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RESEARCH ARTICLE

ON THE EXISTENCE OF ψ^α BOUNDED SOLUTION FOR A NON-HOMOGENOUS FUZZY FIRST ORDER MATRIX DIFFERENCE SYSTEM WITH AN APPLICATION TO TWO-POINT BOUNDARY VALUE PROBLEM

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ABSTRACT

This paper presents a criteria for the existence of at least one ψ^α bounded solution on N for the non-homogenous fuzzy difference equation of first order. We also present a criteria for the existence and uniqueness of solutions to non-homogenous fuzzy two-point boundary value problems.

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INTRODUCTION

Difference equations occur as a natural description of observed evolution phenomena. The aim of this paper is to present a criteria for the existence of at least one ψ^α-bounded solution of the system of fuzzy non- homogenous first order difference equation

$$x(n+1) = A(n)x(n) + b_n \tag{1.1}$$

where A is a (kxk) discrete matrix and x is a k-vector. b_n is a column matrix of first order

k. We assume that the system (1.1) has atleast one ψ^α-bounded solution on N for every ψ^α summable matrix function b_n on N. Existence of ψ-bounded solutions of linear system of differential equations on time scales are established in [Charyulu et al., 2019]. This theory infact unifies both continuous and discrete systems in a single frame work. Our main goal in this paper is to establish ψ^α-bounded solutions of fuzzy linear system of first order difference equations and obtain the existing results as a particular case [Diamandescu, 2014]. We also present a set of sufficient conditions for the fuzzy first order difference system to completely controllable and observable. The main advantage of our approach is difference inclusions and hence is unique of its kind. The results established in [Negotia, 1975] are used as a tool to establish our main results in this paper

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Note that ψ^{α} is a matrix function which is similar to the concept of dichotomy introduced by Coppel as well as in [Murty. 2008 and Murty. 2009]. In the year 2004, Diamandescu [Diamandescu A. 2004] used the idea of bounded solutions for a non-homogenous first order matrix system (1.1) on ℕ.

PRELIMINARIES

In this section, we introduce notations, definitions and preliminary facts which are used throughout the paper.

Definition 2.1: Let x be a non-empty set. A fuzzy set $\alpha \in x$ s characterized by its membership function.

$A : x \rightarrow [0, 1]$ and $A(x)$ is interpreted as the degree of the membership of element n in A for each $x \in X$.

The value 0(zero) is used to represent complete non-membership, the value 1 is used to represent complete membership and the values in between are used to represent the immediate degrees of membership. The mapping A is also called the membership function of fuzzy set A.

Example: The membership function of the fuzzy set of real numbers close to one can be defined as

$$A(t) = \exp(\beta(t-1))$$

where β is a positive real number.

The membership function close to 0 is defined as

$$A(t) = \frac{1}{1+t^2}.$$

Using this function, we can determine the membership grade of each real number in the fuzzy set, which signifies the degree to which that number is close to zero. For instance, the number 2 is assigned a grade 0.025, the number 1 a grade of 0.5 and the number 0 a grade of 1. See [Sailaja, 2019]

In the case of discrete systems we have the following notions.

Definition 2.2: Let x be a non-empty set. A fuzzy set on A in X is characterized by its membership function $A : x \rightarrow [t_0, t_1]$ and A (n) is interpreted as a degree of the membership of elements n in fuzzy set A, for each $n \in N$ For $0 \le \alpha \le 1$, we define $[y]^{\alpha} = \{n \in N : y(n) \ge \alpha\}$ it follows that the α-level sets $[y]^{\alpha} \in E^N$ It is a well known fact that

$$[g(y, \bar{y})]^{\alpha} = g[y^{\alpha}, \bar{y}^{\alpha}]$$

for all $y, \bar{y} \in E^n, 0 \le \alpha \le 1$ and $\bar{y}$ is a discrete function. The Fuzzy set A, the membership function of the fuzzy set of natural numbers close to one can be defined as

$$A(n) = exp - \beta(n-1)$$

where $n \in N$ and β is a positive real number. The membership close to zero is defined as $A(n) = \frac{1}{1+n^2}$ Using this function, we can determine the membership grade of each natural number in the fuzzy set, which signifies the degree of which the number is close to 0

Definition 2.3: Let $u_i^{\alpha}(n)$ be the level set of $u_i(n)$, then we define

$$u_i^{\alpha}(n) = \{u_1^{\alpha}(n), u_2^{\alpha}(n), \dots, u_k^{\alpha}(n)\}.$$

Definition 2.4: A fuzzy number in parametric form is represented by $(u_{\alpha}^{-}, u_{\alpha}^{+})$, where

$u_{\alpha}^{-} = min[u]^{\alpha}$ and $u_{\alpha}^{+} = max[u]^{\alpha}$ $0 \le \alpha \le 1$

Definition 2.5: We define the zadeh's extention principle by definition

$$A[u, v]^{\alpha} = A[u^{\alpha}, v^{\alpha}], 0 \le \alpha \le 1.$$

Definition 2.6 : Let $u_i(n) \in E^1 (i = 1,2, \dots, k)$ and define

$\hat{u}_i^{(n)} = (u_1(n), u_2(n), \ldots, u_k(n))$

$= \{(\tilde{u}_1(n), \tilde{u}_2(n), \ldots, \tilde{u}_k(n))\} = \tilde{u}_i^{\alpha}(n), \alpha \in [0,1]$

$= \{u_1^{(\alpha)}(n), u_2^{(\alpha)}(n), \ldots, u_k^{(\alpha)}(n)\}$, for $\alpha \in [0,1]$

For the proofo fthe next theorem, we refer to Negotia and Relescu [2009].

Theorem 2.1: If $u_i \in E^k$, then

1. $[u]^{(\alpha)} \in p_k[N_{n_0}^+]^{kxk}$ for all $0 \le \alpha \le 1$;
2. $[u]^{(\alpha_2)} \subset [u]^{(\alpha_1)}$ for all $0 \le \alpha_1 \le \alpha_2 \le 1$;
3. If α_k is a non-decreasing sequence converging to $\alpha > 0$, then $[u]^{\alpha} = \cap_{k\ge1} [u]^{\alpha_k}$

Conversely, if $\{A^{\alpha}: 0 \le \alpha \le 1\}$ is a family of subsets of R^k satisfying (1) - (3) above then there exists a $u_i \in E^k$ such that
$[u]^{\alpha} = A^{\alpha}$ for $0 \le \alpha \le 1$ and
$[u]^0 = u_0 A^{\alpha} \subset A^0$ for $0 \le \alpha \le 1$
Let E^k be the Euclidian k- space for $x = (x_1, x_2, \ldots, x_k)^T \in E^k$
Let $\|x\|$ be the normal of x denoted by
$\|x\| = \max\{|x_1|, |x_2|, \ldots, |x_K|\}$

where T denotes transpose of the column matrix.

Let $\psi_i: N \to (0, \infty), i = 1,2, \ldots, k$ and the matrix function
$\bar{\psi}^{\alpha} = diag\ \{\psi_1^{\alpha}, \psi_2^{\alpha}, \ldots, \psi_k^{\alpha}\}, 0 \le \alpha \le 1$,
then $\psi_i(n)$ is invertible for each $n \in N$

Definition 2.7: Let function $f: N \to E^k$ is said to be ψ^{α} summable on N if $\sum_{n=1}^{\infty}\|\psi^{\alpha}(n) f(n)\|$ is convDefinition 2.8: A function $F: N \to E^k$ is said to be ψ^{α} summable on N if $\sum_{n=1}^{\infty}\|\psi^{\alpha}(n) F(n)\|$ is convergent for all $[0, 1]$

Now, we consider the two-point inclusions

$$y(n+1) \in A(n)y(n) + b_n \tag{2.1}$$

$$My(n_0) + Ny(n_f) \in \alpha_n \tag{2.2}$$

for $n \in [0, N]$, where M and N are constant square matrices of order k and let $\tilde{y}^{\alpha_1}$ and $\tilde{y}^{\alpha_2}$ be the solution sets of (2.1) satisfying (2.2), then

$\tilde{y}_1^{\alpha}(n+1) \in A(n)\tilde{y}^{\alpha_1}(n) + b_n$
$M\tilde{y}_1^{\alpha}(n_0) + N\tilde{y}^{\alpha_1}(n_f) \in \alpha_n$

and

$\tilde{y}^{\alpha_2}(n+1) \in A(n)\tilde{y}^{\alpha_2}(n) + b_n$
$M\tilde{y}^{\alpha_2}(n_0) + N\tilde{y}^{\alpha_2}(n_f) \in \alpha_n$
Clearly $\tilde{y}^{\alpha_1} \subset \tilde{y}^{\alpha_2}$ and hence $\tilde{y}^{\alpha_1}(n) \subset \tilde{y}^{\alpha_2}(n)$.

Lemma 2.1: Let $< \alpha_k >$ be non-decreasing sequence converging to $\alpha > 0$, then
$\hat{y}^{\alpha}(n) = \cap_{k\ge1} \hat{y}^{\alpha_k}(n)$
Proof: Let $\hat{u}^{\alpha_k}(n) = \hat{u}_1^{\alpha_k} \times \hat{u}_2^{\alpha_k} \times \ldots \times \hat{u}_k^{\alpha_k}$ and argent.

$\hat{u}^{\alpha}(n) = \hat{u}_1^{\alpha} \times \hat{u}_2^{\alpha} \times \ldots \times \hat{u}_k^{\alpha}$

Consider the inclusions

$$\hat{y}(n+1) \in A(n)\hat{y}(n) + b_n \hat{u}^{\alpha_k}(n) \quad (2.3)$$

$$\hat{y}(n+1) \in A(n)\hat{y}(n) + b_n \hat{u}^{\alpha}(n) \quad (2.4)$$

Let $\hat{y}^{\alpha_k}$ and $\hat{y}^{\alpha}$ be the solution sets of (2.3) and (2.4) respectively. Since $\mu_i(n)$ is a fuzzy set and from Theorem 2.1, we have $u_i^{\alpha} = \cap_{k\geq 1} \hat{u}_i^{\alpha_k}$

Consider

$$\hat{u}^{\alpha}(n) = \hat{u}_1^{\alpha} \times \hat{u}_2^{\alpha} \times \ldots \times \hat{u}_k^{\alpha}$$
$$= \cap_{k\geq 1} \hat{u}_1^{\alpha_k} \times \hat{u}_2^{\alpha_k} \times \ldots \times \hat{u}_k^{\alpha_k}$$
$$\geq \cap_{k\geq 1} \hat{u}^{\alpha_k}(n)$$

and the rest of the proof follows [9 Yan Wu, 2020].

Theorem 2.2: Let $A(n) \in E^{k\times k}$ be invertible for all $n \in N$ Then the difference equation

$$x(n+1) = A(n)x(n) + b_n \quad (2.5)$$

has atleast one ψ^{α}-bounded solution of N for every ψ^{α}summable function $\bar{b}$ on N; if and only if for any fundamental matrix $y(n)$ of equation.

$$x(n+1) = A(n)x(n) \quad (2.6)$$

there exists a positive constant k such that

$$\left|\bar{\psi}(n) y_n p_0 y^{-1}(k+1)\psi^{-1}(k)\right| \leq k \text{ for } 0 \leq k+1 \leq n \quad (2.7)$$

and $\left|\bar{\psi}(n) y_n p_1 y^{-1}(k+1)\psi^{-1}(k)\right| \leq k$ for $0 \leq n \leq k+1$

Proof: For the proof of the theorem, we refer [10] [Han Y 2007 and Kasi Viswanadh V. Kanuri 2020]

Theorem 2.3: Suppose that a) the fundamental matrix $y(n)$ of (2.6) satisfies the condition (2.7) for some $k > 0$ and Further the function $b: N \to E^k$ is ψ^{α}-summable on N. Then every ψ^{α}-bounded solution

- of (2.5) satisfies

$$\lim_{n\to\infty} \|\psi(n)x_n\| = 0$$

For the proof of the theorem, we refer to [2 Murty, K. N., 2013 and Murty, K. N., 2011].

MAIN RESULT

In this section we shall be concerned with our main result namely the existence of ψ^{α} bounded solutions of matrix fuzzy difference system in addition to establishing existence and uniqueness criteria for two-point boundary value problems. We consider

$$\bar{x}(n+1) = A(n)\bar{x}(n) + b_n \quad (3.1)$$

and

$$Mx(n_0) + Nx(n_f) = \alpha \quad (3.2)$$

where M and N are constant matrices of order k. we assume that the homogenous boundary value problem, has a trivial solution. This will enable us that the characteristic matrix.

$$D = M\sum_{j=0}^{\infty} \psi(n_0 - j - 1) + N\sum_{j=0}^{\infty} \psi(n_f - j - 1)$$

is non-singular. Hence we have

$$\tilde{x}(n) \in x(n_0) + \sum_{j=0}^{\infty} \psi(n-j-1)u(j)$$

Substituting the general form of $\tilde{x}(n)$ in the boundary condition matrix, we get

$$Dx(n_0) + \left[M \textstyle\sum_{j=0}^{\infty} \psi(n_0 - j - 1) + N \sum_{j=0}^{\infty} \psi(n_f - j - 1)\right] u_j(\alpha) \quad (3.3)$$

$$x(n_0)E - D^{-1}\alpha_j + D_{-1}\left[M \sum_{j=0}^{\infty} \psi(n_0 - j - 1) + N \sum_{j=0}^{\infty} \psi(n_f - j - 1)\right] u_j(\alpha)$$

$$\sum_{k=1}^{\infty} \| \Psi^{\alpha}(k) f(k) \| \leq \frac{\in}{2k}$$

for $\geq n_1$.

Hence $\lim_{n\to\infty} \|\psi^{\alpha}(n)x(n)\| \to 0$ and hence $\psi^{\alpha}(n)x(n)$s ψ^{α} summable

This is true for all $\alpha \in [0,1]$ and the proof is complete.

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