS provides insignificant results and it is not robust to outlier. To overcome this problem, many researchers have proposed several alternative parameter estimation methods based on m-estimation in regression. The purpose of this study is, to compare the performance of multivariate control chart based on various non robust as well as robust parameter estimation procedures. The performance of these control charts are evaluated to simulation study

Key Words:

Statistical Process Control, ARL,  
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INTRODUCTION

Statistical quality control tools help the quality managers to improve the quality of the products by reducing the process variation. In 1924 Shewhart developed the first control chart, which is the most important quality control tools of Statistical process monitoring. This is usually used for monitoring the process location or scale or both. One of the main purposes of the control charts is to categories between special (assignable and non special (chance) causes of variation. If the non special cause of variation are presence in the process then the process is said to be under statistical control. Otherwise, the process is said to be out of statistical control and if the process is out of control then control chart play an vital role to detect and eliminate the special cause of variation Consequently, return the process to the in-control state .So control charts are effective statistical process control tools and are widely used in controlling the quality of processes or products in many manufacturing and service industries. In certain practical scenario, the quality of a product is characterized by the relationship between a response variable and one or more explanatory variables.

Constructing control charts for detecting whether or not this relationship has changed over time is known as profile monitoring. It was recently developed in quality control and is becoming more popular. Profile monitoring refers to statistical process monitoring of manufacturing process or product that its measurements are summarized by Profiles or curves. In the literature of profile monitoring is the functional relationship is usually referred to as profile. The relationship, can be linear, nonlinear, or even a complicated model, is referred to as a profile. So far, several methods have been proposed for monitoring simple linear profiles. In detail about the Profile monitoring see Woodall *et al.* (2004), Woodall (2007) and Noorossana *et al.* (2011). In applications of control charting, it is useful to distinguish between Phase I and Phase II methods and applications. In Phase I, a control chart is applied on a set of historical data to detect whether a process has been under statistical control or not. The Primary goal in Phase I are to understand the variation in a process and evaluate the process stability after dealing with assignable causes, to estimate the in-control values of the parameters of control chart. However, the main goal of Phase II is to quickly detect the parameter changes from in-control value. The most common type of profile is a simple linear. Simple linear profile monitoring was first investigated by Kang and Albin (2000)

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in which they studied two examples of different situations for which product profiles are of interest. The first example involved aspartame (an artificial sweetener), which is characterized by the amount of dissolved aspartame per liter of water at different levels of temperature. In this case, there is a desired functional relationship between the amount of dissolved aspartame and the temperature. The second example is a semiconductor manufacturing application involving calibration of a mass flow controller in which the performance of the process is characterized by a linear function. Mestek *et al.* (1994) and Stover and Brill (1998) gave similar calibration applications. Kang and Albin (2000) proposed Phase I and Phase II control chart methods for monitoring a process for which the quality of a product is characterized by a linear relationship. Stover and Brill (1998) also considered the Phase I problem, while Brill (2001) considered possible extensions to more general relationships than a linear one. The monitoring of linear profiles is very closely related to the control charting of regression-adjusted variables as proposed by Mandel (1969), Zhang (1992), Hawkins (1991, 1993), Wade and Woodall (1993), and Hauck *et al.* (1999). In these approaches a regression model is often used to account for the effect of an input variable X on the output variable Y when monitoring a particular stage of a manufacturing process. EWMA/R and Hotelling's  $T^2$  control charts were proposed by Kang and Albin (2000) to monitor different parameters. Mahmoud and Woodall (2004) proposed a scheme for multiple linear regression in phase I structure. Walker and Wright (2002) used additive models to represent the curves of interest in the monitoring of density profiles of particleboard. Jin and Shi (2001) used wavelets to monitor "waveform signals" for diagnosis of process faults. The use of linear functions as responses in designed experiments has also been studied recently. See, for example, Miller (2002) and Nair *et al.* (2002). Noorossana *et al.* (2004) introduced a multivariate cumulative sum (MCUSUM) control chart and R chart. The non-normality in the simple linear profiles is discussed by Noorossana *et al.* (2004) Zou *et al.* Mahmoud *et al.* (2007) and Yeh and Zerehsaz, 2003 that used the control charts based on change point model for monitoring the simple linear profiles. The method proposed by Kim *et al.*, (2003) Croarkin and Varner (1982) based on Shewhart method was comparatively studied by Gupta *et al.* (2007) Integrated MCUSUM and chi-square control charts are introduced by Noorossana and Amiri (2007). Further Woodall (2007) provided a comprehensive review on profile monitoring. For recursive residuals, control chart was proposed by Zou *et al.* (2007) while control chart for mixed model in linear profiles was developed by Jensen *et al.* (2008). Problem of within autocorrelation in the model of Jensen *et al.* (2008) was eliminated by Soleimani *et al.* (2009) Moreover, likelihood ratio-based control chart for monitoring the simple linear profile was proposed by Zhang *et al.* (2009) Saghaei *et al.* (2009) designed an approach based on CUSUM chart while Mahmoud *et al.* (2010) proposed the study for monitoring simple linear profiles when sample size cannot be greater than one or two. Many practical applications of profile monitoring in industry are introduced by the many researchers, Stover and Brill (1998), Kim *et al.* (2003), Mestek *et al.* (2004) Mahmoud and Woodall (2004), Mahmoud *et al.* (2007), Zou *et al.* (2007), Zhang J *et al.* (2009), Zhu and Lin (2010), Amiri *et al.* (2010) Also they have been proposed some methods which are investigated in phase I for monitoring simple linear profiles. Mestek *et al.* (1994) used a  $T^2$  control chart in combination with principal component analysis (PCA) approach to monitor a simple linear profile in calibration application. Stover and Brill (1998) studied multivariate  $T^2$  control chart and PCA-based control scheme for monitoring simple linear profiles. Also Kang and Albin (2000) proposed two methods such as  $T^2$  and EWMA/R for monitoring simple linear profiles. Mahmoud and Woodall, (2004) suggested the use a global F-test to monitor the regression coefficients in conjunction with a univariate control chart for monitoring error standard deviation in Phase I. Mahmoud *et al.* (2007) proposed a method to monitor linear profiles based on a likelihood ratio statistic. Zhu and Lin (2010) proposed a shewhart-type control chart for monitoring slopes of linear profiles in both Phases I and II. Models such as multiple linear profiles and polynomial profiles are investigated by authors such as Zou *et al.* (2007), Kazemzadeh *et al.* (2008, 2009), and Mahmoud (2008). More complicated models such as nonlinear profiles and multivariate linear profiles are studied by Williams *et al.* (2007a,b), Vaghefi *et al.* (2009), Noorossana *et al.* (2010a,b), and Eyvazian *et al.* (2011). Zhang *et al.* (2012) proposed nonparametric control chart scheme based on profile error. The general approach for monitoring nonlinear profile was given by Chuang *et al.* (2013). Zhang *et al.* (2014) proposed shewhart type control chart to detect shape changes from linear profile to quadratic profile. More recently Noorossana *et al.* (2015) introduced method for monitoring linear profiles under random effect. And the first control chart based on student t statistic was developed by Yang Zhang *et al.* (2017) using supplementary run rules to detect prespecified changes in linear profile. In the proposed paper our aim is to study of the influence of various M estimation methods for monitoring linear profiles using Hotelling's  $T^2$  Control Chart in presence of outliers

**Simple Linear Regression:** The Simple linear regression model is represented by the following equation (Montgomery, *et al.*, 2001)

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i \quad i = 1, 2, \dots, n \quad (1)$$

Where the intercept  $\beta_0$  and slope  $\beta_1$  are unknown constants and the term  $\epsilon$  is called random error components. In additionally we assume that errors are uncorrelated. According to Neter *et al.* (2005) equation (1) is said to be simple, as it represent the relationship between a quality characteristic and control variable is linear in the parameter because none of the parameter appears as superscript or is being multiplied or divided by other parameters. The error are assumed to be normal and independently distributed with mean zero and unknown variance  $\sigma^2$ . This assumption is important for estimating regression line and must checked later to validate the estimated model (Montgomery *et al.* 2001, Neter *et al.* 2005). Montgomery *et al.* (2001) argue that the variable x should be seen as a control variable and measured with negligible error. While the variable y is seen as a random variable. Thus there is a probability distribution for y every real value of x is normal with mean is

$$E(y/x) = \beta_0 + \beta_1 x \quad (2)$$

and the variance is

$$V(y/x) = (\beta_0 + \beta_1 x) = \sigma^2 \quad (3)$$

and the  $X_i$ 's are independent random variables with probability distribution not containing  $\beta_0, \beta_1$  and  $\sigma^2$ . Form the sample  $j$  may obtained the estimators of the profile parameters  $\beta_0, \beta_1$  by using the ordinary least square(OLS) method (e.g.see Draper and Smith[2])

$$\beta_1 = \frac{S_x}{S_x} \text{ and } \beta_0 = \bar{y} - \beta_1 \bar{x}$$

Where  $\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$ ;  $\bar{y} = \frac{\sum_{i=1}^n y_i}{n}$ ;  $S_x = \sum_{i=1}^n x_i y_i - n\bar{x}\bar{y}$  and

$$S_x = \sum_{i=1}^n (x_i - \bar{x})^2 = \sum_{i=1}^n x_i^2 - n\bar{x}^2$$

Also there is the assumption that the sample statistics  $\beta_0$  and  $\beta_1$  are normally distributed with mean  $\beta_0$  and  $\beta_1$  and variances  $\sigma_0^2$  and  $\sigma_1^2$  where

$$\sigma_0^2 = \sigma^2 \left( \frac{1}{n} + \frac{\bar{x}^2}{S_x} \right); \sigma_1^2 = \frac{\sigma^2}{S_x}$$

and the mean squared error is an unbiased estimator of  $\sigma^2$  may be given as

$$MSE = \frac{\sum_{i=1}^n r_i^2}{n-2} \tag{4}$$

Where  $r_i = y_i - \hat{y}_i$  is the  $i^{th}$  regression residual and  $\hat{y}_i$  is the  $i^{th}$  fitted regression line.

**Robust Regression:** An important statistical tool routinely applied in most sciences is regression analysis. And almost all regression analyses the method of least squares is used for the estimation of the unknown regression parameters. But this method work well in presence of some assumptions such as normality of the error distribution. When outliers are present in the data, the estimates of the parameters providing by this method are not resistant. Hence Robust regression is an important tool have been developed as an improvement to least square estimation to provide resistant results in the presence of outliers. Therefore It can be used to detect outliers and to provide resistant (stable) results in the presence of outliers. Many methods have been developed for these problems. Many researchers have worked in this field and described the methods of robust estimators. The class of robust estimators includes Huber (P.J.Huber in 1964,1973), Andrews (Andrews *et al.* 1972), Hampel (Hampel *et al.* 1986), Tukey (Beaten and Tukey 1974),logistic and Fair (Fair, RC 1974), Talwar (Hinich, M. J. and Talwar, P. P. 1975), Welsch (Dennis, J. E. and Welsch, R. E. ,1976), Qadir Beta function (Qadir, M. F. 1996), Insha (InshaUllaha *et al.* 2006), , GGW (Koller and Stahel (2011), Cauchy, Ramasy (Ramsey J.O. ,1977), Theilsen (Theil H 1950, Sprent P 1993 ), Mallows (Mallows ,1975) ,Bell ( Bell,R,M. 1980). In this paper we consider M-estimation based on Huber,GGW,Mallows and Hample estimators and its comparision is made with OLS estimator

**The T<sup>2</sup> Control chart:** Assume that the  $j$  th random sample collected over time is  $(x_i, y_i), i = 1, 2, \dots, n_j$ . When the process is in statistical control, then the relationship between the response variable and the predictor variable is:

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i = 1, 2, \dots, n \tag{5}$$

Where  $\epsilon_i$  is an independent identically distributed (i.i.d) as a standard normal random variable. When the parameters  $\beta_0, \beta_1$  and  $\sigma^2$  are unknown, they can be estimated from Phase I sample data, but in Phase II we assume they are known. The least squares estimates for  $\beta_0$  and  $\beta_1$  for sample  $j$  are

$$\beta_0 = \bar{y}_j - \beta_{1j} \bar{x} \text{ and } \beta_{1j} = \frac{S_{X(j)}}{S_x} \tag{6}$$

Where  $\bar{y}_j = n^{-1} \sum_{i=1}^n y_{ij}$  ;  $\bar{x}_i = n^{-1} \sum_{i=1}^n x_i$

$$S_{X(j)} = \sum_{i=1}^n y_{ij} (x_i - \bar{x}) \text{ and } S_x = \sum_{i=1}^n (x_i - \bar{x})^2$$

The sample statistics  $\beta_{0j}$  and  $\beta_{1j}$  are normally distributed with means  $\beta_0$  and  $\beta_1$  and Variances

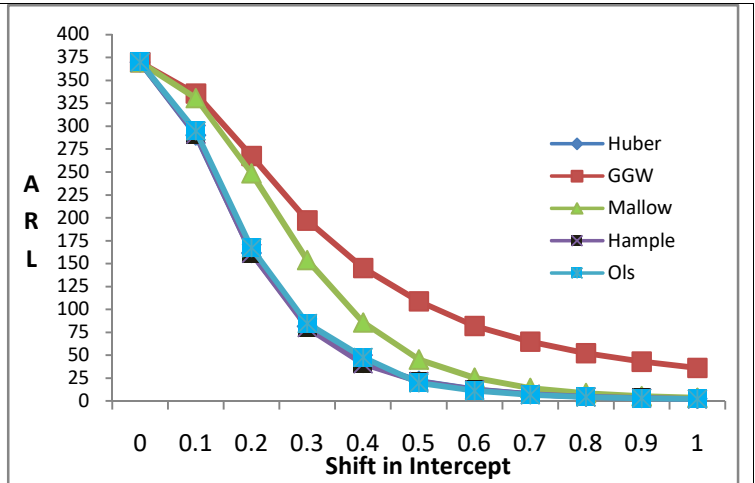
$$\sigma_0^2 = \sigma^2 (n^{-1} + \bar{x}^2 S_x^{-1}) \text{ and } \sigma_1^2 = \sigma^2 S_x^{-1} \tag{7}$$

respectively. Furthermore,  $\beta_{0j}$  and  $\beta_{1j}$  are dependent with covariance

$$\sigma_0^2 = \sigma^2 \bar{X} S_x^{-1} \tag{8}$$

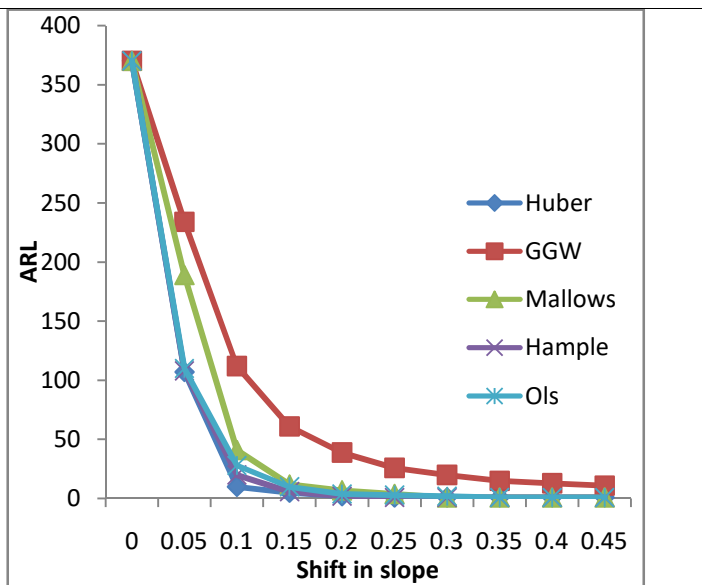
**Table.1 ARL values of T2 control chart for different methods under shift in intercept  $\beta_0$**

Shift d1	Huber	GGW	Mallow	Hample	Ols
0	370	370	370	370	370
0.1	290	336	331	291	295
0.2	162	267	249	161	167
0.3	81	197	154	80	85
0.4	42	145	86	41	48
0.5	22	109	45	21	20
0.6	13	82	25	12	11
0.7	8	65	14	7	7
0.8	5	52	8	5	4
0.9	3	43	5	3	3
1	2	36	4	2	2



**Table 2. ARL values of T2 control chart for different methods under shift in Slope  $\beta_1$**

Shift- d2	Huber	GGW	Mallows	Hample	Ols
0	370	370	370	370	370
0.05	107	234	189	108	110
0.1	10	112	41	20	28
0.15	5	61	12	6	10
0.2	2	39	7	2	4
0.25	1	26	4	1	3
0.3	1	20	1	1	2
0.35	1	15	1	1	1
0.4	1	13	1	1	1
0.45	1	11	1	1	1



The estimate of  $\sigma^2$  is

$$M_j = (n - 2)^{-1} \sum_{i=1}^n e_i^2 \tag{9}$$

where  $e_i$  is the difference between the observed and predicted values; that is,

$$e_i = Y_i - \beta_0 - \beta_1 X_i \tag{10}$$

For sample  $j$ , express the sample slope and intercept in Eq. (6) as the vector  $Z_j = (\beta_{0j}, \beta_{1j})^T$ . Then the expected value  $Z$  and the variance-covariance matrix of the vector  $Z_j$  are

$$Z = (\beta_0, \beta_1)^T \quad \text{and} \quad \Sigma = \begin{bmatrix} \sigma_0^2 & \sigma_0 \sigma_1 \\ \sigma_0 \sigma_1 & \sigma_1^2 \end{bmatrix} \tag{11}$$

The elements of  $\Sigma$  are defined in equation (3) and (4). The charting statistic

$$T_j^2 = (Z_j - Z)^T \Sigma^{-1} (Z_j - Z) \tag{12}$$

is called Hotelling's  $T^2$  statistic.  $T^2$  follows a chi-square distribution with 2 degrees of freedom. The upper control limit is  $UCL = \chi^2_{2, \alpha} E$  where  $\alpha$  is the overall probability of type I error.

**Simulation Study:** The average run length (*ARL*) is a good tool to evaluate the performance of a statistical process control chart. The *ARL* is the average number of points that must be plotted before a point indicates an out-of-control condition (Montgomery, 2005). When a process control chart is set up, it is desirable that it produces a large *ARL* when the process is in control while smaller *ARL* values are preferred when the process is out-of-control. A large in-control *ARL* reduces the false alarms while a small out-of-control *ARL* indicates quick detection of a change. Since evaluating *ARL* values is not elementary, let us consider the univariate shewhart control chart for the purpose of illustrating how the *ARL* is calculated. In this case, it is well-known that the run length follows a geometric distribution. Thus, its expected value is

$$ARL = \frac{1}{\alpha}$$

Where  $\alpha$  is the probability that any point exceeds the control limits. For instance, when the process is in-control with  $\alpha = 0.0027$ , then the in-control *ARL* (called *ARL<sub>0</sub>*) equals 370, which means that the control chart signals a false (out of-control) alarm on average every 370 plotted points even though the process is in-control. When the process is out-of-control, it is expected that more chart points will go out of the control limits. Thus the out-of-control *ARL* (called *ARL<sub>1</sub>*) will be smaller than *ARL<sub>0</sub>*.

In this section we compare the *ARL* performance of Hotelling  $T^2$  control chart based on seventeen estimation methods. We consider the model  $y = 2 + 3X + \varepsilon$ . Where  $\varepsilon \sim N(0,1)$  and  $x$  follows normal with mean 5 and variance  $\frac{5}{3}$ . In this proposed work the parameters of the control chart are designed to have the same in control *ARL* of 370. Hence the parameter of control chart are designed under in control conditions and they doesn't depend on the type of shift. We use 10000 simulation runs to calculate the out of control *ARL* under the shift in the intercept and slope and standard deviation using R software.

## Conclusion

In this paper we investigated the performance of the multivariate control chart based on various estimation methods under linear trend. Shift with positive rates in phase II monitoring linear profiles. In this paper we investigated the performance of the Hotelling  $T^2$  control chart based on various estimation methods under linear trend. Shift with positive rates in phase II monitoring linear profiles. From the Table No.1, the results shows that  $T^2$  based on Huber and Hampel M- estimator shows excellent performance in detection of small, moderate and large shift in intercept than ordinary least square (OLS) and However,  $T^2$  based on OLS estimator perform better in the rest of GGW and Mallow. but  $T^2$  based on mallow estimator best performance as compared to GGW estimator in detection of small, moderate and large shift in intercept. From the Table No.2, the results shows that  $T^2$  based on Huber and Hampel M- estimator shows excellent performance in detection of small, moderate than ordinary least square (OLS) and However,  $T^2$  based on OLS estimator perform better than Mallow in detection of small and moderate shift in slope parameter. and except GGW all are equally perform better in detecting large shift and  $T^2$  based on GGW estimator shows worst performance in detection of small, moderate and large shift in slope.

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