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RESEARCH ARTICLE

A NEW SUB CLASS OF MEROMORPHICALLY CONVEX FUNCTIONS WITH NEGATIVE AND FIXED SECOND COEFFICIENTS

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ABSTRACT

In this paper, we introduce and study a subclass $\Lambda_k(\alpha,\beta,A,B,\lambda)$ of meromorphic univalent functions. We obtain coefficients inequalities, extreme points, distortion and growth bounds, radii of meromorphically starlikeness and meromorphically convexity for this class. Further it is shown that this class is closed under convex linear combination.

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INTRODUCTION

Let Σ denote the class of functions of the form

$$f(z)=\frac{1}{z}+\sum_{n=1}^{\infty}a_n z^n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.1)$$

Which are analytic in the punctured open unit disk

$$D^*=\{z:z\in C,0<|z|<1\}=D\setminus\{0\}$$

with a simple pole at the origin and residue 1 there.

Let Σ_k denote the subclass of Σ consisting of functions f(z) which are convex with respect to the origin, i.e. satisfying the condition:

$$R\left\{-\left(1+\frac{zf''(z)}{f'(z)}\right)\right\}>0.(z\in D^*)\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.2)$$

Let $\Sigma_k(\alpha)$ denote the subclass of Σ consisting of functions f(z) which are convex of order α ,i.e. satisfying the condition

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$$R\left\{-\left(1+\frac{zf''(z)}{f'(z)}\right)\right\} > \alpha.(z \in D^*; 0 \le \alpha < 1) \quad \text{.................................(1.3)}$$

and similar other classes of meromorphically univalent functions have been defined and studied by Altintas et al.[1], Aouf[2,3], Ganigi and Uralegaddi[6],Uralegaddi[9],Uralegaddi and Ganigi[10] and others.

Let $\Sigma_k(\alpha, A, B)$ denote the class of functions f(z) in Σ which satisfy the condition

$$\left| \frac{\frac{zf''(z)}{f'(z)}+2}{B\left(1+\frac{zf''(z)}{f'(z)}\right)+[B+(A-B)(1-\alpha)]} \right| < 1 \quad \text{.............................(1.4)}$$

$$(z \in D^*, 0 \le \alpha < 1; -1 \le A < B \le 1; 0 < B \le 1)$$

We note that $\Sigma_k(\alpha, -1, 1) = \Sigma_K(\alpha)$.

Let Λ denote the subclass of Σ consisting of functions of the form

$$f(z) = \frac{1}{z} - \sum_{n=1}^{\infty} |a_n| z^n \quad \text{..................................(1.5)}$$

Now in the following definition, we define a subclass $\Lambda_k(\alpha, \beta, A, B)$ for functions in the class Σ .

Definition 1.1: A function f(z) defined by (1.5) is in the class $\Lambda_k(\alpha, \beta, A, B)$ if it satisfies the condition

$$\left| \frac{\frac{zf''(z)}{f'(z)}+2}{B\left(1+\frac{zf''(z)}{f'(z)}\right)+[B+(A-B)(1-\alpha)]} \right| < \beta \quad \text{...........................(1.6)}$$

$$(z \in D^*, 0 \le \alpha < 1; 0 < \beta \le 1; -1 \le A < B \le 1; 0 < B \le 1)$$

For the class $\Lambda_k(\alpha, \beta, A, B)$,the following characterization was given bySrivastava et al. [8].

Theorem 1.1: Let the function f(z) defined by (1.5) be analytic in D* .Then $f(z) \in \Lambda_K(\alpha, \beta, A, B)$ if and only if

$$\sum_{n=1}^{\infty} n\{(n+1) + \beta[Bn + (B-A)\alpha + A]\}|a_n| \le (B-A)\beta(1-\alpha). \quad \text{...(1.7)}$$

For a function f(z) defined by (1.5) and in the class $\Lambda_k(\alpha, \beta, A, B)$,Theorem 1.1 yields

$$a_1 \le \frac{(B-A)\beta(1-\alpha)}{2+\beta[B+(B-A)\alpha+A]} \quad \text{...(1.8)}$$

Hence we may take

$$a_1 = \frac{(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A]}, \lambda(0 < \lambda < 1) \quad \text{...(1.9)}$$

Motivated by the works of Aouf and Darwish[4],Aouf and Joshi[5],Sivasubramanian et al.[7], we now introduce the following class of functions and use the similar techniques to prove our results.

Let class $\Lambda_k(\alpha,\beta,A,B,\lambda)$ be the subclass of $\Lambda_k(\alpha,\beta,A,B)$ consisting of functions of the form

$$f(z)=\frac{1}{z}-\frac{(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A\}}z-\sum_{n=2}^{\infty}n\{(n+1)+\beta[Bn+(B-A)\alpha+A]\}a_n z^n \quad\text{.........................(1.10)}$$

where $0<\lambda<1.$

In this paper, we obtain coefficient inequalities, extreme points, distortion and growth bounds, radii of meromorphically starlikeness and meromorphically convexity for the class $\Lambda_k(\alpha,\beta,A,B,\lambda)$ by fixing the second coefficient. Further it is shown that the class $\Lambda_k(\alpha,\beta,A,B,\lambda)$ is closed under convex linear combinations.

2.Cofficients Inequalities

Theorem 2.1: Let the function f(z) is defined by (1.10). then $f(z)\in\Lambda_k(\alpha,\beta,A,B,\lambda)$

iff

$$\sum_{n=2}^{\infty}n\{(n+1)+\beta[Bn+(B-A)\alpha+A]\}|a_n|\le(B-A)\beta(1-\alpha)(1-\lambda). \quad\text{..(2.1)}$$

The result is sharp.

Proof: By putting

$$a_1=\frac{(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A]},(0<\lambda<1) \quad\text{...(2.2)}$$

in (1.7),the result is easily derived. The result is sharp for the function

$$f(z)=\frac{1}{z}-\frac{(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A\}}z-\frac{(B-A)\beta(1-\alpha)(1-\lambda)}{n\{(n+1)+\beta[Bn+(B-A)\alpha+A]\}}z^n,n\ge 2. \quad\text{.........................(2.3)}$$

Corollary 2.2: If the function f(z) defined by (1.10) is in the class $\Lambda_k(\alpha,\beta,A,B,\lambda)$

then

$$a_n\le\frac{(B-A)\beta(1-\alpha)(1-\lambda)}{n\{(n+1)+\beta[Bn+(B-A)\alpha+A]\}},n\ge 2. \quad\text{...(2.4)}$$

The result is sharp for the function f(z) given by (2.3).

3. DISTORTION THEOREMS

Theorem 3.1: If the function f(z) defined by (1.10) is in the class $\Lambda_k(\alpha,\beta,A,B,\lambda)$.

Then for $0<|z|=r<1$, we have

$$\frac{1}{r}-\frac{(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A]}r-\frac{(B-A)\beta(1-\alpha)(1-\lambda)}{2\{3+\beta[2B+(B-A)\alpha+A]\}}r^2$$
$$\le|\mathrm{f(z)}|\le\frac{1}{r}+\frac{(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A]}r+\frac{(B-A)\beta(1-\alpha)(1-\lambda)}{2\{3+\beta[2B+(B-A)\alpha+A]\}}r^2 \quad\text{........................(3.1)}$$

Where equality holds true for the function

$$\mathrm{f(z)} = \frac{1}{z} - \frac{(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A]} z - \frac{(B-A)\beta(1-\alpha)(1-\lambda)}{2\{3+\beta[2B+(B-A)\alpha+A]\}} z^2 \quad \text{……………………(3.2)}$$

and

$$\frac{1}{r^2} - \frac{(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A]} - \frac{(B-A)\beta(1-\alpha)(1-\lambda)}{\{3+\beta[2B+(B-A)\alpha+A]\}} r$$
$$\le |\mathrm{f}'(\mathrm{z})| \le \frac{1}{r^2} + \frac{(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A]} + \frac{(B-A)\beta(1-\alpha)(1-\lambda)}{\{3+\beta[2B+(B-A)\alpha+A]\}} r \quad \text{…………………(3.3)}$$

Proof: Since $f(z) \in \Lambda_K(\alpha, \beta, A, B, \lambda)$,then from theorem (2.1)

$$a_n \le \frac{(B-A)\beta(1-\alpha)(1-\lambda)}{n\{(n+1)+\beta[Bn+(B-A)\alpha+A]\}}, n \ge 2. \quad \text{…………………………………………(3.4)}$$

Then for $0 < |z| = r < 1$

$$|f(z)| \le \frac{1}{|z|} + \frac{(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A]}|z| + \sum_{n=2}^{\infty} a_n |z|^n$$

$$\le \frac{1}{r} + \frac{(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A]} r + r^2 \sum_{n=2}^{\infty} a_n$$

$$\le \frac{1}{r} + \frac{(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A]} r + \frac{(B-A)\beta(1-\alpha)(1-\lambda)}{2\{3+\beta[2B+(B-A)\alpha+A]\}} r^2 \quad \text{...........................(3.5)}$$

and

$$|f(z)| \ge \frac{1}{|z|} - \frac{(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A]}|z| - \sum_{n=2}^{\infty} a_n |z|^n$$

$$\ge \frac{1}{r} - \frac{(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A]} r - r^2 \sum_{n=2}^{\infty} a_n$$

$$\ge \frac{1}{r} - \frac{(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A]} r - \frac{(B-A)\beta(1-\alpha)(1-\lambda)}{2\{3+\beta[2B+(B-A)\alpha+A]\}} r^2 \quad \text{...........................(3.6)}$$

Thus (3.5) and (3.6) together yield (3.1).

Further more,from theorem 2.1,it follows that

$$na_n \le \frac{(B-A)\beta(1-\alpha)(1-\lambda)}{\{3+\beta[2B+(B-A)\alpha+A]\}}, n \ge 2. \quad \text{.......................................(3.7)}$$

Then for 0 <|z| = r < 1 and using(3.7),we obtain

$$|f'(z)| \le \frac{1}{|-z^2|} + \frac{(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A]} + \sum_{n=2}^{\infty} na_n|z|^{n-1}$$

$$\le \frac{1}{r^2} + \frac{(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A]} + \frac{(B-A)\beta(1-\alpha)(1-\lambda)}{\{3+\beta[2B+(B-A)\alpha+A]\}} r \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3.8)$$

$$\text{and} \qquad |f'(z)| \ge \frac{1}{|-z^2|} - \frac{(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A]} - \sum_{n=2}^{\infty} na_n|z|^{n-1}$$

$$\ge \frac{1}{r^2} - \frac{(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A]} - \frac{(B-A)\beta(1-\alpha)(1-\lambda)}{\{3+\beta[2B+(B-A)\alpha+A]\}} r \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3.9)$$

Thus (3.8) and (3.9) together yield (3.3).

4. CLOSURE THEREMS

Theorem 4.1: If

$$f_1(z) = \frac{1}{z} - \frac{(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A]} z \qquad \ldots\ldots\ldots\ldots\ldots\ldots(4.1)$$

$$f_n(z) = \frac{1}{z} - \frac{(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A]} z - \sum_{n=2}^{\infty} \frac{(B-A)\beta(1-\alpha)(1-\lambda)}{n\{(n+1)+\beta[nB+(B-A)\alpha+A]\}} z^n, n \ge 2. \qquad \ldots\ldots\ldots\ldots(4.2)$$

Then $f(z) \in \Lambda_K(\alpha, \beta, A, B, \lambda)$ if and only if it can be expressed in the form

$$f(z) = \sum_{n=1}^{\infty} \mu_n f_n(z) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.3)$$

$$\text{where } \mu_n \ge 0 \text{ and } \sum_{n=1}^{\infty} \mu_n = 1.$$

Proof: From (4.2) and (4.3), we have

$$f(z) = \sum_{n=1}^{\infty} \mu_n f_n(z) = \mu_1 f_1(z) + \sum_{n=2}^{\infty} \mu_n f_n(z)$$

$$= \frac{1}{z} - \frac{(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A]} z - \sum_{n=2}^{\infty} \frac{(B-A)\beta(1-\alpha)(1-\lambda)\mu_n}{n\{(n+1)+\beta[nB+(B-A)\alpha+A]\}} z^n.$$

$$\text{Since} \sum_{n=2}^{\infty} \frac{(B-A)\beta(1-\alpha)(1-\lambda)\mu_n}{n\{(n+1)+\beta[nB+(B-A)\alpha+A]\}} n\{(n+1)+\beta[nB+(B-A)\alpha+A]\}$$

$$= \sum_{n=2}^{\infty} (B-A)\beta(1-\alpha)(1-\lambda)\mu_n$$

$$= (B-A)\beta(1-\alpha)(1-\lambda)\sum_{n=2}^{\infty} \mu_n$$

$$\le (B-A)\beta(1-\alpha)(1-\lambda).$$

So from theorem (2.1) it follows that $f(z) \in \Lambda_K(\alpha, \beta, A, B, \lambda)$

Conversely let $f(z) \in \Lambda_K(\alpha, \beta, A, B, \lambda)$.
Since

$$a_n \le \frac{(B-A)\beta(1-\alpha)(1-\lambda)}{n\{(n+1)+\beta[Bn+(B-A)\alpha+A]\}}, n \ge 2.$$

Setting

$$\mu_n = \frac{n\{(n+1)+\beta[Bn+(B-A)\alpha+A]\}}{(B-A)\beta(1-\alpha)(1-\lambda)} a_n$$

and

$$\mu_1 = 1 - \sum_{n=2}^{\infty} \mu_n$$

It follows that

$$f(z) = \sum_{n=1}^{\infty} \mu_n f_n(z).$$

This complete the proof.

Theorem 4.2: The class $f(z) \in \Lambda_K(\alpha, \beta, A, B, \lambda)$ is closed under convex linear combinations.

Proof: Suppose the function f(z) be given by (1.10) and let the function g(z) be given by

$$g(z) = \frac{1}{z} - \frac{(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A]} z - \sum_{n=2}^{\infty} |b_n| z^n, n \ge 2.$$

Assuming that f(z) and g(z) are in the class $\Lambda_K(\alpha, \beta, A, B, \lambda)$, it is enough to prove that the function h(z) defined by $h(z) = \mu f(z) + (1-\mu) g(z), 0 \le \mu \le 1$.

is also in the class $\Lambda_K(\alpha, \beta, A., B, \lambda)$ .Since

$$h(z) = \frac{1}{z} - \frac{(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A]} z - \sum_{n=2}^{\infty} |\mu a_n + (1-\mu) b_n| z^n.$$

We observe that

$$\sum_{n=2}^{\infty} n\{(n+1)+\beta[Bn+(B-A)\alpha+A]\} |\mu a_n + (1-\mu) b_n| \le (B-A)\beta(1-\alpha)(1-\lambda)$$

with the aid of theorem 2.1.Thus $h(z) \in \Lambda_k(\alpha, \beta, A, B, \lambda)$.

5.RADIUS OF STARLIKENESS AND CONVEXITY

Theorem 5.1: Let the function f(z) defined by (1.10) be in the class $\Lambda_K(\alpha, \beta, A, B, \lambda)$,then we have

(i) f is meromorphically starlike of order $\delta (0 \le \delta < 1)$ in the disk $|z| < r_1(\alpha, \beta, A, B, \lambda, \delta)$ where

$r_1(\alpha, \beta, A, B, \lambda, \delta)$ is the largest value for which

$$\frac{(3-\delta)(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A]}r^2+\frac{(n+2-\delta)(B-A)\beta(1-\alpha)(1-\lambda)}{n\{(n+1)+\beta[Bn+(B-A)\alpha+A]\}}r^{n+1}\le 1-\delta, n\ge 2. \quad ....(5.1)$$

(ii)f is meromorphically convex of order $\delta(0\le\delta<1)$ in the disk $|z|<r_2(\alpha,\beta,A,B,\lambda,\delta)$ where $r_2(\alpha,\beta,A,B,\lambda,\delta)$ is the largest value for which

$$\frac{(3-\delta)(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A]}r^2+\frac{(n+2-\delta)(B-A)\beta(1-\alpha)(1-\lambda)}{\{(n+1)+\beta[Bn+(B-A)\alpha+A]\}}r^{n+1}\le 1-\delta, n\ge 2. \quad ....(5.2)$$

Proof: It is enough to show that

$$\left|\frac{zf'(z)}{f(z)}+1\right|\le 1-\delta, |z|<r_1.$$

Thus we have

$$\left|\frac{zf'(z)}{f(z)}+1\right|=\left|\frac{-\frac{2(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A}z-\sum_{n=2}^{\infty}(n+1)a_nz^n}{\frac{1}{z}-\frac{(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A}z-\sum_{n=2}^{\infty}a_nz^n}\right| \quad ....(5.3)$$

Hence (5.3) holds ture if

$$\frac{2(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A}r^2+\sum_{n=2}^{\infty}(n+1)a_nr^{n+1}$$
$$\le(1-\delta)\left[1-\frac{(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A}r^2-\sum_{n=2}^{\infty}a_nr^{n+1}\right],$$

or,

$$\frac{(3-\delta)(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A]}r^2+\sum_{n=2}^{\infty}(n+2-\delta)a_nr^{n+1}\le(1-\delta).$$

and it follows that from (2.1),we may take

$$a_n\le\frac{(B-A)\beta(1-\alpha)(1-\lambda)}{n\{(n+1)+\beta[Bn+(B-A)\alpha+A]\}}, n\ge 2.$$

For each fixed r, we choose the positive integer $n_1=n_1(r_1)$ for which

$$\frac{(n+2-\delta)}{n\{(n+1)+\beta[Bn+(B-A)\alpha+A]\}}r^{n+1}$$

is maximal. Then it follows that

$$\sum_{n=2}^{\infty}(n+2-\delta)a_nr^{n+1}\le\frac{(n_1+2-\delta)(B-A)\beta(1-\alpha)(1-\lambda)}{n_1\{(n_1+1)+\beta[Bn_1+(B-A)\alpha+A]\}}r^{n_1+1}$$

Then f(z) is starlike of order δ in $0<|z|<r_1(\alpha,\beta,A,B,\lambda,\delta)$ provided that

$$\frac{(3-\delta)(B-A)\beta(1-\alpha)\lambda}{2+\beta[B+(B-A)\alpha+A]}r^2+\frac{(n_1+2-\delta)(B-A)\beta(1-\alpha)(1-\lambda)}{n_1\{(n_1+1)+\beta[Bn_1+(B-A)\alpha+A]\}}r^{n_1+1}\le 1-\delta.$$

We find the value $r_1 = r_1(\alpha, \beta, A, B, \lambda, \delta)$ and the corresponding integer $n_1 = n_1(r_1)$ so that

$$\frac{(3-\delta)(\mathrm{B}-\mathrm{A})\beta(1-\alpha)\lambda}{2+\beta[\mathrm{B}+(\mathrm{B}-\mathrm{A})\alpha+\mathrm{A}]} r^2 + \frac{(n_1+2-\delta)(B-A)\beta(1-\alpha)(1-\lambda)}{n_1\{(n_1+1)+\beta[Bn_1+(B-A)\alpha+A]\}} r^{n_1+1} = 1-\delta.$$

It is the value for which the functionf(z) is starlike in $0 < |z| < r_1$.

(ii) In a similar manner, we can prove our result providing the radius of meromorphically convexity of order $\delta (0 \le \delta < 1)$ for the function $\Lambda_K(\alpha, \beta, A, B, \lambda)$.

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