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RESEARCH ARTICLE

# AN EOQ MODEL FOR TIME DEPENDENT DETERIORATING ITEM WITH IMPRECISE SEASONAL TIME

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**ABSTRACT**

Here, inventory model of a deteriorating item during its seasonal time is considered where lifetime of an item has an upper limit. Rate of deterioration increases with time and depends on the duration of lifetime left. It is assumed that duration of the season of the item is imprecise in nature in non-stochastic sense, i.e., fuzzy in nature. Demand of the item is price dependent and unit cost of item is time dependent. Unit cost is a decreasing function at the beginning of the season and an increasing function at the end of the season and is constant during the remaining part of the season. The model is formulated to maximize the total proceeds out of the system from the planning horizon which is fuzzy in nature. As optimization of fuzzy objective is not well defined, using credibility measure of fuzzy event, an approach is proposed for comparison of two fuzzy objectives and using this approach a dominance based Particle Swarm Optimization (PSO) algorithm is proposed to find marketing decision for the decision maker (DM). In particular case when planning horizon is crisp, the model is also solved. The models are illustrated with some numerical examples and some sensitivity analyses have been made.



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## INTRODUCTION

For items like potato, onion, cabbage, cauliflower etc., it is normally observed that price of the item decreases with time at the beginning of the production season due to availability in the market and reaches to a minimum value. Price of the item remains constant at this minimum value during the major part of the season due to sufficient availability of the item in the market and towards the end of the season due to scarcity, cost again increases gradually and reaches its off season value. This price remains stable during the remaining part of the year. A considerable number of research works have been done for seasonal products by several researchers Zhou *et al*. (2004); Chen and Chang (2007); Panda *et al*. (2008); Banerjee and Sharma 2010A, 2010B). In most of these research works, it is assumed that price of the item decreases with time or demand increases with time. But the above mentioned real life phenomenon of a seasonal product is overlooked by the researchers. Another shortcoming of these research work is the assumption that the duration of the season of such products as crisp value. Although, the duration of the season for an item is finite but it varies from year to year due to environmental changes. So, it is worthwhile to assume this duration as a fuzzy parameter. Occurrence of fuzzy seasonal time leads to optimization problem with fuzzy objective function. In the last two decades extensive research work has been done on inventory control problems in fuzzy environment (Lee *et al*. (1991); Maiti *et al*. (2014); Lam and Wong (1996); Roy and Maiti (2000); Mondal and Maiti (2002); Kao and Hsu (2002); Bera *et al*. (2012). These problems considered different inventory parameters as fuzzy numbers which render fuzzy objective function. As optimization in fuzzy environment is not well defined some of these researcher transform the fuzzy parameters as equivalent crisp number or crisp interval and then the objective function is transformed to an equivalent crisp number/interval (Maiti and Maiti, 2007; Bera *et al*., 2012). Some of the researchers (Mandal and Maiti, 2002) set the fuzzy objective as fuzzy goal whose membership function as a linear/non-linear fuzzy number and try to optimize this membership function using Bellman Zadeh's principle (Bellman and Zadeh, 1970). Maiti and Maiti (2006) propose a technique where instead of objective function pessimistic return of the fuzzy objective is optimized. They use necessity measure on fuzzy event to determine this pessimistic return and propose fuzzy simulation process to find this return function. Recently, Maiti (2008, 2011) proposes a technique where possibility/necessity measure of objective function (fuzzy profit) on fuzzy goal is optimized to find optimal decision. All these studies transform the fuzzy objective of the

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problem to an equivalent crisp objective and solution of the reduced problem is taken as approximate solution of the fuzzy problem. But there exist always some error in such approximation. Again in present day competitive market, an erroneous inventory decision may invite a huge loss in business. So modeling of present day inventory control problems must be very realistic and a methodology is required which can deal with fuzzy objective function directly without reducing it to crisp form. Again, most of the seasonal products have finite lifetime and are deteriorating in nature (Mahata and Goswami, 2010). Rate of deterioration increases with time and actually depends on the amount of lifetime left. Rate of deterioration becomes 100% when age of product covers the lifetime. In the literature, there are several investigations for deteriorating items such as Jaber *et al*. (2009); Yadav *et al.* (2011); Sana (2011) and others. Most of the inventory articles are developed with constant deterioration. But the deterioration mentioned earlier, deterioration increases with time as stress of units on others causes damage. According to the author's best knowledge, very few articles have been published incorporating time varying deterioration Sarkar (2011).

Use of soft computing techniques for inventory control problems is a well established phenomenon. Several authors use Genetic Algorithm (GA) in different forms to find marketing decisions for their problems. Pal *et al*. (2009) uses GA to solve an EPQ model with price discounted promotional demand in an imprecise planning horizon. Roy *et al*. (2009) used a GA with varying population size to solve a production inventory model with stock dependent demand incorporating learning and inflationary effect in a random planning horizon. Bera and Maiti (2012) used GA to solve multi-item inventory model incorporating discount. Maiti *et al*. (2009) used GA to solve inventory model with stochastic lead time and price dependent demand incorporating advance payment. Mondal *et al*. (2002) uses a dominance based GA to solve a production-recycling model with variable demand, demand-dependent fuzzy return rate. Combining the features of GA and PSO a hybrid algorithm PSGA is used by Guchhait *et al*. (2013) to solve an inventory model of a deteriorating item with price and credit linked fuzzy demand. All these soft computing techniques are not capable to deal with fuzzy objective directly.

From the above discussion it is clear that there are some lacunas in fuzzy inventory models of deteriorating items, specially for seasonal products. In this research work an attempt has been made to reduce these lacunas. The aim of this research work is fourfold:

The aim of this research work is fourfold:

- Firstly to model price of a seasonal product as a function $f_1(t)$ of time which decreases monotonically for a duration $H_1$ at the beginning of the season and reaches a minimum value $f_1(H_1)$. The price remains at this value $f_1(H_1)$ during a period $H_2$. Then it again follows an increasing function $f_2(t)$ and after a period $H_3$ it reaches the off season value, i.e., $f_1(0)=f_2(H_1+H_2+H_3)$.
- Secondly to model the season length $(H_1+H_2+H_3)$ as imprecise parameter.
- Thirdly for such a realistic inventory model, rate of deterioration as increasing function of time which actually depends on the lifetime of the item.
- At length to introduce an approach which can deal with fuzzy optimization problem, without reducing the objective function to any deterministic form.

Here, inventory model for a deteriorating seasonal product is developed whose demand depends upon the unit cost of the product. Unit cost of the product is time dependent. During the beginning of the period as availability of the item gradually increases, unit cost decreases monotonically with time and reaches a constant value when availability of the item becomes stable. Unit cost remains constant until the items availability again decreases towards the end of the season. Then as availability decreases, unit cost gradually increases and reaches its value as it was at the beginning of the season and then the season ends. Here exponential increasing and decreasing rate of unit cost function is considered. Rate of deterioration $\theta$ of the item increases with time and is of the form $\theta=[1/(1+R-t)]$, where R is the lifetime of the product, t is the time passed after the arrival of the units in the inventory.

Clearly as $t \to R, \theta \to 1$ , i.e., when t=R, all units in the inventory will be spoiled. It is assumed that time horizon of the season is fuzzy in nature. In fact three parts in which unit cost function can be divided are considered as fuzzy number. The model is formulated to maximize the total proceeds out of the system which is fuzzy in nature. Using credibility measure of fuzzy events, fuzzy objectives due to different solutions are compared and a PSO is used following this comparison approach of objectives to find marketing decision for the DM. In a particular case when planning horizon is crisp the model is also solved using same PSO. The models are illustrated with some numerical examples and some sensitivity analyses have been presented.

**Mathematical Preliminaries**

$\Re$ represents the set of real numbers $\widetilde{A}$ and $\widetilde{B}$ be two fuzzy numbers with membership Let functions $\mu_{\widetilde{A}}$ and $\mu_{\widetilde{B}}$ respectively. Then taking degree of uncertainty as the semantics of fuzzy number, according to Liu and Iwamura (1998A, 1998B):

$$\mathrm{Pos}(\widetilde{A} * \widetilde{B})=\sup\{\min(\mu_{\widetilde{A}}(x), \mu_{\widetilde{B}}(y)), x, y\in \Re, x * y\} \quad (1)$$

Where the abbreviation Pos represent possibility and * is any one of the relations >, <, =, ≤, ≥. Analogously if $\widetilde{B}$ is a crisp number, say b, then

$$\mathrm{Pos}\, \widetilde{A} * b)=\sup\{\mu_{\widetilde{A}}(x), x\in \Re, x * b\} \quad (2)$$

On the other hand necessity measure of an event ( $\widetilde{A} * \widetilde{B}$ ) is a dual possibility measure. The grade of necessity of an event is the grade of impossibility of the opposite event and is defined as

$$\mathrm{Nes}(\widetilde{A} * \widetilde{B})=1-\mathrm{Pos}\,\overline{(\widetilde{A} * \widetilde{B})} \quad (3)$$

Where the abbreviation Nes represents measure and $\overline{(\widetilde{A} * \widetilde{B})}$ represents complement of the event $\widetilde{A} * \widetilde{B}$.

Similarly credibility measure of an $\widetilde{A} * \widetilde{B}$ is denoted by Cr ($\widetilde{A} * \widetilde{B}$) and is defined as (Liu and Liu, 2003)

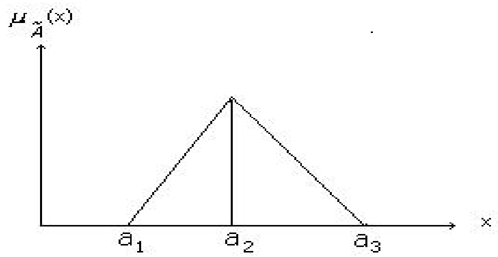
$$Cr(\widetilde{A} * \widetilde{B})=[Pos(\widetilde{A} * \widetilde{B})+Nes(\widetilde{A} * \widetilde{B})]/2 \quad (4)$$

If $\widetilde{A}$ , $\widetilde{B} \in \Re$ and $\widetilde{C} = f(\widetilde{A}, \widetilde{B})$ where $f: \Re \times \Re \rightarrow \Re$ be binary operation then membership function $\mu_{\widetilde{C}}$ of $\widetilde{C}$ can be obtained using Fuzzy Extension Principle (Zadeh, 1965, 1973) as

$$\mu_{\widetilde{C}}(z) = \sup\{\min(\mu_{\widetilde{A}}(x), \mu_{\widetilde{B}}(y)), x, y \in \Re, \ and \ z = f(x,y), \forall z \in \Re\} \quad (5)$$

**Triangular Fuzzy Number (TFN):** A TFN $\widetilde{A}$ is specified by the triplet ($a_1$, $a_2$, $a_3$) and is defined by its continuous membership function $\mu_{\widetilde{A}} : X \rightarrow [0,1]$ as follows (cf. Fig-1):

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & if \quad a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & if \quad a_2 \le x \le a_3 \\ 0 & otherwise \end{cases}$$



Triangular Fuzzy Number
Fig-1

Using these definitions the following lemmas can be derived

**Lemma1: If** $\widetilde{A}$ =( $a_1$, $a_2$, $a_3$) and $\widetilde{B}$ =($b_1$, $b_2$, $b_3$) be TFNS then

$$Cr(\widetilde{A} > \widetilde{B}) = \begin{cases} 1 & if \quad a_1 \ge b_3 \\ \dfrac{b_3 + 2(a_2 - b_2) - a_1}{2(b_3 - b_2 + a_2 - a_1)} & if \quad b_2 \le a_2, a_1 \le b_3 \\ \dfrac{a_3 - b_1}{a_3 - a_2 + b_2 - b_1} & if \quad a_2 \le b_2, a_3 \ge b_1 \\ 0 & otherwise \end{cases}$$

**Dominance based Particle Swarm Optimization technique**

During the last decade, nature inspired intelligence becomes increasingly popular through the development and utilization of intelligent paradigms in advance information systems design. Among the most popular nature inspired approaches, when task is to optimize with in complex decisions of data or information, PSO draws significant attention. Since its introduction a very large number of applications and new ideas have been realized in the context of PSO (Najafi *et al*., 2009; Marinakis and Marinaki, 2010). But till now PSO is not significantly used to solve inventory control problems (Guchhait *et al*., 2013). A PSO normally starts with a set of solutions (called swarm) of the decision making problem under consideration. Individual solutions are called particles and food is analogous to optimal solution. In simple terms, the particles are flown through a multi-dimensional search space, where the position of each particle is adjusted according to its own experience and that of its neighbors. The particle *i* has a position vector ($X_i(t)$), velocity vector ($V_i(t)$), the position at which the best fitness $X_{pbesti}(t)$ encountered by the particle so far and the best position of all particles $X_{gbest}(t)$ in current generation *t*. In generation $(t+1)$, the position and velocity of the particle are changed to $X_i(t+1)$ and $V_i(t+1)$ using following rules:

$$V_i(t+1) = wV_i(t) + \mu_1 r_1 (X_{pbesti}(t) - X_i(t)) + \mu_2 r_2 (X_{gbest}(t) - X_i(t)) \quad (6)$$

$$X_i(t+1) = X_i(t) + V_i(t+1) \quad (7)$$

The parameters $\mu_1$ and $\mu_2$ are set to constant values, which are normally taken as 2, $r_1$ and $r_2$ are two random values uniformly distributed in [0,1], $w$ ($0<w<1$) is inertia weight which controls the influence of previous velocity on the new velocity. Here ($X_{pbesti}(t)$) and ($X_{gbest}(t)$) are normally determined by comparison of objectives due to different solutions. So for optimization problem involving crisp objective the algorithm works well. The algorithm can be tactically used to optimize fuzzy objective also where a solution is said to be best solution among a set of solutions if fuzzy objective due to the solution dominates the fuzzy objectives of other solutions of the set. For maximization problem a objective $\widetilde{Z}_1$ may dominate other objective $\widetilde{Z}_2$ if Cr $(\widetilde{Z}_1 > \widetilde{Z}_2) > 0.5$. This is a valid fuzzy comparison as Cr $(\widetilde{Z}_1 > \widetilde{Z}_2)$ + Cr $(\widetilde{Z}_1 \ge \widetilde{Z}_2) = 1$. If objective value due to solution $X_i$ dominates objective value due to solution $X_j$, we say that $X_i$ dominates $X_j$. Using this dominance property PSO can be used to optimize crisp optimization problem. This form of the algorithm is named as dominance based PSO (DBPSO) and the algorithm takes the following form. In the algorithm $V_{max}$ represent maximum velocity of a particle, $B_{il}(t)$ and $B_{iu}(t)$ represent lower and upper boundary of the *i*-th variable respectively. check_constraint ($X_i(t)$) function check whether solution $X_i(t)$ satisfies the constraints of the problem or not. It returns 1 if the solution $X_i(t)$ satisfies the constraints of the problem otherwise it returns 0.

**Proposed DBPSO**

1. Initialize $\mu_1, \mu_2$, *w, N* and Maxgen.
2. Set iteration counter *t=0* and randomly generate initial swarm *P(t)* of *N* particles (solutions).
3. Determine objective value of each solution $X_i(t)$ and find $X_{gbest}(t)$ using dominance property.
4. Set initial velocity $V_i(t)$, $\forall X_i(t) \in P(t)$ and set $X_{pbesti}(t)=X_i(t)$, $\forall X_i(t) \in P(t)$.
5. While (t<Maxgen) do

6. For i=1:N do
7. $V_i(t+1) = wV_i(t) + \mu_1 r_1 (X_{pbesti}(t) - X_i(t)) + \mu_2 r_2 (X_{gbest}(t) - X_i(t))$
8. If $(V_i(t+1)>V_{max})$ then set $V_i(t+1)=V_{max}$.
9. If $(V_i(t+1)<-V_{max})$ then set $V_i(t+1)=-V_{max}$
10. $X_i(t+1)=X_i(t)+V_i(t+1)$
11. If $(X_i(t+1)>B_{iu}(t))$ then set $X_i(t+1)=B_{iu}(t)$.
12. If $(X_i(t+1)<B_{il}(t))$ then set $X_i(t+1)=B_{il}(t)$.
13. If check_constraint $(X_i(t+1))=0$
14. Set $X_i(t+1)=X_i(t)$, $V_i(t+1)=V_i(t)$
15. Else
16. If $X_i(t+1)$ dominates $X_{pbesti}$ $(t)$ then set $X_{pbesti}$ $(t+1)=X_i(t+1)$.
17. If $X_i(t+1)$ dominates $X_{gbest}$ $(t)$ then set $X_{gbest}$ $(t+1)=X_i(t+1)$.
18. End If.
19. End For.
20. Set $t=t+1$.
21. End While.
22. Output: $X_{gbest}(t)$.
23. End Algorithm

**Different procedures of DBPSO**

**(a) Representation of solutions:** A n dimensional real vector $X_i=(x_{i1}, x_{i2},\ldots\ldots,x_{in})$, is used to represent i-th solution, where $x_{i1}, x_{i2},\ldots\ldots, x_{in}$ represent n decision variables of the decision making problem under consideration.

**(b) Initialization:** N such solutions $X_i=(x_{i1}, x_{i2},\ldots\ldots,x_{in})$, i=1,2,….,N, are randomly generated by random number generator within the boundaries for each variable $[B_{jl}, B_{ju}]$, j=1,2,……,n. Initialize (P(0)) sub function is used for this purpose.

**(c) Dominance property:** For crisp maximization problem, a solution $X_i$ dominates a solution $X_j$ if objective value of $X_i$ is greater than that of $X_j$. For the fuzzy optimization problem a solution $X_i$ dominates a solution $X_j$ if Cr $(\tilde{Z}_i > \tilde{Z}_j) > 0.5$ where $\tilde{Z}_i$ and $\tilde{Z}_j$ are objective values of $X_i$ and $X_j$ respectively. It is valid fuzzy comparison operators as Cr $(\tilde{Z}_i > \tilde{Z}_j) +$ Cr $(\tilde{Z}_i \geq \tilde{Z}_j) = 1$.

**(d) Implementation:** With the above function and values the algorithm is implemented using C-programming language. Different parametric values of the algorithm used to solve the model are as follows (Engelbrech, 2005), $\mu_1 = 1.49618$, $\mu_2 = 1.49618$, $w = 0.7298$.

**Assumptions and notations for the proposed model**

The following notations and assumptions are used in developing the models.

(i) Inventory system involves only one item.
(ii) Time horizon *(H)* is finite and $H=H_1+H_2+H_3$.
(iii) Unit cost, i.e., purchase price p(t) is a function of t and is of the form

$$p(t) = \begin{cases} be^{-ct} & for\ 0 \le t \le H_1 \\ be^{-cH_1} & for\ H_1 \le t \le H_1 + H_2 \\ Ae^{\frac{cH_1(t-H_1-H_2)}{H_3}} & for\ H_1 + H_2 \le t \le H_1 + H_2 + H_3 \end{cases}$$

where A= $be^{-cH_1}$

(iv) Selling price *s(t)* is mark-up *m* of *p(t)* and *m* takes the values $m_1$, $m_2$ and $m_3$ during $(0,H_1)$, $(H_1, H_1+H_2)$ and $(H_1+H_2, H_1+H_2+H_3)$ i.e. $s(t)=m[=m_1, m_2, m_3]\ p(t)$.
(v) Demand is a function of selling price *s(t)* and is of the form $D(t)= \frac{D_0}{[s(t)]^\gamma} = \frac{D_1}{[p(t)]^\gamma}$ where $D_1 = \frac{D_0}{m^\gamma}$, $D_0>0$
(vi) The lead time is zero.
(vii) Deterioration rate $\theta(t)$ is a function of time where $\theta(t) = \frac{1}{1+R+T_{j-1}-t}$ where R is the maximum lifetime of the product. This form of deterioration comes from the fact that as $(t-T_{j-1}) \rightarrow$ R, $\theta(t) \rightarrow 1$ i.e rate of deterioration tends to 100%.
(viii) $T_i$ is the total time that elapses up to and including the *i*-th cycle $(i=1,2,\ldots,n_1+n_2+n_3)$ where $n_1+n_2+n_3$ denotes the total number of replenishment to be made during the interval $(0, H_1+H_2+H_3)$ and $T_0=0$.
(ix) $n_1$ is the number of replenishment to be made during $(0,H_1)$ at $t=T_0, T_1,\ldots\ldots, T_{n_1-1}$ So, there are $n_1$ cycles in this duration. As purchase cost decreases during this session, so demand increases. Hence, successive cycle length must decrease. Here, $\alpha$ is the rate of reduction of successive cycle length and $t_1$ is the first cycle length. So, i-th cycle length $t_i=t_1-(i-1)\alpha$.

$$T_i = \sum_{j=1}^{i} t_j = it_1 - \alpha\frac{i(i-1)}{2},\ i = 1,2,\ldots.., n_1. Clearly, T_{n_1} = H_1$$

$$Thus,\ n_1 t_1 - \alpha\frac{n_1(n_1-1)}{2} = H_1$$

$$\Rightarrow \alpha = \frac{2(n_1 t_1 - H_1)}{n_1(n_1-1)} \tag{8}$$

Here, $t_1$ is decision variable.

(x) $n_2$ be the number of replenishment to be made during $(H_1, H_1+H_2)$. Since purchase cost is constant, demand is also constant during this interval. So, all the sub-cycle length in this interval is assumed as constant. Replenishment are done at

$$t = T_{n_1}, T_{n_1+1}, \ldots\ldots.., T_{n_1+n_2-1}\ where\ T_{n_1+j} = T_{n_1} + (j-1)\frac{H_2}{n_2},\ j = 1,2,\ldots,n_2$$

(xi) $n_3$ is the number of replenishment to be made during $(H_1+H_2, H_1+H_2+H_3)$. During this interval, purchase cost increases, as a result demand decreases. So, the duration of placing of order gradually increases. Here, $\beta$ be the rate of increase of cycle length. Let $t_1'$ be the initial cycle length. Then i-th cycle length $t_i'=t_1'+(i-1)\ \beta$. $Thus,\ t'_{n_3} = t'_1 + (n_3-1)\beta.$ Orders are made at $t = T_{n_1+n_2}, T_{n_1+n_2+1,\ldots\ldots..}, T_{n_1+n_2+n_3-1}$ Where

$$T_{n_1+n_2+i} = T_{n_1+n_2} + \sum_{j=1}^{i} t'_j = H_1 + H_2 + it'_1 + \beta\frac{i(i-1)}{2}$$

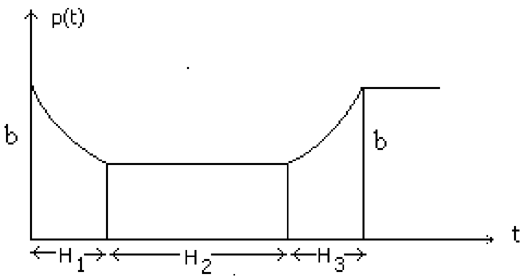
$$Clearly,\ T_{n_1+n_2+n_3} = H_1 + H_2 + H_3$$

$$H_1 + H_2 + n_3 t'_1 + n_3(n_3-1)\beta/2 = H_1 + H_2 + H_3$$

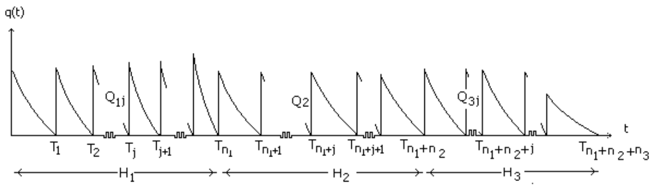
$$\Rightarrow \beta = \frac{2(H_3 - n_3 t'_1)}{n_3(n_3-1)} \qquad (9)$$

(x) $c_h$ is the holding cost per unit/unit time.
(xi) $c_0$ is the ordering cost.
(xii) $Q(T_i)$ is the order quantity at $t=T_i$.
(xiii) $q(t)$ is the inventory level at time t.
(xiv) Shortages are not allowed.
(xv) Z is the total profit from the planning horizon.

A wavy bar (~) is used with this symbol to represent corresponding fuzzy numbers when required.




Pictorial representation of p(t)
Fig-2




Inventory situation of the model
Fig-3

**Model development and analysis**

In the development of the model, it is assumed that at the beginning of every $j$-th cycle $[T_{j-1}, T_j]$, an amount $Q1_j$ units of item is ordered. As lead time negligible, replenishment of an item occurs as soon as order is made. Item is sold during the cycle and inventory level reaches zero at time $t=T_j$. Then order for next cycle is made. Here, selling price is a markup of initial purchase cost for each cycle. The inventory situation and the purchase cost are shown in Fig-2 and Fig-3.

**Formulation of the model in crisp environment**

This part is formulated in three phases.

**Formulation for first phase (i.e., $0 \le t \le H_1$):** Duration of $j$-th ( $1 \le j \le n_1$ ) cycle is $[T_{j-1}, T_j]$ where $T_{j-1} = jt_1 - \alpha j(j-1)/2$ at the beginning of the cycle inventory level is $Q1_j$. So, the governing differential equation of the model in the presence of deterioration of the item during $T_{j-1} \le t \le T_j$ is given by

$$\frac{dq(t)}{dt} + \theta(t)\,q = -D_j \qquad (10)$$

where $D_j = \dfrac{D_1}{\left(m_1 b e^{-cT_{j-1}}\right)^{\gamma}}$ and $\theta(t) = \dfrac{1}{1+R+T_{j-1}-t}$

Solving the above differential equation using the initial condition at $t=T_j$, $q(t)=0$, we get

$$q(t) = (1+R+T_{j-1}-t)D_j \log\left(\frac{1+R+T_{j-1}-t}{1+R+T_{j-1}-T_j}\right) \qquad (11)$$

When

$$t = T_{j-1},\ Q1_j = q(T_{j-1}) = (1+R)D_j \log\left(\frac{1+R}{1+R+T_{j-1}-T_j}\right) \qquad (12)$$

So, the holding cost for jth ($1 \le j \le n_1$) cycle, $H1_j$ is given by

$$H1_j = c_h \int_{T_{j-1}}^{T_j} q(t)\,dt$$

$$= c_h D_j\left[\frac{1}{4}\left\{\left(1+R+T_{j-1}-T_j\right)^2 - (1+R)^2\right\} + \frac{(1+R)^2}{2}\log\left(\frac{1+R}{1+R+T_{j-1}-T_j}\right)\right]$$

Thus, the total holding cost during $(0, H_1)$, HOC1, is given by

$$HOC1 = \sum_{j=1}^{n_1} H1_j \qquad (13)$$

Total purchase cost during $(0, H_1)$, PC1, is given by

$$PC1 = \sum_{j=1}^{n_1}\left[Q1_j\, p(T_{j-1})\right]$$

$$= \sum_{j=1}^{n_1}\left[(1+R)D_j \log\left(\frac{1+R}{1+R+T_{j-1}-T_j}\right)p(T_{j-1})\right] \qquad (14)$$

where $p(T_{j-1}) = be^{-cT_{j-1}}$

Total ordering cost during $(0, H_1)$, OC1, is given by

$$OC1 = \sum_{j=1}^{n_1}\left[c_{o1} + c_{o2}Q1_j\right] \qquad (15)$$

where $Q1_j$ is given by (12)

Selling price for j-th ($1 \le j \le n_1$) cycle $SP1_j$, is given by

$$SP1_j = m_1 p(T_{j-1}) \int_{T_{j-1}}^{T_j} D_j\,dt$$

$$= m_1 p(T_{j-1}) D_j (T_j - T_{j-1})$$

Total selling price during *(0, $H_1$),* SP1, is given by

$$SP1 = \sum_{j=1}^{n_1} SP1_j \tag{16}$$

**Formulation of second phase (i.e., $H_1 \le t \le H_1 + H_2$):** In the second phase, the purchase price of an item remains constant. So, the demand of customer is taken as constant. During of *j*-th ($n_1 \le j \le n_1 + n_2$) cycle is $[T_{j-1}, T_j]$. The governing differential equation of the model of deteriorating item during $T_{j-1} \le t \le T_j$ is given by

$$\frac{dq(t)}{dt} + \theta(t)\,q = -D_j \tag{17}$$

where $D_j = \dfrac{D_1}{\left(m_2 b e^{-cH_1}\right)^{\gamma}}$ and $\theta(t) = \dfrac{1}{1 + R + T_{j-1} - t}$

Solving the above differential equation using the initial condition t=$T_j$, q(t)=0, we get

$$q(t) = (1 + R + T_{j-1} - t) D_j \log\left(\frac{1 + R + T_{j-1} - t}{1 + R + T_{j-1} - T_j}\right) \tag{18}$$

When

$$t = T_{j-1},\ Q2_j = q(T_{j-1}) = (1+R) D_j \log\left(\frac{1+R}{1 + R + T_{j-1} - T_j}\right) \tag{19}$$

So, the holding cost for j-th ($n_1 \le j \le n_1 + n_2$) cycle, $H2_j$, is given by

$$H2_j = c_h \int_{T_{j-1}}^{T_j} q(t)\, dt$$

$$= c_h D_j \left[\frac{1}{4}\left\{(1 + R + T_{j-1} - T_j)^2 - (1+R)^2\right\} + \frac{(1+R)^2}{2} \log\left(\frac{1+R}{1 + R + T_{j-1} - T_j}\right)\right]$$

Thus, the total holding cost during ($H_1$, $H_1$+$H_2$), *HOC2*, is given by

$$HOC2 = \sum_{j=n_1+1}^{n_1+n_2} H2_j \tag{20}$$

Total purchase cost during *($H_1$, $H_1$+$H_2$), PC2*, is given by

$$PC2 = \sum_{j=n_1+1}^{n_1+n_2} \left[Q2_j\, p(T_{j-1})\right]$$
$$= \sum_{j=n_1+1}^{n_1+n_2} \left[(1+R) D_j \log\left(\frac{1+R}{1 + R + T_{j-1} - T_j}\right) p(T_{j-1})\right] \tag{21}$$

where $p(T_{j-1}) = b e^{-cH_1}$

Total ordering cost during ($H_1$, $H_1$+$H_2$), *OC2*, is given by

$$OC2 = \sum_{j=n_1+1}^{n_1+n_2} \left[c_{o1} + c_{o2} Q2_j\right] \tag{22}$$

where $Q2_j$ is given by (19)

Selling price for j-th ($n_1 \le j \le n_1 + n_2$) cycle *$SP2_j$*, is given by $SP2_j = m_2 p(T_{j-1}) \int_{T_{j-1}}^{T_j} D_j\, dt$

$$= m_2 p(T_{j-1}) D_j (T_j - T_{j-1})$$

Total selling price during *($H_1$, $H_1$+$H_2$), SP2,* is given by

$$SP2 = \sum_{j=n_1+1}^{n_1+n_2} SP2_j \tag{23}$$

**Formulation of third phase**

**(i.e., $H_1 + H_2 \le t \le H_1 + H_2 + H_3$):** In the second phase, duration of *j*-th ($n_1 + n_2 \le j \le n_1 + n_2 + n_3$) cycle is $[T_{j-1}, T_j]$

where

$$T_j = H_1 + H_2 + (j - n_1 - n_2) t_1' + (j - n_1 - n_2)(j - n_1 - n_2 - 1)\beta / 2$$

and at the beginning of cycle inventory level is *Q3j* . So, instantaneous state q(t) of deteriorating item during $T_{j-1} \le t \le T_j$ is given by

$$\frac{dq(t)}{dt} + \theta(t)\,q = -D_j \tag{24}$$

where $D_j = \dfrac{D_1}{\left(m_3 A e^{\frac{cH_1}{H_3}(T_{j-1} - H_1 - H_2)}\right)^{\gamma}}$ ,

$\theta(t) = \dfrac{1}{1 + R + T_{j-1} - t}$ and $A = b e^{-H_1}$

Solving the above differential equation using the initial condition t=$T_j$, q(t)=0, we get

$$q(t) = (1 + R + T_{j-1} - t) D_j \log\left(\frac{1 + R + T_{j-1} - t}{1 + R + T_{j-1} - T_j}\right) \tag{25}$$

When

$$t = T_{j-1},\ Q3_j = q(T_{j-1}) = (1+R) D_j \log\left(\frac{1+R}{1 + R + T_{j-1} - T_j}\right) \tag{26}$$

So, the holding cost for j-th ( $n_1 + n_2 \le j \le n_1 + n_2 + n_3$ ) cycle, $H3_j$ , is given by

$$H3_j = c_h \int_{T_{j-1}}^{T_j} q(t)\, dt$$

$$= c_h D_j \left[ \frac{1}{4}\left\{ (1+R+T_{j-1}-T_j)^2 - (1+R)^2 \right\} + \frac{(1+R)^2}{2} \log\left( \frac{1+R}{1+R+T_{j-1}-T_j} \right) \right]$$

Thus, the total holding cost during

( $H_1 + H_2 \le t \le H_1 + H_2 + H_3$ ), HOC3,

is given by $HOC3 = \sum_{j=n_1+1}^{n_1+n_2} H3_j$ (27)

Total purchase cost during ( $H_1 + H_2 \le t \le H_1 + H_2 + H_3$ ),

PC3, is given by

$$PC3 = \sum_{j=n_1+n_2+1}^{n_1+n_2+n_3} \left[ Q3_j\, p(T_{j-1}) \right]$$

$$= \sum_{j=n_1+n_2+1}^{n_1+n_2+n_3} \left[ (1+R) D_j \log\left( \frac{1+R}{1+R+T_{j-1}-T_j} \right) p(T_{j-1}) \right]$$

(28)

where $p(T_{j-1}) = Ae^{\frac{cH_1}{H_3}(T_{j-1}-H_1-H_2)}$, $A = be^{-H_1}$

Total ordering cost during ( $H_1 + H_2 \le t \le H_1 + H_2 + H_3$ ), *OC3*, is given by $OC3 = \sum_{j=n_1+n_2+1}^{n_1+n_2+n_3} \left[ c_{o1} + c_{o2} Q3_j \right]$ (29)

where $Q3_j$ is given by (26)

Selling price for j-th ( $n_1 + n_2 \le j \le n_1 + n_2 + n_3$ ) cycle , *SP3*$_j$ is given by $SP3_j = m_3 p(T_{j-1}) \int_{T_{j-1}}^{T_j} D_j\, dt$

$= m_3 p(T_{j-1}) D_j (T_j - T_{j-1})$

Total selling price during ( $H_1 + H_2 \le t \le H_1 + H_2 + H_3$ ), *SP3,*

is given by $SP3 = \sum_{j=n_1+n_2+1}^{n_1+n_2+n_3} SP3_j$ (30)

Thus, total profit Z, for this model over the planning horizon ( $H_1 + H_2 + H_3$ ), is given by

*Z=(SP1+SP2+SP3)-(PC1+PC2+PC3)-(HOC1+HOC2+HOC3)-(OC1+OC2+OC3)* (31)

**Mathematical Model**: According to the above discussion, as lifetime of the product is R, so, no cycle should exceed R which implies $t_1 \le R, H_2 / n_2 \le R, t'_{n_3} \le R.$ Therefore, the problem reduces to determine the decision variables $t_1, t'_1, m_1, m_2, m_3, n_1, n_2$ *and* $n_3$ . The problem becomes

Maximize Z Subject to $t_1 \le R, H_2 / n_2 \le R, t'_{n_3} \le R.$ (32)

This constrained optimization problem is solved using proposed DBPSO for crisp objective function

**Mathematical Model in Fuzzy Environment:** As discussed in introduction section, in real life phase intervals $H_1$, $H_2$ and $H_3$ are imprecise in nature i.e $\tilde{H}_1, \tilde{H}_2$ and $\tilde{H}_3$ respectively, then the profit function Z reduces to the fuzzy number $\tilde{Z}$ whose membership function is a function of the decision variables $t_1$, $t_1'$, $m_1$, $m_2$, $m_3$, $n_1$, $n_2$ and $n_3$. Also the last cycle length $t'_{n3}$ becomes imprecise $\tilde{t}'_{n3}$ . So, in this case problem reduces to the following fuzzy optimization problem

Maximize $\tilde{Z}$ Subject to $t_1 \le R, \tilde{H}_2 / n_2 \le R, \tilde{t}'_{n3} \le R$ (33)

If $\tilde{H}_1, \tilde{H}_2$ and $\tilde{H}_3$ are considered as TFNs *($H_{11}$, $H_{12}$ ,$H_{13}$), ($H_{21}$, $H_{22}$ ,$H_{23}$)* and *($H_{31}$, $H_{32}$ ,$H_{33}$)* respectively, then $\tilde{Z}$ becomes a TFN ($Z_1$, $Z_2$, $Z_3$), where $Z_i$=value of Z for $H_1=H_{1i}$, $H_2=H_{2i}$, $H_3=H_{3i}$, i=1,2,3. In this case $\tilde{t}'_{n3}$ also becomes a TFN ( $t'_{n31}$, $t'_{n32}$, $t'_{n33}$ ). So it is an obvious assumption that fuzzy constraints should necessarily hold. The problem reduces to

Maximize $\tilde{Z} = (Z_1, Z_2, Z_3)$ Subject to

$t_1 \le R, \tilde{H}_2 / n_2 \le R, \tilde{t}'_{n3} \le R$

(34)

This constraint optimization problem is solved using proposed DBPSO.

**Numerical Experiments**

**Results obtained for crisp environment:** To illustrate the model following hypothetical set of data is used. This data set is taken for items like rice, potato, wheat, onion, cabbage, cauliflower, etc, whose demand exists in the market throughout the year. When new crops come in the market, then its price gradually decreases during some weeks (say $H_1$) and reaches a lowest level. This minimum price prevails for few weeks (say $H_2$). Then again it gradually increases during few weeks (say $H_3$) and reaches its normal value. This normal price prevails remaining part of the year. For an item like potato, values of $H_1$, $H_2$ and $H_3$ are about 5 weeks, 15 weeks, 7 weeks in the state of West Bengal, India. Normal price of the item throughout the year is about \$3 for a 10 kg bag. Lowest price of it in the season is about \$2 for a 10 kg bag. Keeping this real life situation in mind the following data set is fixed to illustrate the modes in crisp environment. In the data set 10 kg of the item is considered as one unit item, one week is considered as unit time and costs are represented in \$.

b=10, c=0.2, $H_1$=5(weeks), $H_2$=(15 weeks), $H_3$=7(weeks), $D_0$=1500, $\gamma$=2.5, $c_h$=0.5, $c_{01}$=10, $c_{02}$=0.5, R=3.

For the above parametric values, results are obtained via DBPSO for different values of $n_1$, $n_2$, $n_3$ and presented in Table-1. It is found that profit is maximum for $n_1=3$, $n_2=13$, $n_3=4$.

**Table 1. Results obtained for crisp model via DBPSO**

| $n_1$ | $n_2$ | $n_3$ | $m_1$ | $m_2$ | $m_3$ | $t_1$ | $t'_1$ | Profit($) |
|---|---|---|---|---|---|---|---|---|
| 2 | 11 | 3 | 2.966 | 2.493 | 3.013 | 2.848 | 2.015 | 267.660 |
| 2 | 11 | 4 | 2.965 | 2.494 | 2.577 | 2.841 | 1.407 | 272.087 |
| 2 | 11 | 5 | 2.967 | 2.493 | 2.378 | 2.841 | 1.067 | 270.095 |
| 2 | 12 | 3 | 2.968 | 2.431 | 3.013 | 2.843 | 2.016 | 270.222 |
| 2 | 12 | 4 | 2.967 | 2.431 | 2.577 | 2.843 | 1.407 | 274.648 |
| 2 | 12 | 5 | 2.969 | 2.431 | 2.378 | 2.843 | 1.067 | 272.656 |
| 2 | 13 | 3 | 2.967 | 2.380 | 3.014 | 2.842 | 2.015 | 270.998 |
| 2 | 13 | 4 | 2.966 | 2.380 | 2.577 | 2.481 | 1.404 | 275.424 |
| 2 | 13 | 5 | 2.968 | 2.380 | 2.377 | 2.843 | 1.068 | 273.432 |
| 2 | 14 | 3 | 2.968 | 2.338 | 3.014 | 2.842 | 2.014 | 270.347 |
| 2 | 14 | 4 | 2.951 | 2.339 | 2.580 | 2.843 | 1.398 | 274.771 |
| 2 | 14 | 5 | 2.967 | 2.339 | 2.377 | 2.846 | 1.067 | 272.781 |
| 2 | 15 | 3 | 2.966 | 2.303 | 3.013 | 2.843 | 2.014 | 268.537 |
| 2 | 15 | 4 | 2.965 | 2.303 | 2.578 | 2.841 | 1.408 | 272.964 |
| 2 | 15 | 5 | 2.964 | 2.303 | 2.378 | 2.845 | 1.069 | 270.972 |
| 3 | 11 | 3 | 2.431 | 2.494 | 3.014 | 2.050 | 2.015 | 273.615 |
| 3 | 11 | 4 | 2.431 | 2.493 | 2.578 | 2.052 | 1.408 | 278.042 |
| 3 | 11 | 5 | 2.431 | 2.493 | 2.378 | 2.051 | 1.068 | 276.050 |
| 3 | 12 | 3 | 2.431 | 2.431 | 3.015 | 2.051 | 2.015 | 276.177 |
| 3 | 12 | 4 | 2.430 | 2.431 | 2.577 | 2.052 | 1.408 | 280.603 |
| 3 | 12 | 5 | 2.430 | 2.431 | 2.378 | 2.050 | 1.069 | 278.611 |
| 3 | 13 | 3 | 2.430 | 2.380 | 3.015 | 2.048 | 2.016 | 276.953 |
| 3 | 13 | 4 | 2.432 | 2.380 | 2.577 | 2.051 | 1.408 | 281.379 |
| 3 | 13 | 5 | 2.429 | 2.280 | 2.377 | 2.052 | 1.067 | 279.387 |
| 3 | 14 | 3 | 2.431 | 2.338 | 3.013 | 2.051 | 2.015 | 276.302 |
| 3 | 14 | 4 | 2.431 | 2.338 | 2.577 | 2.050 | 1.407 | 280.728 |
| 3 | 14 | 5 | 2.431 | 2.339 | 2.378 | 2.051 | 1.070 | 278.737 |
| 3 | 15 | 3 | 2.430 | 2.303 | 3.015 | 2.049 | 2.015 | 274.492 |
| 3 | 15 | 4 | 2.429 | 2.304 | 2.577 | 2.051 | 1.404 | 278.919 |
| 3 | 15 | 5 | 2.432 | 2.304 | 2.378 | 2.051 | 1.068 | 276.927 |
| 4 | 11 | 3 | 2.242 | 2.493 | 3.013 | 1.588 | 2.011 | 272.806 |
| 4 | 11 | 4 | 2.243 | 2.493 | 2.577 | 1.589 | 1.409 | 277.232 |
| 4 | 11 | 5 | 2.242 | 2.493 | 2.377 | 1.589 | 1.069 | 275.240 |
| 4 | 12 | 3 | 2.242 | 2.431 | 3.014 | 1.587 | 2.014 | 275.368 |
| 4 | 12 | 4 | 2.243 | 2.432 | 2.577 | 1.588 | 1.404 | 279.794 |
| 4 | 12 | 5 | 2.243 | 2.431 | 2.377 | 1.588 | 1.068 | 277.802 |
| 4 | 13 | 3 | 2.090 | 2.397 | 3.186 | 1.882 | 1.671 | 271.915 |
| 4 | 13 | 4 | 2.243 | 2.380 | 2.577 | 1.587 | 1.406 | 280.570 |
| 4 | 13 | 5 | 2.243 | 2.380 | 2.377 | 1.589 | 1.067 | 278.578 |
| 4 | 14 | 3 | 2.242 | 2.338 | 3.015 | 1.588 | 2.015 | 275.493 |
| 4 | 14 | 4 | 2.242 | 2.578 | 2.338 | 1.589 | 1.408 | 279.919 |
| 4 | 14 | 5 | 2.243 | 2.339 | 2.378 | 1.588 | 1.070 | 277.927 |
| 4 | 15 | 3 | 2.149 | 2.438 | 2.815 | 1.657 | 1.069 | 268.540 |
| 4 | 15 | 4 | 2.242 | 2.303 | 2.576 | 1.585 | 1.408 | 278.109 |
| 4 | 15 | 5 | 2.242 | 2.304 | 2.378 | 1.590 | 1.069 | 276.118 |

For above parametric values, results are obtained for different values of $\gamma$ and presented in Table-2. It is observed that as $\gamma$ increases, profit decreases due to decrease of demand which agrees with reality. It is also found that as $\gamma$ increases for same values of $n_1$, $n_2$ and $n_3$, $t_1$ increases but $t'_1$decreases. Moreover, $m_1$, $m_2$ and $m_3$ also decrease with increase of $\gamma$. It happens because as $\gamma$ increases demand decreases in each cycle and demand is minimum when purchase cost is maximum. Again, purchase cost is maximum in first and last cycle of the whole planning horizon. As demand decreases length of first and lat cycle increases as a result $t_1$ increases and $t'_1$ decreases. Again as demand decreases due to increase of $\gamma$ to keep the demand high markup of selling price $m_1$, $m_2$ and $m_3$ also decreases. All these observations agree with reality.

For the above parametric values, results are obtained for different values of R and presented in Table-3. It is observed that as R increases profit increases. It happens because increase of R, i.e., increase of lifetime of the product, decreases rate of deterioration which in turn increases profit.

**Table 2. Results obtained for crisp model due to different $\gamma$**

| $\gamma$ | $n_1$ | $n_2$ | $n_3$ | $m_1$ | $m_2$ | $m_3$ | $t_1$ | $t'_1$ | Profit($) |
|---|---|---|---|---|---|---|---|---|---|
| 2.40 | 4 | 14 | 4 | 2.308 | 2.405 | 2.652 | 1.568 | 1.422 | 408.060 |
| 2.42 | 4 | 14 | 4 | 2.294 | 2.391 | 2.639 | 1.574 | 1.421 | 379.875 |
| 2.44 | 4 | 14 | 4 | 2.280 | 2.377 | 2.622 | 1.575 | 1.417 | 353.036 |
| 2.45 | 4 | 14 | 4 | 2.275 | 2.371 | 2.614 | 1.577 | 1.414 | 340.100 |
| 2.46 | 3 | 13 | 4 | 2.459 | 2.407 | 2.606 | 2.042 | 1.415 | 327.583 |
| 2.48 | 3 | 13 | 4 | 2.444 | 2.394 | 2.592 | 2.048 | 1.412 | 303.918 |
| 2.50 | 3 | 13 | 4 | 2.432 | 2.380 | 2.577 | 2.051 | 1.408 | 281.379 |
| 2.52 | 3 | 13 | 4 | 2.418 | 2.368 | 2.564 | 2.056 | 1.403 | 259.909 |
| 2.54 | 3 | 13 | 4 | 2.405 | 2.356 | 2.550 | 2.063 | 1.401 | 239.454 |
| 2.55 | 3 | 12 | 4 | 2.398 | 2.400 | 2.543 | 2.063 | 1.399 | 229.602 |
| 2.56 | 3 | 12 | 4 | 2.391 | 2.393 | 2.537 | 2.066 | 1.397 | 220.124 |
| 2.58 | 3 | 12 | 4 | 2.379 | 2.382 | 2.524 | 2.074 | 1.395 | 201.842 |
| 2.60 | 3 | 12 | 4 | 2.368 | 2.370 | 2.512 | 2.078 | 1.392 | 184.421 |

**Table 3. Results obtained for crisp model due to different R**

| R | $n_1$ | $n_2$ | $n_3$ | $m_1$ | $m_2$ | $m_3$ | $t_1$ | $t'_1$ | Profit($) |
|---|---|---|---|---|---|---|---|---|---|
| 2.70 | 3 | 13 | 4 | 2.494 | 2.418 | 2.652 | 2.035 | 1.426 | 268.028 |
| 2.80 | 3 | 13 | 4 | 2.470 | 2.405 | 2.625 | 2.042 | 1.419 | 272.725 |
| 2.90 | 3 | 13 | 4 | 2.450 | 2.392 | 2.601 | 2.045 | 1.412 | 277.169 |
| 3.00 | 3 | 13 | 4 | 2.432 | 2.380 | 2.577 | 2.051 | 1.408 | 281.379 |
| 3.10 | 3 | 13 | 4 | 2.412 | 2.370 | 2.557 | 2.054 | 1.401 | 285.375 |
| 3.20 | 3 | 13 | 4 | 2.395 | 2.359 | 2.537 | 2.060 | 1.394 | 289.172 |
| 3.30 | 3 | 13 | 4 | 2.378 | 2.349 | 2.519 | 2.065 | 1.390 | 292.784 |
| 3.40 | 3 | 13 | 4 | 2.364 | 2.341 | 2.502 | 2.067 | 1.384 | 296.226 |
| 3.50 | 3 | 12 | 4 | 2.352 | 2.376 | 2.486 | 2.070 | 1.380 | 299.586 |

**Results obtained for fuzzy environment:** To illustrate the proposed inventory models, following input data are considered. In this case also hypothetical data set is used and source of this data has been discussed for crisp model. For crisp model it was considered that unit price of the item decreases during the period $H_1$=5 weeks, but in reality it is about 5 weeks which is fuzzy in nature. Due to this reason here $H_1$ is considered as TFN (4.75, 5, 5.2). Following the same argument other parameters are fixed and the data set are presented below. In the data set fuzzy numbers are considered as TFN types.

b=10, c=0.2, $\widetilde{H}_1$=(4.75, 5, 5.2), $\widetilde{H}_2$=(14.5,15, 15.4), $\widetilde{H}_3$=(6.8, 7, 7.3), $D_0$=1500, $\gamma$=2.5, $c_h$=0.5, $c_{01}$=10, $c_{02}$=0.5, R=3.

For the above parametric values, results are obtained via DBPSO for different values of $n_1$, $n_2$, $n_3$ and presented in Table-4. It is found that profit is maximum for $n_1=3$, $n_2=13$, $n_3=4$.

For the above parametric values, results are obtained for different values of $\gamma$ and presented in Table-5. In this case also same trend of result is obtained as found in crisp model.

For the above parametric values, results are obtained for different values of R and presented in Table-6. As expected in this case also same trend of result is obtained as in crisp model, i.e., profit increases with increase of R, which agrees in reality.

## Practical Implications

The present models have the following practical usages:

**Table 4. Results obtained for fuzzy model via DBPSO**

| $n_1$ | $n_2$ | $n_3$ | $m_1$ | $m_2$ | $m_3$ | $t_1$ | $t_1'$ | $Z_1$ (\$) | $Z_2$ (\$) | $Z_3$ (\$) |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 11 | 3 | 2.574 | 2.308 | 3.791 | 2.847 | 2.262 | 228.737 | 256.368 | 278.658 |
| 2 | 11 | 4 | 2.87 | 2.494 | 2.577 | 3.000 | 1.407 | 240.271 | 271.818 | 297.868 |
| 2 | 11 | 5 | 2.966 | 2.493 | 2.377 | 2.841 | 1.069 | 238.189 | 270.095 | 296.923 |
| 2 | 12 | 3 | 2.968 | 2.31 | 3.014 | 2.841 | 2.016 | 238.976 | 270.222 | 295.011 |
| 2 | 12 | 4 | 2.966 | 2.430 | 2.577 | 2.842 | 1.407 | 241.978 | 274.648 | 301.697 |
| 2 | 12 | 5 | 2.967 | 2.430 | 2.378 | 2.839 | 1.066 | 239.207 | 272.656 | 300.876 |
| 2 | 13 | 3 | 2.988 | 2.380 | 3.014 | 3.000 | 2.013 | 237.730 | 270.730 | 297.110 |
| 2 | 13 | 4 | 2.966 | 2.380 | 2.578 | 2.841 | 1.407 | 241.415 | 275.424 | 303.684 |
| 2 | 13 | 5 | 2.965 | 2.380 | 2.378 | 2.843 | 1.069 | 238.633 | 273.432 | 302.874 |
| 2 | 14 | 3 | 2.966 | 2.339 | 3.015 | 2.843 | 2.014 | 236.591 | 270.347 | 297.400 |
| 2 | 14 | 4 | 2.958 | 2.338 | 2.583 | 2.603 | 1.396 | 239.687 | 274.204 | 302.874 |
| 2 | 14 | 5 | 2.967 | 2.339 | 2.378 | 2.844 | 1.068 | 236.810 | 272.782 | 303.280 |
| 2 | 15 | 3 | 2.987 | 2.303 | 3.015 | 3.000 | 2.016 | 233.068 | 268.269 | 296.643 |
| 2 | 15 | 4 | 2.967 | 2.304 | 2.577 | 2.841 | 1.408 | 236.749 | 272.964 | 303.216 |
| 2 | 15 | 5 | 2.964 | 2.303 | 2.378 | 2.840 | 1.068 | 233.985 | 270.972 | 302.386 |
| 3 | 11 | 3 | 2.430 | 2.493 | 3.014 | 2.050 | 2.016 | 242.198 | 273.615 | 298.644 |
| 3 | 11 | 4 | 2.429 | 2.494 | 2.578 | 2.051 | 1.408 | 245.186 | 278.041 | 305.349 |
| 3 | 11 | 5 | 2.431 | 2.494 | 2.378 | 2.051 | 1.071 | 242.412 | 276.050 | 304.531 |
| 3 | 12 | 3 | 3.845 | 2.399 | 3.081 | 2.405 | 1.867 | 232.191 | 265.057 | 291.268 |
| 3 | 12 | 4 | 2.429 | 2.431 | 2.578 | 2.050 | 1.408 | 246.207 | 280.603 | 309.301 |
| 3 | 12 | 5 | 2.430 | 2.431 | 2.377 | 2.051 | 1.068 | 243.433 | 278.611 | 308.481 |
| 3 | 13 | 3 | 2.445 | 2.406 | 2.875 | 2.341 | 1.868 | 240.726 | 275.220 | 302.273 |
| 3 | 13 | 4 | 2.431 | 2.380 | 2.577 | 2.050 | 1.407 | 245.644 | 281.379 | 311.285 |
| 3 | 13 | 5 | 2.430 | 2.380 | 2.378 | 2.049 | 1.068 | 242.875 | 279.387 | 310.461 |
| 3 | 14 | 3 | 2.430 | 2.339 | 3.014 | 2.051 | 2.015 | 240.828 | 276.302 | 304.994 |
| 3 | 14 | 4 | 2.431 | 2.338 | 2.578 | 2.051 | 1.407 | 243.822 | 280.728 | 311.691 |
| 3 | 14 | 5 | 2.430 | 2.339 | 2.378 | 2.051 | 1.070 | 241.046 | 278.737 | 310.877 |
| 3 | 15 | 3 | 2.430 | 2.304 | 3.015 | 2.051 | 2.014 | 237.986 | 274.492 | 304.116 |
| 3 | 15 | 4 | 2.429 | 2.303 | 2.578 | 2.049 | 1.408 | 240.989 | 278.919 | 310.806 |
| 3 | 15 | 5 | 2.430 | 2.303 | 2.378 | 2.052 | 1.070 | 238.208 | 276.927 | 309.995 |
| 4 | 11 | 3 | 2.242 | 2.494 | 3.015 | 1.589 | 2.013 | 240.351 | 272.806 | 298.787 |
| 4 | 11 | 4 | 20244 | 20493 | 2.577 | 1.589 | 1.406 | 243.350 | 277.232 | 305.482 |
| 4 | 11 | 5 | 2.243 | 2.493 | 2.378 | 1.588 | 1.067 | 240.577 | 275.240 | 304.663 |
| 4 | 12 | 3 | 2.242 | 2.431 | 3.013 | 1.589 | 2.014 | 241.366 | 275.368 | 302.745 |
| 4 | 12 | 4 | 2.245 | 2.431 | 2.578 | 1.587 | 1.408 | 244.361 | 279.794 | 309.448 |
| 4 | 12 | 5 | 2.243 | 2.431 | 2.377 | 1.588 | 1.068 | 241.587 | 277.802 | 308.627 |
| 4 | 13 | 3 | 2.243 | 2.380 | 3.013 | 1.588 | 2.015 | 240.809 | 276.144 | 304.727 |
| 4 | 13 | 4 | 2.244 | 2.380 | 2.587 | 1.588 | 1.407 | 243.798 | 280.570 | 311.432. |
| 4 | 13 | 5 | 2.243 | 2.380 | 2.378 | 1.587 | 1.068 | 241.029 | 278.578 | 310.608 |
| 4 | 14 | 3 | 2.243 | 2.339 | 3.014 | 1.589 | 2.015 | 238.983 | 275.493 | 305.137 |
| 4 | 14 | 4 | 2.243 | 2.338 | 2.577 | 1.587 | 1.407 | 241.987 | 279.919 | 311.826 |
| 4 | 14 | 5 | 2.243 | 2.338 | 2.378 | 1.588 | 1.069 | 239.207 | 277.927 | 311.016 |
| 4 | 15 | 3 | 2.243 | 2.303 | 3.007 | 1.596 | 2.016 | 236.144 | 273.682 | 304.245 |
| 4 | 15 | 4 | 2.242 | 2.303 | 2.579 | 1.587 | 1.408 | 239.142 | 278.109 | 310.954 |
| 4 | 15 | 5 | 2.242 | 2.303 | 2.379 | 1.590 | 1.069 | 236.363 | 276.118 | 310.141 |

**Table 5. Results obtained due to different $\gamma$ for fuzzy model via DBPSO**

| $\gamma$ | $n_1$ | $n_2$ | $n_3$ | $m_1$ | $m_2$ | $m_3$ | $t_1$ | $t_1'$ | $Z_1$ (\$) | $Z_2$(\$) | $Z_3$(\$) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.40 | 4 | 14 | 4 | 2.307 | 2.405 | 2.653 | 1.569 | 1.421 | 362.311 | 408.060 | 446.593 |
| 2.42 | 4 | 14 | 4 | 2.294 | 2.391 | 2.637 | 1.571 | 1.417 | 335.816 | 379.875 | 416.973 |
| 2.44 | 4 | 14 | 4 | 2.281 | 2.378 | 2.620 | 1.576 | 1.415 | 310.601 | 353.036 | 388.757 |
| 2.46 | 3 | 13 | 4 | 2.458 | 2.406 | 2.607 | 2.040 | 1.413 | 289.050 | 327.583 | 359.851 |
| 2.48 | 3 | 13 | 4 | 2.444 | 2.394 | 2.591 | 2.044 | 1.409 | 266.814 | 303.918 | 334.977 |
| 2.50 | 3 | 13 | 4 | 2.431 | 2.380 | 2.577 | 2.050 | 1.407 | 245.644 | 281.379 | 311.285 |
| 2.52 | 3 | 13 | 4 | 2.416 | 2.368 | 2.564 | 2.058 | 1.405 | 225.484 | 259.909 | 288.713 |
| 2.54 | 3 | 13 | 4 | 2.404 | 2.356 | 2.550 | 2.061 | 1.401 | 206.298 | 239.454 | 267.187 |
| 2.56 | 3 | 12 | 4 | 2.391 | 2.393 | 2.537 | 2.066 | 1.399 | 189.422 | 220.124 | 245.715 |
| 2.58 | 3 | 12 | 4 | 2.380 | 2.382 | 2.525 | 2.069 | 1.396 | 172.281 | 201.842 | 226.476 |
| 2.60 | 3 | 12 | 4 | 2.368 | 2.370 | 2.512 | 2.077 | 1.392 | 155.946 | 184.421 | 208.142 |

**Table 6. Results obtained due to different R for fuzzy model via DBPSO**

| R | $n_1$ | $n_2$ | $n_3$ | $m_1$ | $m_2$ | $m_3$ | $t_1$ | $t_1'$ | $Z_1$(\$) | $Z_2$(\$) | $Z_3$(\$) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.90 | 3 | 13 | 4 | 2.494 | 2.392 | 2.652 | 2.046 | 1.411 | 241.911 | 277.169 | 306.583 |
| 3.00 | 3 | 13 | 4 | 2.431 | 2.380 | 2.577 | 2.050 | 1.407 | 245.644 | 281.379 | 311.285 |
| 3.10 | 3 | 13 | 4 | 2.412 | 2.369 | 2.555 | 2.055 | 1.402 | 249.192 | 285.375 | 315.735 |
| 3.20 | 3 | 13 | 4 | 2.394 | 2.359 | 2.537 | 2.059 | 1.394 | 252.559 | 289.172 | 319.966 |
| 3.30 | 3 | 13 | 4 | 2.379 | 2.350 | 2.520 | 2.062 | 1.390 | 255.767 | 292.784 | 323.993 |
| 3.40 | 3 | 13 | 4 | 2.364 | 2.340 | 2.502 | 2.066 | 1.387 | 258.822 | 296.226 | 327.826 |
| 3.50 | 3 | 13 | 4 | 2.351 | 2.376 | 2.486 | 2.071 | 1.379 | 263.050 | 299.586 | 330.440 |

- It is applicable for the inventory control of seasonal goods like tomato, cabbage, cauliflower, potato, paddy, wheat, pulses etc whose demand exists throughout the year and their price found stable about half of the year. But at the beginning of the production season their price gradually decreases to a stable lowest value for a period. This lowest price persists for a period and then again gradually increases to normal price of the year. The model is developed for these types of items during their seasonal period.
- Optimization of fuzzy objectives is not properly defined. So to deal with problem involving fuzzy objective one can compare the objectives due to different solutions by credibility measure of fuzzy events and then optimization can be done by any soft computing technique like PSO.
- The methodology used for the formulation and determination of solution is quiet general and can be useable on any inventory control/ supply chain/ optimization problem in fuzzy environment.

**Conclusion**

Here, a real-life inventory model for deteriorating seasonal product is developed whose demand depends upon the unit cost of the product in fuzzy environment. Unit cost of product is time dependent. Lifetime of each item is finite and rate of deterioration depend on the age of the item. Unique contribution of the paper is fourfold:

- Using credibility measure of fuzzy event and PSO an approach is followed which can deal with constrained fuzzy optimization problem without taking crisp equivalent of the fuzzy problem.
- The model is developed for such items like food grains, pulses, potato, onion etc., whose stable demand exists in the market throughout the year but it fluctuates for a part of the year when they are produced in the field. Here modeling is done for such products during their season of grown. These items are normally stored in cold storage and when bought in the market items are fully deteriorated after a finite time R, which is considered here as lifetime of the product. For the best of author's knowledge none have considered this type of inventory model.
- Here for the first time unit cost of an item is modeled following real life situation, which gradually decreases with time during grown of the item in the field, then it retains the lowest value for a period and again gradually increases with time to normal price of the year. Though it is found for above mentioned items in every year, inventory practitioners overlooked this real life phenomenon.
- It is assumed that time horizon of the season is fuzzy in nature. For the first time season of an item is considered as a combination of three imprecise intervals. In fact three parts in which unit cost function can be divided are considered as fuzzy numbers, which agree with reality.

At length, though the model is formulated in fuzzy environment, demand or lifetime/deterioration of the product is not considered as imprecise in nature, though it is appropriate for these types of products. In fact, consideration of fuzzy demand or deterioration the inventory model leads to fuzzy differential equation for formulation of the model. Using proposed solution approach one cannot consider imprecise demand which is the major limitation of the approach. So, further research work can be done incorporating fuzzy demand and or deterioration in the imprecise planning horizon. Though the model is presented in crisp environment and fuzzy, it can be formulated in stochastic, fuzzy-stochastic environment.

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