ISSN: 0975-833X

Available online at http://www.journalcra.com

International Journal of Current Research
Vol. 8, Issue, 10, pp.40456-40458, October, 2016

INTERNATIONAL JOURNAL OF CURRENT RESEARCH

RESEARCH ARTICLE

SOME CONSTRAINS ON ABSORPTION AND RIGHT REGULAR SEMIRINGS

*Amala, M.

Department of Applied Mathematics, Yogi Vemana University, Kadapa, Andhra Pradesh, India

ARTICLE INFO

Article History:
Received 27th July, 2016
Received in revised form
12th August, 2016
Accepted 18th September, 2016
Published online 30th October, 2016

Key words:

Rectangular Band,
Square Regular,
Positive Totally Ordered,
Zeroid.

ABSTRACT

In this paper we have shown that If S is a zero-square and a Right regular semiring. Here 0 is an additive identity, then axa = 0 for all a, x in S and (S, +) is a band.

Citation: Amala, M. 2016. "Some constrains on absorption and right regular Semirings", *International Journal of Current Research,* 8, (10), 40456-40458.

INTRODUCTION

The theory of semigroups had essentially two origins. One was an attempt to generalize both group theory and ring theory to the algebraic system consisting of a single associated operation which from the group theoretical point of view omits the axioms of the existence of the identities and inverses and from the ring theoretical point of view omits the additive structure of the ring. Semirings are used for modeling social network analysis, Economics, queuing theory, computation of bio-polymers, penalty theory in artificial intelligence, computation theory, modern control theory of psychological phenomenon. Semirings are used for physical theory on cognitive processes. S.Gosh studied the class of idempotent semiring. He revealed that S is distributive lattice if S is an idempotent commutative semiring satisfying the absorption equality a + ax = a for all a, x in S. This paper is concerned with structures of Absorption semiring. In first section, structures of Absorption semiring are given. In last section, we also introduce the notion of Right regular semiring as a generalization of regular semiring. Sen, Ghosh & Mukhopadhyay studied the congruences on inverse semirings with the commutative additive reduct and Maity improved this to the regular semirings with the set of all additive idempotents a bi-semilattice. The study of regular semigroups has yielded many interesting results.

Corresponding author: Amala, M.,
Dept. of Applied Mathematics, Yogi Vemana University, Kadapa, Andhra Pradesh, India.

These results have applications in other branches of algebra and analysis. We will see the interrelations between different semirings. We characterize zerosumfree semiring. A semiring S is a Right regular semiring, if S satisfies the identity a + xa + a = a for all a, x in S.

PRELIMINARIES

Definition 2.1:

A semiring S is Absorption if it satisfies the condition a + ax = a for all a, x in S.

Definition 2.2:

A semiring S is Positive Rational Domain (PRD) if (S, •) is an abelian group.

Definition 2.3:

A semigroup (S, +) is left (right) singular if it satisfies the identity a + x = a (a + x = x) for all a, x in S.

Definition 2.4:

A semigroup (S, +) is lateral zero if axc = x.

Definition 2.5:

A semigroup (S, +) is a band if a + a = a for all a in S.

Definition 2.6:

A semigroup (S, •) is left (right) singular if it satisfies the identity ax = a (ax = x) for all a, x in S.

Definition 2.7:

A semiring S is zero-square if $a^2 = 0$, for all a in S. Here zero is multiplicative zero.

Definition 2.8:

A semiring S is zerosumfree if a + a = 0, for all a in S. Here zero is an additive identity.

2.A Study on Absorption Semiring:

Theorem 3.1: Assume that S is an Absorption semiring and PRD. Then

- õ (S, +) is a band.
- õ 1 is additively rectangular element.

Proof: (i) Since S is an Absorption semiring and PRD $a + aa^{-1} = a$ for all a, a^{-1} in S

Again S is PRD this implies a + 1 = a ì **(1)**

Which implies $a^2 + a = a^2$

$\Rightarrow a + a^2 + a = a + a^2$
$\Rightarrow$ a + a = a for all a in S
Therefore (S, +) is a band
(ii) Given that a + a.a = a for all a in S
$\Rightarrow a (1 + a) = a.1$
$\Rightarrow 1 + a = 1$
$\Rightarrow 1 + a + 1 = 1$

Thus 1 is additively rectangular element

Theorem 3.2: If S is an Absorption semiring, (S, +) and (S, •) are left cancellative, then (S, +) is right singular semigroup.

Proof: By hypothesis S is an Absorption semiring then a + ax = a

Which implies $a + ax + a^2 = a + a^2$

Using (S, +) left cancellation law in above we get $ax + a^2 = a^2$
$\Rightarrow a (x + a) = a.a$
Using (S, •) left cancellation law in above we obtain x + a = a

Therefore x + a = a for all a, x in S

Hence (S, +) is right singular semigroup

Theorem 3.3: If S is an Absorption semiring and (S, +) is lateral zero, then for every three elements a, b, c in S, $a^n c^{n-1} + b = a^n c^{n-1}$ for all n ≥ 1.

Proof: Given S is an Absorption semiring then a + ab = a for all a, b in S
$\Rightarrow ac + abc = ac$
Since (S, +) is lateral zero abc = b then above equation implies
$ac + b = ac$
$\Rightarrow a + ac + b = a + ac$
$\Rightarrow a + b = a$
$\Rightarrow ac + bc = ac$
$\Rightarrow a^2c + abc = a^2c$
$\Rightarrow a^2c + b = a^2c$ ì **(1)**
$\Rightarrow a^3c^2 + abc = a^3c^2$
$\Rightarrow a^3c^2 + b = a^3c^2$ ì **(2)**

Continuing in a similar manner as above we get $a^n c^{n-1} + b = a^n c^{n-1}$

A Study on Right regular semiring

Proposition 4.1: Let S be a semiring in which (S, +) is a band and (S, •) is right singular semigroup. Then S is a Right regular semiring.

Proof: Assume that (S, +) is a band then a + a = a for all a in S

By hypothesis (S, •) is a right singular semigroup, xa = a for all a, x in S
$\Rightarrow a + xa = a + a$
$\Rightarrow a + xa = a$
$\Rightarrow a + xa + a = a + a$
$\Rightarrow a + xa + a = a$

Thus S is Right regular semiring

Proposition 4.2: Suppose S is a zerosumfree semiring containing the additive identity zero. Then S is a Right regular semiring if and only if xa = a for all a, x in S.

Proof: By hypothesis S is a zerosumfree semiring a + a = 0 for all a in S

Since S is Right regular semiring, a + xa + a = a for all a, x in S

$\Rightarrow a + xa + a + a = a + a \Rightarrow a + xa + 0 = 0$
$\Rightarrow a + a + xa = a + 0$
$\Rightarrow 0 + xa = a$
$\Rightarrow xa = a$

To prove the converse part let us consider xa = a

Adding a to both sides of above equation we get a + xa = a + a
$\Rightarrow a + xa = 0$
$\Rightarrow a + xa + a = 0 + a$
$\Rightarrow a + xa + a = a$

Therefore a + xa + a = a for all a, x in S

Hence S is a Right regular semiring

Theorem 4.3: If S is a Right regular semiring, (S, •) is left permutable and band, then a + ax + a = a for all a, x in S.

Proof: By hypothesis S is Right regular then $a + a^2 + a = a$ for all a in S

$\Rightarrow a^2 + axa + a^2 = a^2$

+	0	a	x
a	0	a	x
a	a	a	a
x	x	x	x

Since (S, •) is left permutable a (xa) = a (ax) for all a, x in S and also (S, •) is band, $a^2 = a$ then above equation becomes

a + a (ax) + a = a

$\Rightarrow a + a^2x + a = a$

$\Rightarrow$ a + ax + a = a

Therefore a + ax + a = a for all a, x in S

Theorem 4.4: If S is a zero-square and a Right regular semiring. Here 0 is an additive identity, then

- õ axa = 0 for all a, x in S.
- õ (S, +) is a band.

Proof: (i) By hypothesis S is Right regular semiring then a + xa + a = a

$\Rightarrow$ a (a + xa + a) = a.a

$\Rightarrow a^2 + axa + a^2 = a^2$

Using the definition of zero-square semiring, $a^2 = 0$ in above we get

0 + axa + 0 = 0

$\Rightarrow$ axa = 0

Therefore axa = 0 for all a, x in S

By hypothesis we have a + xa + a = a for all a, x in S

$\Rightarrow a + a^2 + a = a$ for all a in S

$\Rightarrow$ a + 0 + a = a

$\Rightarrow$ a + a = a for all a in S

Therefore (S, +) is a band

Example 4.5: A semiring S = {0, a, x} with addition and $S^2 = \{0\}$ is given as follows.

REFERENCES

Haixuan Yang and S. Ponizovskii, 1998. "Identities of Regular Semigroup Rings", Semigroup Forum Vol. 56 293- 295 Springer-Verlag New York Inc.

Jonathan S. Golan, 1999. "Semirings and their Applications", *Kluwer Academic Publishers*.

Jonathan S. Golan, 1992. "The theory of Semirings with applications in Mathematics and omputer Science", Pitman Monographs and surveys in pure and applied mathematics, Vol – 54, Zongamann House, Burnt Mill, Harlow, Essex CM 20 JE. England: Longman Scientific and Technical.

Jonathan S. Golan, 1999. "Semirings and Affine Equations over Them: Theory and Applications", Kluwer Academic Publishers.

Kanchan Jana, 2011."Quasi *K*-Ideals In *K*-Regular And Intra *K*-Regular Semirings", Pu. M.A. Vol. 22 (2011), No.1, Pp. 65-74.

Kehayopulu, N.1977. "On a characterization of regular duo le Semigroups", Maths. konica 7, 181-186.

Nambooripad, K.S.S.1975. "Structure of regular semigroups I fundamental regular semigroups", Semigroup Forum, Vol.9, 354-363.

Satyanarayana, M. 1985. "On the additive semigroup of ordered semirings". Semigroup Forum, 31, 193-199.

Satyanarayana, M.1981. "On the additive semigroup structure of semirings", Semigroup Forum, Vol.23, 7-14.

Sen, M.K. and Maity, S.K. 2004. "Semirings Embedded in a Completely Regular Semiring", Acta Univ. Palacki. Olomuc, Fac. rer. nat., Mathematica 43 141–146.

Vasantha Kandasamy, W.B. "Smarandache Semirings, Semifields and Semivector Spaces", Dept. of Mathematics, Indian Institute of Technology, Madras.

****