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RESEARCH ARTICLE

PROPERTIES OF SG*α-CLOSED SETS IN TOPOLOGY

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ABSTRACT

In this paper we introduce a new class of sets namely, sg*α-closed sets in topological spaces. For these sg*α-closed sets,we define and study their neighbourhoods. **Mathematics Subject** Classifications (2010): 54A05, 54B05.

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1. INTRODUCTION

In (Levine, 1970), Levine generalized the concept of closed set to generalized closed sets. Bhattacharya and Lahari (1987) generalized the concept of closed sets to semi-generalized closed sets via semi-open sets. In this paper we generalized the concept of closed sets to semi-generalized closed sets via αg*s-open sets called semi generalized star α-closed (in short sg*α-closed) sets in topological spaces and study some of their relationship and their properties. Furthermore, the notion of sg*α-neighbourhood, sg*α-limit points, sg*α-derived sets,sg*α-closure sg*α-interior and sg*α-R_o as well as weakly sg*α-R_o spaces are presented.

2. Preliminaries

Throughout this paper (X, τ), (Y, σ) and (Z, η) represent topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a space (X, τ) the closure and interior of A with respect to τ are denoted by Cl(A) and Int(A) respectively. The complement of A is denoted by A^c or X-A.

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Definition 2.1: A subset A of a topological space (in short, TS) X is called a

- semi-open set (Levine, 1963) if $A \subseteq cl(int(A))$ and a semi-closed set (Crossley , 1972) if $int(cl(A) \subseteq A$.
- α-open set (8) if $A \subseteq int(cl(int(A)))$ and a α-closed set (Njåstad, 1965) if $cl(int(cl(A))) \subseteq A$.
- The complement of semi-open set is called semi-closed (Levine., 1963) set of a space X. The family of all semi-open (resp. pre-open, semi-preopen) sets of a space X is denoted by SO(X) and that of semi-closed sets of X is denoted by SF(X).

Definition 2.2 (Crossley, 1972**):** For a subset A of X,

- The intersection of all semi-closed subsets of X containing A is called semi-closure of A and is denoted by sCl(A).
- Semi-interior of A is the union of all semi-open sets contained in A in X and is denoted by sInt(A).

Definition 2.3: A subset A of a TS X is known as

- Generalized closed (briefly g-closed) (5) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

- Generalized-semi closed (briefly gs-closed) (1) if scl(A) $\subseteq$ U whenever A$\subseteq$U and U is open in X.
- α-Generalized-closed (briefly αg-closed) set (6) if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
- Generalized α-closed (briefly gα-closed) set (7) if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is α-open in X.
- αg*s set (9) if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is gs-open in X. and the collection of αg*s-closed sets in X is denoted by αg*sC(X).

3. Properties of sg*α-closed sets

We, present basic results of semi generalized star α- closed sets in this section.

Definition 3.1: A subset A of X is termed as semi generalized star α-closed (in short sg*α-closed) set if sCl(A)$\subseteq$ U whenever A$\subseteq$U and U is αg*s-open in X. The family of entire sg*α-closed members of X is labeled as SG*αC(X).

Example 3.2: Take X = {a, b, c} and τ = {X, ϕ, {a}, {b}, {a, b}}. Here SO(X)= { X, ϕ, {a}, {b}, {a, b}, {a, c}, {b, c}}. αC(X)= {X, ϕ, {a},{c}, {a, c}, {b, c}}. GSO(X)= {X, ϕ, {a}, {b}, {a, b}, {a, c}, {b, c}}. αg*sC(X)= {X, ϕ, {c}, {a,c}, {b, c}}. So, SG*αC(X)= {X, ϕ, {a}, {b}, {c},{a, c}, {b, c}}.

Theorem 3.3: Each closed set is sg*α-closed set still converse is not true.

Proof: Allow K be a closed set in X. Note that sCl(K) $\subseteq$ Cl(K) holds good as well as Cl(K) = K as K is closed. So if A $\subseteq$ G where G is αg*s-open set in X. Subsequently sCl(A) $\subseteq$ G. Thereupon A is sg*α-closed set in X.

Example 3.4: In 3.2, {a} and {b} are sg*α-closed sets but not closed sets in X.

Theorem 3.5: Each gα-closed, αg-closed set is sg*α-closed s*et al*though converse is false.

Proof: Allow A be a αg-closed set in X. Allow A $\subseteq$ U, where U is open as well as it is α-open set which in turn it is αg*s-open. Again αcl(A) $\subseteq$ U. Note that sCl(A) $\subseteq\alpha$cl(A). Consequently, sCl(A) $\subseteq$ U. Hence A is sg*α-closed set in X.

Example 3.6: In example 3.2, {b} is sg*α-closed although it isn't gα-closed as well as αg-closed.

Theorem 3.7: Each α-closed is αg-closed and hence sg*α-closed set though reverse is not true.

Proof: Trivial.

Example 3.8: In example 3.2, {a} is sg*α-closed still not α-closed.

Theorem 3.9: Each sg*α-closed is gs-closed set though contrarily false.

Proof: Authorize M be a sg*α-closed set in X. Make O be an open set and so it is αg*s-open set such that M $\subseteq$ O. Hence sCl(M) $\subseteq$ O. Consequently, M is gs-closed set in X.

Example 3.10: Consider a topology τ = {X, ϕ, {a},{a, b}}on X={a, b, c, d} and SG*αC(X)= {X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}}. gs-closed sets are {X, ϕ, {b},{c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}, {a, b, c}}. As we can see that {a, c} is gs-closed yet it isn't sg*α-closed set.

Theorem 3.11: Each αg*s-closed is sg*α-closed set though contrarily false.

Proof: Authorize M be a αg*s-closed set in X. Make O be an αg*s-open set and so it is gs-open. Again αcl(A) $\subseteq$ U. Note that sCl(A) $\subseteq\alpha$cl(A). Hence sCl(M) $\subseteq$ O. Consequently, M is sg*α-closed set in X.

Example 3.12: In 3.2 {b} is sg*α-closed set but it is not αg*s-closed.

Remark 3.13: The intersection of two sg*α-closed sets is again sg*α-closed set however union of two sg*α-closed sets isn't sg*α-closed.

Example 3.14: In 3.2 , {a} and {b} are sg*α-closed sets but their union {a, b} is not sg*α-closed.

Definition 3.15: A TS X is termed as sg*$\alpha T_{1/2}$-space whenever each sg*α-closed set is closed.

4. sg*α-Neighbourhoods

Definition 4.1: A subset P of a TS X is named as semi generalized star α-neighbourhood (in short sg*α-nhd) of a point k of X if there arises a sg*α-open set U so that k $\in$ U $\subseteq$ P. The collection of entire sg*α-nhds of x $\in$ X is termed sg*α-nhd system of x and is labeled as sg*α-N(x).

Theorem4.2: Enable p be any arbitrary point of a TS X. At that time sg*α-N(x) satisfies succeeding properties.

- sg*α-N(p) $\neq\phi$
- Whenever N $\in$sg*α-N(p) then p $\in$N.
- Whenever N $\in$sg*α-N(p) and N $\subset$ M at that time M $\in$sg*α-N(p).

Proof: (i) By the reason of each p $\in$ X, X is a sg*α-open set. Therefore x $\in$ X $\subset$ X, implicit X is sg*α-nhd of p, hence X $\in\alpha$-sg-N(x). Accordingly, sg*α-N(p)$\neq\phi$.

- Given N $\in$sg*α-N(p), implicit N is a sg*α-nhd of p, which indicates there is a sg*α-open set G so as p $\in$G $\subset$N. This impart, x $\in$N.
- Given N $\in\alpha$-sg-N(p) implicit there is a sg*α-open set G in such a manner p $\in$ G $\subset$ N. And N $\subset$ M, which implicit p $\in$G $\subset$M. This shows that M $\in$sg*α-N(x).

Theorem 4.3: Let A be a member of a TS X. Thereupon A is sg*α-open iff A contains a sg*α-nhd of each of its points.

Proof: Allow A be a sg*α-open set in X. Make x$\in$ A, which imparts x $\in$A $\subseteq$ A. So A is sg*α-nhd of x. Hence A contains a sg*α-nhd of each of its points. Contrarily, A contains a sg*α-nhd of each of its points.

For each x ∈ A there arises a neighbourhoodN_x of x such that x ∈N_x⊆ A. By the definition of sg*α-nhd of x, there is a sg*α-open set G_x such that x ∈G_x⊆N_x⊆ A. Now we shall prove that A = ∪ {G_x: x ∈ A}. Let x ∈ A. Then there is sg*α-open set G_x such that x ∈G_x. Therefore, x ∈∪{G_x: x ∈ A} which implies A ⊆∪{G_x: x∈ A}. Now let y∈{G_x: x ∈ A} so that y ∈ some G_x for some x∈ A and hence y∈ A. Hence, ∪{G_x: x ∈ A} ⊆ A. Hence A = ∪{G_x: x ∈ A}. Also each G_x is a sg*α-open set. And hence A is a sg*α-open set.

Theorem 4.4: Whenever A is a sg*α-closed subset of X and x ∈ X – A, accordingly there is an sg*α-nhd N of x so that N ∩ A =ϕ.

Proof: Assuming that A is a sg*α-closed set in X, then X – A is a sg*α-open set. By the Theorem 4.3, X – A contains a sg*α-nhd of each of its points. Which intimate that, there is an sg*α-nhd N of x so as N ⊆ X-A. That is, no point of N belongs to A and hence N ∩ A = ϕ.

Definition 4.5: A point x∈X is termed as sg*α-limit point of A iff each sg*α-nhd of x contains a point of A different from x. That is (N – {x}) ∩ A ≠ϕ, for each sg*α-nhd N of x. Also equivalently iff each sg*α-open set G comprising x contains a point of A other than x. The collection of entire sg*α-limit points of A is named as sg*α-derived set of A and is labeled as sg*α-d(A).

Theorem4.6 : Enable subsets A, B of X and A ⊆ B implies sg*α-d(A) ⊆ sg*α-d(B).

Proof: Enable x∈ sg*α-d(A) implies x is a sg*α-limit point of A that is each sg*α-nhd of x contains a point of A other than x. As A ⊆ B, each sg*α-nhd of x contains a point of B other than x. Consequently x is a sg*α-limit point of B. That is x∈ sg*α-d(B). Hence sg*α-d(A) ⊆ sg*α-d(B).

Theorem 4.7: A subset P of X is sg*α-closed iff sg*α-d(P) ⊆P.

Proof: Whenever P is sg*α-closed set. That is X-P is sg*α-open. Now we prove that sg*α-d(P) ⊆ A. Allow x∈ sg*α-d(P) which intend x is a sg*α-limit point of P, that is each sg*α-nhd of x contains a point of P different from x. Now think x∉P so that x∈ X-P, which is sg*α-open and by definition of sg*α-open sets, there is a sg*α-nhd N of x in such a manner N ⊆ X-P. From this we conclude that N contains no point of P, which is a contradiction. Therefore x∈A and hence sg*α-d(P) ⊆ P. Contrarily assume that sg*α-d(P) ⊆ P and we will prove that P is a sg*α-closed set in X or X-P is sg*α-open set. Ensure x be an arbitrary point of X-P, so that x∉ P which imparts that x∉ sg*α-d(A). That is there exists a sg*α-nbd N of x which consists of only points of X – P. This means that X-P is sg*α-open. And hence P is sg*α-closed set in X.

Theorem4.8: Each sg*α-derived set in X is sg*α-closed.

Proof: Permit A be a member of X and sg*α-d(A) is sg*α-derived set of A. By Theorem 4.7, sg*α-d(A) is sg*α-closed iff sg*α-d(α-sg-d(A)) ⊆ sg*α-d(A) . That is each sg*α-limit point of sg*α-d(A) belongs to sg*α-d(A).

Now allow x be a sg*α-limit point of sg*α-d(A). That is x∈ sg*α-d(sg*α-d(A)). So that there is a sg*α-open set G containing x such that {G – {x}} ∩ sg*α-d(A) ≠ϕ which imparts {G – {x}} ∩ A ≠ϕ, as each sg*α-nhd of an element of sg*α-d(A) has at least one point of A. Hence x is a sg*α-limit point of A. That is x belongs to sg*α-d(A). So x∈ sg*α-d(sg*α-d(A)) implicit x∈ sg*α-d(A). Accordingly sg*α-d(A) is sg*α-closed set in X.

Theorem 4.9: The following properties are true for A, B ⊂ X

- sg*α-d(ϕ) = ϕ.
- Whenever A ⊂ B then sg*α-d(A) ⊂sg*α-d(B).
- Whenever q ∈sg*α-d(A) then q ∈sg*α-d(A−{q}).
- sg*α-d(A) ∪sg*α-d(B) ⊂sg*α-d(A ∪B).
- sg*α-d(A∩B) ⊂sg*α-d(A)∩ sg*α-d(B).

Proof: (i) Authorize q ∈ X and G be a sg*α-open involving q. Then (G −{q}) ∩ ϕ = ϕ. This suggest x ∉sg*α-d(ϕ). Accordingly for any q ∈ X, q is not sg*α-limit point of ϕ. Hence sg*α-d(ϕ) = ϕ.

- Allow q ∈sg*α-d(A). Afterwards G∩(A−{q}) ≠ϕ, for each sg*α-open set G involving q. As A ⊂ B, implies G∩(B−{q}) ≠ϕ. This impart q ∈sg*α-d(B). Thereupon, q ∈sg*α-d(A) implies q ∈sg*α-d(B). Therefore, sg*α-d(A) ⊂sg*α-d(B).
- Let q ∈sg*α-d(A). Then G∩(A−{x}) ≠ϕ, for each sg*α-open set G containing x. This intimate that each sg*α-open set G including q, contains at least one point different from q of A−{q}. Therefore x ∈sg*α-d(A−{x}).
- Since A ⊂ A∪B and B ⊂ A∪B and by (ii), sg*α-d(A) ⊂sg*α-d(A ∪ B) and sg*α-d(B) ⊂sg*α-d(A ∪ B). Hence, sg*α-d(A) ∪sg*α-d(B) ⊂sg*α-d(A∪B).
- Since A∩B ⊂ A and A∩B ⊂ B and by (ii), sg*α-d(A∩B) ⊂sg*α-d(A) and sg*α-d(A∩B) ⊂sg*α-d(B). Therefore sg*α-d(A∩B) ⊂sg*α-d(A) ∩ sg*α-d(B).

Theorem 4.10: Whenever A is member of X, then A∪sg*α-d(A) is sg*α-closed set.

Proof: To prove A ∪sg*α-d(A) is sg*α-closed set, it is sufficient to prove X − (A ∪sg*α-d(A)) is α-sg-open. Whenever X − (A ∪sg*α-d(A))= ϕ, then it is clearly sg*α-open set. Enable X − (A ∪sg*α-d(A)) ≠ϕ and x ∈ X − (A ∪sg*α-d(A)), implies x ∉A ∪sg*α-d(A). This impart x ∉ A and x ∉sg*α-d(A). Now x ∉sg*α-d(A), which indicates x is not sg*α-limit point of A. Therefore, there is a sg*α-open set G so that G ∩ (A −{x}) = ϕ. As x ∉A, implies G ∩ A = ϕ. This suggest x ∈ G ⊂ X − A——(1). Again G is sg*α-open set and G ∩ A = ϕ implies no point of G can be α-sg-limit point of A. This follows G∩ sg*α-d(A) = ϕ, implies x ∈ G ⊂ X− sg*α-d(A)——(2). From (1) and (2), x ∈ G ⊂ (X −A)∩(X − sg*α-d(A)) = X −(A∪sg*α-d(A)). That is x ∈ G ⊂ X−(A∪sg*α-d(A)). This impart X−(A∪sg*α-d(A)) is sg*α-sg-nhd of each of its points. By theorem 4.4, X−(A∪sg*α-d(A)) is α-sg-open as well as A∪sg*α-sg-d(A) is α-sg-closed set.

Theorem 4.11: For A ⊂ X. Then A is sg*α-closed set iffsg*α-d(A) ⊂ A.

Proof: Imagine A is sg*α-closed set, in that case sg*α-d(A) = ϕ, then the result is trivial. Whenever sg*α-d(A) ≠ϕ then x ∈sg*α-d(A), implies G∩(A−{x}) ≠ϕ for each sg*α-open set G containing x. Assuming that x ∉ A later x ∈ X−A. As A is α-sg-closed and X − A is sg*α-open set containing x and not containing any other point of A. Which is contradiction to x ∈sg*α-d(A), therefore x ∈ A. For this reason, x ∈sg*α-d(A) implies x ∈ A. Hence sg*α-d(A) ⊂ A. On the other hand, sg*α-d(A) ⊂ A. To prove A is sg*α-closed set; it is similar to prove X −A is sg*α-open set. Enable x ∈ X −A implies x ∉ A. In view of sg*α-d(A) ⊂ A, implies x ∉sg*α-d(A), which impart there is an sg*α-open set G containing x thereby G∩(A−{x}) = ϕ. That is G∩A = ϕ as x ∉ A, implies, x ∈ G ⊂ X−A. Therefore, X−A is sg*α-nhd of x. As x is arbitrary X −A is α-sg-nhd of each of its points. By theorem 4.4, X−A is sg*α-open set. Hence A is sg*α-closed set.

5. On sg*α-closure and sg*α-interior operators

Definition 5.1: Consider X be a TS and Q⊆X. The set of intersection of entire sg*α-closed sets including Q is named sg*α-closure of Q and is labelled as sg*αCl(Q).

Theorem 5.2: For members A, B of X, the listed properties hold:

- sg*αCl(X)=X and sg*αCl(ϕ)=ϕ.
- Whenever A⊆B, then sg*αCl(A) ⊆ sg*αCl(B)
- sg*αCl(P)⋃ sg*αCl(Q) ⊆ sg*αCl(P⋃Q)
- sg*αCl(A⋂B) ⊆ sg*αCl(A)⋂ sg*αCl(B)
- sg*αCl(sg*αCl(A)) = sg*αCl(A)
- A is sg*α-closed iff sg*αCl(A)=A.

Theorem 5.3: For A⊆X, then x∈ sg*αCl(A) iff G⋂A ≠ ϕ for each sg*α-open set G containing x.

Proof: Necessity, enable x∈ sg*αCl(A) for any x ∈X. Expect there is a sg*α-open set G comprising x so that G⋂A=ϕ. Then A⊂X-G. As X-G is sg*α-closed set comprising A, we have sg*αCl(A)⊂X-G, which indicates x∉ sg*αCl(A). This is contradiction to hypothesis. Hence G⋂A ≠ ϕ. Contrarily, presume x∉ sg*αCl(A). There exist a sg*α-closed set F involving A so that x∉F. Then x∈X-F and X-F is sg*α-open. Also (X-F)⋂A=ϕ. This is contradiction to the hypothesis. Therefore x∈ sg*αCl(A).

Definition 5.4: For a TS X and S⊂X the union of entire sg*α-open sets included in S is termed as sg*α-interior of S and is labelled as sg*αInt(A).

Theorem 5.5: A and B be members of TS X. Then the listed results hold:

- sg*αInt(X)=X and sg*αInt(ϕ)=ϕ.
- Whenever A⊆B, then sg*αInt(A) ⊆ sg*αInt(B)
- sg*αInt(A)⋃ sg*αInt(B) ⊆ sg*αInt(A⋃B)
- sg*αInt(A⋂B) ⊆ sg*αInt(A)⋂ sg*αInt(B)
- sg*αInt(sg*αInt(A))= sg*αInt(A)
- A is sg*α-open iff sg*αInt(A)=A.

Theorem 5.6: For a member A of X, the listed results hold:

- sg*αCl(X-A)= X- sg*αInt(A)
- sg*αInt(X-A) = X- sg*αCl(A)
- sg*αInt(A)= X- sg*αCl(X-A)
- sg*αCl(A)=X- sg*αInt(X-A)

Proof: (1) Allow x∈X-sg*αInt(A). So x∉ sg*αInt(A), implies for each sg*α-open set U comprising x we have U⋂(X-A)≠ ϕ. Thus, x∈sg*αCl(X-A). Hence X-sg*αInt(A) ⊂ sg*αCl(X-A). Contrarily, allow x∈X- sg*αCl(A). So U⊄ A for each sg*α-open set U comprising x. Hence x∉ sg*αInt(A), implies x∈X- sg*αInt(A). This indicates sg*αCl(X-A) ⊂ X- sg*αInt(A).Therefore, X- sg*α-sgInt(A) = sg*αClX-A)

(2)Enable x∈X- sg*αCl(A). So x∉ sg*αCl(A), implies for each α-sg-open set U including x we have U ⋂A=ϕ. This impart x∈U⊂A^c, so x∈ sg*αInt(A^c) or x∈ sg*αInt(X-A). Therefore we have X- sg*αCl(A) ⊂ sg*αInt(X-A) Contrarily, make x∈ sg*αInt(X-A). Then there is an sg*α-open set U including x so that x∈U⊂X-A. Hence U⋂A=ϕ, x∉ sg*αCl(A), implies x∈X- sg*αCl(A). This indicates sg*αInt(X-A) ⊂ X- sg*αCl(A).Therefore, X- sg*αCl(A) = sg*αInt(X-A)
(3) Replacing A by X-A in (2) we result (3)
(4) Replacing A by X-A in (2) we result (4)

6. sg*α-R_oSPACES

Definition 6.1: Let A be a subset of a TSX. The sg*α-kernel of A , labeled as sg*α-ker (A) is defined to be the set sg*α-ker (A) = ∩{U: A ⊆ U and U is sg*α-open in X}

Definition 6.2: Let x be a point of a TSX. The sg*α-kernel of x , labeled as sg*α-ker ({x}) is defined to be the set sg*α-ker ({x}) = ∩ {U: x ∈ U and U is sg*α-open in (X,τ)}

Lemma 6.3: Let X be a TS and x ∈X. Then sg*α-ker (A) = {x ∈ X: sg*αCl ({x}) ∩ A ≠ ∅ }.

Proof: Let x ∈ sg*α-ker (A) and suppose sg*αCl ({x}) ∩ A = ∅. Hence x ∉X – sg*αCl ({x}) which is a sg*α-open set including A. This is absurd, as x ∈ sg*α-ker (A).Hence sg*αCl ({x}) ∩ A ≠ ∅. Contrarily, let sg*αCl ({x}) ∩ A ≠ ∅ and assume that x ∉ sg*α-ker (A). Then there is a αpg-open set U including A and x ∉ U. Let y ∈ sg*αCl ({x}) ∩ A. Hence , U is a sg*α-nhd of y in which x ∉ U. By this contradiction , x ∈ sg*α-ker (A) and the claim.

Definition 6.4: ATS X is named as semi generalized star α-R_o (in short , sg*α-R_o) space iff for each sg*α-open set G and x ∈ G implies sg*αCl ({x}) ⊆ G.

Lemma 6.5: Let X be a TS and x ∈X. Then y ∈ sg*α-ker ({x}) iff x ∈ sg*αCl ({y}).

Proof: Suppose that $y \notin$ sg*α-ker ({x}). Then there exists a sg*α-open set V comprising x such that $y \notin V$. Therefore we have $x \notin$ sg*αCl ({y}). The proof of converse can be done similarly.

Lemma 6.6: The following results are similar for any points x and y in a TS X:

- sg*α-ker ({x}) $\neq$ sg*α-ker({y})
- sg*αCl ({x}) $\neq$ sg*αCl ({y}).

Proof: (i) →(ii).Suppose that sg*α-ker ({x}) ≠ sg*α-ker ({y}) , then there exists a point z in X such that z ∈ sg*α-ker ({x}) and z ∉ sg*α-ker ({y}). From z ∈ sg*α-ker ({x}) it follows that {x} ∩ sg*αCl ({z}) ≠ ∅which implies x ∈ sg*αCl ({z}).By z ∉ sg*α-ker ({y}) , we have {y} ∩ sg*αpgCl ({z}) = ∅. Since x ∈ sg*αCl ({z}) , sg*αCl ({x}) ⊂ sg*αCl ({z}) and {y} ∩ sg*αCl ({x}) =∅. Therefore it follows that sg*αCl ({x}) ≠ sg*αCl ({y}). (ii) → (i). Suppose thatsg*αCl ({x}) ≠ sg*αCl ({y}).There exists a point z in X such that z ∈ sg*αCl ({x}) and z ∉ sg*αCl ({y}).Then there exists a sg*α-open set containing z and therefore x but not y , namely , y ∉ sg*α-ker ({x}). Hence sg*α-ker ({x}) ≠ sg*α-ker ({y}).

Theorem6.7: A TS X is sg*α-R_o space ifffor any x , y in X , sg*αCl ({x}) ≠ sg*αCl ({y}) implies sg*αCl {x}) ∩ sg*αCl ({y}) = ∅.

Proof: Consider X is sg*α-R_o space and x , y ∈ X in that case sg*αCl ({x}) ≠ sg*αCl ({y}).Then there exists a point z ∈ sg*α-ker ({x}) so that z ∉ sg*αCl ({y}) (or z ∈ sg*α-ker({y}) such that z ∉ sg*αCl ({x})).There exists a sg*α-open set V such that y ∉ V and z ∈ V ; hence x ∈ V. Therefore , we have x ∉ sg*αCl ({y}). Thus x ∈X – sg*αCl ({y}) a sg*α-open set, which implies sg*αCl ({x}) ⊆ sg*αCl ({y}) and sg*αCl ({x}) ∩ sg*αCl ({y}) =∅.

Contrarily , let V be a sg*α-open set in X and let x ∈ V. Now we have claim that sg*αCl ({x}) ⊂ V. Make y ∉ V that is , y ∈ X – V. Then x ≠ y as well as x ∉ sg*αCl ({y}).This implies , sg*αCl ({x}) ≠ sg*αCl ({y}). By assumption , sg*αCl{x}) ∩ sg*αgCl ({y}) = ∅. Hence y ∉ sg*αCl ({x}) and therefore sg*αCl ({x})) ⊆ V.

Theorem6.8: A TS X is sg*α-R_o space iff for any x , y in X sg*α-ker ({x}) ≠ sg*α-ker ({y}) implies sg*α-ker ({x}) ∩ sg*α-ker ({y}) =∅.

Proof: Suppose X is sg*α-R_o space. Thus by Lemma 6.6 for any points x , y ∈ X whenever sg*α-ker ({x}) ≠ sg*α-ker ({y}) then sg*αCl ({x}) ≠ sg*αCl ({y}). Now we prove that sg*α-ker ({x}) ∩ sg*α-ker ({y}) = ∅ . Suppose that z ∈ sg*α-ker ({x}) ∩ sg*α-ker ({y}). By Lemma 6.5 and z ∈ sg*α-ker ({x}) implies x ∈ sg*α-ker ({z}). Since x ∈ sg*αCl ({x}) , by Theorem 6.7, sg*αCl ({x}) = sg*αCl ({z}). Similarly , we have sg*αCl ({y}) = sg*αCl ({x}) a contradiction. Hence sg*α-ker ({x}) ∩ sg*α-ker ({y}) = ∅. Conversely , Let X be a topological space such that for any points x and y in X , sg*α-ker({x}) ≠ sg*α-ker ({y}) implies sg*α-ker ({x}) ∩ sg*α-ker ({y}) =∅ . If sg*αCl ({x}) ≠ sg*αCl ({y}) , then by Lemma 6.6 , sg*α-ker ({x}) ≠ sg*α-ker ({y}). Hence sg*α-ker ({x}) ∩ sg*α-ker ({y}) = ∅ implies sg*αCl ({x}) ∩ sg*αCl ({y}) = ∅. Since z ∈ sg*αCl ({x}) implies that x ∈ sg*α-ker ({z}). Therefore sg*α-ker ({x}) = sg*α-ker ({z}). Then z ∈ sg*αCl ({x}) ∩ sg*αCl({y}) implies that sg*α-ker ({x}) = sg*α-ker ({z}) = sg*α-ker ({y}) , a contradiction. Hence sg*αCl ({x}) ∩ sg*αCl ({y}) =∅. Therefore by Theorem 6.7, X is a sg*α-R_o space.

Theorem 6.9: For a TS X the following properties are equivalent:

- X is a sg*α-R_o space.
- x ∈ sg*αCl ({x}) if and only if y ∈ sg*αCl {x}) for any points x and y in X.

Proof: (i) → (ii). Assume that X is a sg*α-R_o space. Let x ∈ sg*αCl ({y}) and U be any sg*α-open set such that y ∈ U. Now by hypothesis x ∈ U. Therefore, every sg*α-open set containing y contains x. Hence y ∈ sg*αCl ({x}).

(ii) → (i). Let V be a sg*α-open set and x ∈ V. If y ∉ V then x ∉ sg*αCl ({x}) and hence y ∉ sg*αCl ({x}). This implies that sg*αCl ({x}) ⊆ V. Hence X is a sg*α-R_o space.

Theorem 6.10: For a topological space X the following properties are equivalent;

- X is a sg*α-R_o space.
- Whenever A is a sg*α-closed , then A = sg*α-ker (A).
- Whenever A is a sg*α-closed as well as x ∈ A , thereupon sg*α-ker ({x}) ⊆ A
- Whenever x ∈X, then sg*α-ker ({x}) ⊆ sg*αCl ({x}).

Proof: (i) → (ii). Let A be sg*α-closed and x ∉A .Thus X – A is a sg*α-open and x ∈X –A . Since X is a sg*α-R_o space , sg*αCl ({x}) ⊆ X – A. Thus sg*αCl({x}) ∩ A =∅ and by the Lemma 6.3 , x ∉ sg*α-ker (A). Therefore sg*α-ker (A) = A.
(ii) → (ii). In general U ⊆ V implies sg*α-ker (U) ⊆ sg*α-ker (V). Therefore sg*α-ker ({x}) ⊆ sg*α-ker (A) = A by (ii).
(iii) → (iv). Since x ∈ sg*αCl ({x}) and sg*αCl ({x}) is sg*α-closed by
(iii) sg*α-ker ({x}) ⊆ sg*αCl ({x}). (iv) → (i). Let x ∈ sg*αCl ({x}) then by the Lemma 6.5 , y ∈ sg*α-ker ({x}). Since x ∈ sg*αCl ({x}) and sg*αCl ({x})is sg*α-closed , by (iv) we obtain y ∈ sg*α-ker ({x}) ⊆ sg*αCl ({x}).Therefore x ∈ sg*αCl ({y}) implies y ∈ sg*αCl ({x}) .The converse is obvious and X is a sg*α-R_o space.

Definition 6.14: A TS X is termed as

- sg*α-C_o whenever for x , y ∈ X with x ≠ y , there exists a sg*α-open set G such that sg*αCl ({G}) contains one of x and y but not other.
- sg*α-C_1 whenever for x , y ∈ X with x ≠ y , there exist sg*α-open sets G and H such that x ∈ sg*αCl (G) , y ∈ sg*αCl (H) but x ∉ sg*αCl (H) ,y ∉ sg*αCl (G).
- weakly sg*α-C_o whenever ∩ sg*α-ker ({x}) = ∅.
- weakly sg*α-R_o whenever ∩ { sg*αCl ({x}) / x ∈X }

Theorem 6.15: A topological space X is weakly sg*α-R_o if and only if sg*α-ker ({x}) ≠ X for x ∈X

Proof: Necessity: Assume that there is a point x_o in X with sg*α-ker ({x_o}) = X. Then X is the only sg*α-open set containing x_o. This implies that x_o∈ sg*α-ker ({x}) for every x ∈X. Hence x_o ∈∩ { sg*αCl ({x}) / x ∈ X} ≠ ∅ , a contradiction.

Sufficiency: If X is not weakly sg*α-R_0, then choose some x_o in X such that x_o ∈∩ { sg*αCl ({x}) / x ∈ X}. This implies that every sg*α-open set containing x_o must contain every point of X. Thus the space X is the unique sg*α-open set containing x_o. Hence sg*α-ker ({x_o}) = X, which is a contradiction. Therefore X is weakly sg*α-R_o.

Theorem 6.16: A space X is weakly sg*α-C_o if and only if for each x ∈X, there exists a proper sg*α-closed set containing y.

Proof: Suppose there is some y ∈ X such that X is the only sg*α-closed set containing y .Let U be any proper sg*α-open subset of X containing a point of x. This implies that X – U ≠ X. Since X – U is sg*α-closed set, we have y ∈ X – U. So, y ∈ U. Thus y ∈∩ { sg*α-ker ({x}) / x ∈ X} for any point x of X , a contradiction. Conversely, suppose X is not weakly sg*α-C_o , then choose y ∈∩ { sg*α-ker ({x}) / x ∈ X}. So y belongs to sg*α-ker ({x}) for any x ∈X. This implies that X is the only sg*α-open set which contains the point y, a contradiction.

Theorem 6.17: Every sg*α-C_o (or sg*α-C_1) space is weakly sg*α-C_o

Proof: Whenever p , q ∈ X such that p ≠ q , where X is a sg*α-C_o space , then without loss of generality , we can assume that there exists a sg*α-open set G such that p ∈ sg*αCl(G) but q ∉ sg*αCl(G) . This implies that G ≠ ∅ . Hence we can choose some z in G. Now sg*αker (z) ∩ sg*α-ker (q) ⊆ G ∩ (sg*αCl (G))^C = sg*αCl (G) ∩ (sg*αCl (G))^C =∅. Therefore ∩ {sg*α-ker ({p}) / p ∈ X} = ∅. Hence the space X is weakly sg*α-C_o.

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