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RESEARCH ARTICLE

THE DOUBLE PRIOR SELECTION FOR THE PARAMETER OF POWER FUNCTION DISTRIBUTION UNDER TYPE-II CENSORING

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ABSTRACT

In this paper a comparison of double priors assumed for the parameter of power distribution is 'made sometimes we may have different prior information's available instead of single prior for the parameter of the power distribution. It may be beneficial to include such different types of information's in the Bayes estimation of the parameter. We have considered three double prior distributions viz. Gamma and Uniform priors, Gamma and Jeffrey's priors, Gamma and priors and only Gamma prior. The results are compared with results based on single Gamma prior. We have derived Bayes estimator for the parameter and reliability of the distribution under squared error loss function based on type-II censored sample. The predictive distribution for future failure time and for the remaining failure times after the first r failures observed have been derived. For each case corresponding equal tail credible intervals are also obtained. A simulation study is done to exemplify the results obtained.

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1. INTRODUCTION

The power function distribution is used as a lifetime distribution model for certain sets of failure data to check the reliability of electrical component Meniconi (1995) used power function distribution over the other life time models like exponential, lognormal, Weibull and showed that power function distribution performs well in reliability and hazard function studies. Statistical properties of Power function distribution are studied by Johnson and Kotz (1970). Moments of order statistics for a Power function distribution were calculated by Malik (1967). Ahsanullah (1974, 1989) has considered estimation of the parameters of a Power function distribution by linear functions of order statistics and by record values. Kapadia (1978) discussed the sample size required to estimate parameter of the power function distribution. Zaka *et al.* (2013, 2014) have used different estimation methods for the parameters of the Power function distribution. In life-testing experiments usually two basic censoring schemes are used viz., (i) Type-I censoring and (ii) Type-II censoring. In Type-I censoring scheme the life test is terminated as soon as the predetermined time for the test is observed. When cost of the test heavily increases with time of the experiment such censoring scheme is used. While in type-II censoring scheme the life test is terminated as soon as predetermined number of failures are observed. Such censoring schemes are used for testing of very costly items. In life testing experiment now a days Bayesian estimation approach is widely used by the statisticians. Saleem *et al*. (2010) considered the Bayesian analysis of the mixture of power function distributions based on complete and censored sample, Munawar and Farooq (2012) have considered Bayesian parameter estimation for Power function distribution. Zarrin *et al*. (2013) have used Bayes estimation for shape parameter of generalized power function distribution. But they have used a single prior distribution for estimation of the parameters. Sometimes we may have different information's about the unknown parameter of the given life time model. In such situation it is more beneficial to include such different information's in the Bayesian setup. Haq and Aslam (2009) have considered double prior selection for the parameter of Poisson distribution for evaluation of posterior variance, posterior predictive variance and posterior predictive probabilities. Patel and Patel (2015 (a, b)) have considered double prior distributions for estimating the parameter and other

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reliability characteristics in case of Rayleigh and Exponential life time models. In this paper the following three different double priors are used and the results based on them are compared with the results based on single prior distribution.

(i) Gamma-Jeffery (non-informative for c =1) double prior
(ii) Gamma-non-informative double prior (for c = 2)
(iii) Gamma-non-informative double prior (for c = 3)
(iv) Gamma prior

The posterior distribution of parameter θ under different type of prior distributions is developed in Section 2. Bayes estimate of θ and reliability at time $t$are derived in Section 3. Section 4 covers Bayes predictive estimation and construction of equal tail credible interval for future observation. In Section 5, Bayes predictive estimation for the remaining $(n \quad r)$ ordered failure time truncated at $x_{(r)}$ is considered along with their equal tail credible interval. A simulation study is carried out to compare the performance of the estimators under different double priors. The estimation is done based on the type-II censored sample from the power function distribution.

The probability density function (pdf) of the power function distribution is given by,

$$f(x,\theta) = \theta x^{\theta-1};\ 0 < x < 1,\quad \theta > 0. \qquad \ldots(1.1)$$

Its Cumulative distribution function (cdf) is,

$$F(x,\theta) = x^{\theta} \qquad ..(1.2)$$

The reliability function at time t is

$$R_{\theta}(t) = 1 - t^{\theta}; 0 < t < 1, \quad \theta > 0 \qquad ...(1.3)$$

2. The posterior distribution of θ under different prior distributions

Let n items are placed on a life test and the test is terminated after the r^{th} failure, $1 \le r \le n$; r is predetermined fixed integer. Consider $X_{(1)}, X_{(2)}, X_{(3)}, \dots, X_{(i)}, \dots, X_{(r)}$ are the r ordered observed failure times. During the test failure are not replaced and the test is continued with the remaining items of the test, such censoring scheme is called Type-II censoring without replacement. The likelihood function under such censoring scheme is given by

$$L(\underline{x},\theta) \propto \prod_{i=1}^{r} f(x_{(i)},\theta).\left[1 - F(x_{(r)},\theta)\right]^{n-r}$$

Using (1.1.) and (1.2), it reduces to,

$$L = L(\underline{x},\theta) \propto \theta^{r} \prod_{i=1}^{r} x_{(i)}^{\theta-1}\left[1 - x_{(r)}^{\theta}\right]^{n-r}$$

$$= \sum_{j=0}^{n-r} \binom{n-r}{j} \theta^{r} \prod_{i=1}^{r-1} x_{(i)}^{\theta-1} . x_{(r)}^{\theta(1+j)-1} (-1)^{j} \qquad ..(2.1)$$

2A. General Non-informative and Gamma Priors:

Let us consider the general non-informative prior distribution of θ is

$$P_{i1}(\theta) = \frac{1}{\theta^{c}},\quad \theta > 0,\quad c \ge 0 \qquad ..(2.2)$$

and the second Prior distribution for θ is gamma distribution given as,

$$P_{i1}(\theta) = \frac{b^{a}}{\lceil a}\theta^{a-1}e^{-b\theta},\quad \theta > 0, a > 0, b > 0. \qquad ..(2.3)$$

Combining (2.2) and (2.3), the double prior distribution for θ can be written as,

$$P_{i}(\theta) \propto P_{i1}(\theta).P_{i2}(\theta)$$

$$\propto \theta^{a-c-1} e^{-b\theta} \; ; \; \theta > 0, a > 0, b > 0, c \geq 0 \qquad \ldots (2.4)$$

For $i = 1$, we take $c = 1$, and we have gamma-Jeffery double prior, given by

$$P_1(\theta) \propto \theta^{a-1} e^{-b\theta} \quad ; \theta > 0, a > 0, b > 0.$$

For $i = 2$, we take $c = 2$, and we have gamma-non-informativedouble prior, given by

$$P_2(\theta) \propto \theta^{a-2} e^{-b\theta} \quad ; \theta > 0, a > 0, b > 0.$$

For $i = 3$, we take $c = 3$,and we have gamma-non-informativedouble prior, given by

$$P_3(\theta) \propto \theta^{a-3} e^{-b\theta} \quad ; \theta > 0, a > 0, b > 0.$$

For $i = 4$, we have only single gamma prior given by

$$P_4(\theta) \propto \theta^{a-1} e^{-b\theta} \quad ; \theta > 0, a > 0, b > 0.$$

2. B The posterior Distribution of θ

The posterior Distribution of $\theta$for given$\underline{x}$ in case of double prior distribution $P_i(\theta)$ can be obtained as,

$$\pi_i(\theta/\underline{x}) \propto L(\underline{x}, \theta). P_i(\theta)$$

$$= \sum_{j=0}^{n-r} \binom{n-r}{j} \theta^r \prod_{i=1}^{r-1} x_{(i)}^{\theta-1} (-1)^j x_{(r)}^{\theta(1+j)-1} \theta^{a+c-1} e^{-b\theta}$$

$$= \sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \theta^r e^{(\theta-1)\sum_{i=1}^{r-1} \log x_{(i)}} e^{\{\theta(1+j)-1\}.\log x_{(r)}} \theta^{a+c-1} e^{-b\theta}$$

$$= \sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} e^{-y_j\theta} \theta^{\alpha_i - 1} e^{-\sum_{i=1}^{r} \log x_{(i)}} \quad ..(2.5)$$

where,

$$y_j = b - \sum_{i=1}^{r-1} \log x_{(i)} - (1+j) \log x_{(r)} \text{ and } \alpha_i = r + a - c \qquad ..(2.6)$$

Hence,

$$\pi_i(\theta/\underline{x}) = \frac{L(\underline{x}, \theta) P_i(\theta)}{\int_\theta L(\underline{x}, \theta) P_i(\theta) d\theta}$$

$$= \frac{\sum_{j=0}^{n-r} \binom{n-r}{j} e^{-y_j\theta} \theta^{\alpha_i - 1} (-1)^j}{\sum_{j=0}^{n-r} \binom{n-r}{j} \frac{\Gamma\alpha_i}{y_j^{\alpha_i}}} \qquad ..(2.7)$$

3. Bayes estimate of θ and reliability $R(t)$ at time t

3 A. Bayes estimate of θ under squared error loss function is given by,

$$\hat{\theta} = E_{\pi_i}(\theta|\underline{x})$$

$$= \int_0^\infty \theta . \pi_i(\theta|\underline{x}). d\theta$$

Using (2.7), we get

$$\hat{\theta} = \frac{\int_0^\infty \theta \sum_{j=0}^{n-r} \binom{n-r}{j} e^{-y_j\theta} \theta^{\alpha_i - 1} (-1)^j d\theta}{D}$$

where,

$$D = \sum_{j=0}^{n-r} \binom{n-r}{j} \frac{\lceil \alpha_i}{y_j^{\alpha_i}} (-1)^j$$

on simplifying we get,

$$\hat{\theta} = \frac{\lceil(\alpha_i + 1)}{\lceil \alpha_i} \frac{\sum_{j=0}^{n-r} \binom{n-r}{j} \frac{(-1)^j}{y_j^{\alpha_i+1}}}{\sum_{j=0}^{n-r} \binom{n-r}{j} \frac{(-1)^j}{y_j^{\alpha_i}}}$$

$$= \frac{\alpha_i \sum_{j=0}^{n-r} \binom{n-r}{j} \frac{(-1)^j}{y_j^{\alpha_i+1}}}{\sum_{j=0}^{n-r} \binom{n-r}{j} \frac{(-1)^j}{y_j^{\alpha_i}}} \quad ..(3.1)$$

Bayes estimate of reliability $R(t)$:

Here, $R(t) = P(X > t) = 1 - F(t)$

$= (1 - t^\theta) = \psi(\theta)$ say

Hence the Bayes estimate of $R(t)$ is given as

$$\hat{R}(t) = E_{\pi_i}[\varphi(\theta)/\underline{x}]$$

$$= \int_0^\infty \varphi(\theta).\pi_i(\theta/\underline{x}) d\theta$$

$$= \frac{\int_0^\infty (1 - t^\theta) \sum_{j=0}^{n-r} \binom{n-r}{j} e^{-y_j\theta} \theta^{\alpha_i - 1} (-1)^j d\theta}{D}$$

$$= 1 - \frac{\sum_{j=0}^{n-r} (-1)^j \int_0^\infty \binom{n-r}{j} e^{-\theta\{y_j - \log t\}} \theta^{\alpha_i - 1} d\theta}{D},$$

Here it should be $y_j > \log t$ or $e^{y_j} > t$.

$$1 - \left\{ \frac{\sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \Big/ (y_j - \log t)^{\alpha_i}}{\sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \Big/ y_j^{\alpha_i}} \right\} \quad ..(3.2)$$

3B. Bayes equal tail credible interval for θ:

Interval $[I_{1i}, I_{2i}]$ is said to be the Bayes equal tail $(1-\gamma)100\%$ credible interval of θ if

$$\int_0^{I_{1i}} \pi_i(\theta|\underline{x}) d\theta = \frac{r}{2} = \int_{I_{2i}}^\infty \pi_i(\theta|\underline{x}).d\theta \quad , \quad 0 < r < 1$$

To obtain I_{1i} and I_{2i}, we consider

$\int_0^{I_{1i}} \pi_i(\theta|\underline{x})\, d\theta = \frac{\gamma}{2}$

$$\int_0^{I_{1i}} \sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} e^{-y_j\theta} \theta^{\alpha_i - 1}\, d\theta = \frac{D\gamma}{2}$$

$$\sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \int_0^{I_{1i}} e^{-y_j\theta} \theta^{\alpha_i - 1} d\theta = \frac{D\gamma}{2}$$

Taking $W = y_j\theta$, we get

$$\sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \int_0^{y_j I_{1i}} \frac{e^{-w} w^{\alpha_i - 1}}{y_j^{\alpha_i}} dw = \frac{D\gamma}{2}$$

$$\sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \frac{1}{y_j^{\alpha_i}} \int_0^{y_j I_{1i}} \frac{e^{-w} w^{\alpha_i - 1}}{\lceil \alpha_i} dw = \frac{D\gamma}{2}$$

$$\sum_{j=0}^{n-r} \frac{\binom{n-r}{j}(-1)^j}{y_j{}^{\alpha_i}} \Gamma(\propto_i)_{y_j I_{1i}} = \frac{r}{2} \sum_{j=0}^{n-r} \frac{(-1)^j \binom{n-r}{j}}{y_j^{\alpha_i}} \qquad ..(3.3)$$

Where $\Gamma(\propto_i)_{y_j I_{1i}}$ denote incomplete gamma integral.

Solving (3.3) for given γ, we can find I_{1i}.

Similarly to obtain I_{2i}, we solve the equation

$$\int_{I_{2i}}^{\infty} \pi_i(\theta|\underline{x}).\, d\theta = \frac{\gamma}{2}$$

$$\int_o^{I_{2i}} \pi_i(\theta|\underline{x}).\, d\theta = 1 - \frac{\gamma}{2}$$

And simplifying it , we get

$$\frac{\sum_{j=0}^{n-r}(-1)^j}{y_i^{\alpha_i}} \Gamma(\propto_i)_{y_j I_{2i}} = \frac{\left(1-\frac{\gamma}{2}\right)\sum_{j=0}^{n-r}(-1)^j}{y_j^{\alpha_i}} \qquad ..(3.4)$$

Solving (3.4) we can get I_{2i} and hence $[I_{1i}, I_{2i}]$ became $(1-\gamma)100\%$ equal tail Bayes credible interval for θ.

Hence we can deduce Bayes equal tail $(1-\gamma)100\%$ credible interval for $R(t)$ as follow:

Since, $P(I_{1i} < \theta < I_{2i}) = \frac{\gamma}{2}$

$$P\left(t^{I_{1i}} < t^{\theta} < t^{I_{2i}}\right) = \frac{\gamma}{2}$$

$$P\left(-t^{I_{1i}} > -t^{\theta} > -t^{I_{2i}}\right) = \frac{\gamma}{2}$$

$$P\left(1 - t^{I_{1i}} < 1 - t^{\theta} < 1 - t^{I_{2i}}\right) = \frac{\gamma}{2}$$

$$P\left(h_{1i} < 1 - t^{\theta} < h_{2i}\right) = \frac{\gamma}{2} \qquad ..(3.5)$$

where,

$$h_{1i} = 1 - t^{I_{2i}} \text{ and } h_{2i} = 1 - t^{I_{1i}} \qquad ..(3.6)$$

gives $(1-\gamma)100\%$ credible interval for $R(t) = (1-t^{\theta})$ at given time t.

4. Bayes predictive estimator and $(1-\gamma)100\%$ equal tail credible interval for future observation

Let Z_i be a future observation which has already survived time $x_{(r)}$. Let $w_i = z_i - x_{(r)}$. According to Howlader and Hossain (1995) given the data $\underline{x}$, the conditional joint pdf of w_i and θ is given by

$$h_i(w_i, \theta/\underline{x}) = f(w_i/\theta).\pi_i(\theta/\underline{x})$$

$$= \frac{\theta w_i^{\theta-1}\sum_{j=0}^{n-r}\binom{n-r}{j} e^{-y_j\theta}\theta^{\alpha_i-1}(-1)^j}{\left(1-x_{(r)}\right)^{\theta}\sum_{j=0}^{n-r}\binom{n-r}{j}\frac{\lceil\alpha_i}{y_j\alpha_i}(-1)^j}$$

$$= \frac{\sum_{j=0}^{n-r}\binom{n-r}{j}\frac{\theta^{\alpha_i}}{w_i}e^{-(y_j-\log w_i)\theta}(-1)^j}{\left(1-x_{(r)}\right)^{\theta}\sum_{j=0}^{n-r}\frac{(-1)^j\binom{n-r}{j}\lceil\alpha_i}{y_j^{\alpha_i}}} \quad ; w_i > 0$$

Integrating out the above expression with respect to θ, the predictive density of w_i is given by,

$$P_i(w_i/x) = \int_0^{\infty} h_i(w_i, \theta/x)d\theta = \frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{1}{w_i}\int_0^{\infty}\frac{\theta_i^{\alpha_i}e^{-(y_j-\log w_i)^{\theta}}}{\left(1-x_{(r)}\right)^{\theta}}d\theta}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\lceil\alpha_i/y_j^{\alpha_i}}$$

$$= \frac{\sum_{j=0}^{n-r}\frac{(-1)^j\binom{n-r}{j}\frac{1}{w_i}\lceil(\alpha_i+1)}{\left(y_j-\log w_i+\ln\left(1-x_{(r)}\right)\right)^{\alpha_i+1}}}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\lceil\alpha_i/y_j^{\alpha_i}}$$

$$= \frac{\frac{\alpha_i}{w_i}\sum_{j=0}^{n-r}\frac{(-1)^j\binom{n-r}{j}}{\left(y_j-\log w_i+\ln\left(1-x_{(r)}\right)\right)^{\alpha_i+1}}}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}/y_j^{\alpha_i}} \quad ; 0 < w_i < 1-x_{(r)} \qquad ..(4.1)$$

Hence the Bayes estimator of w_i under squared error loss function is given by,

$$w_i^* = E\left[W_i/\underline{x}\right]$$

$$= \frac{\alpha_i\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\int_0^{1-x_{(r)}}\frac{1}{\left(y_j-\log w_i+\ln\left(1-x_{(r)}\right)\right)^{\alpha_i+1}}.dw_i}{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}/y_j^{\alpha_i}}$$

Integration can be done by numerically transformation.

Thus the Bayes predictive estimator for a future observation is given by

$$Z_i^* = w_i^* + x_{(r)} \qquad ..(4.2)$$

Now $(1-\gamma)100\%$ equal tail credible interval for w_i can be obtained by solving the equations

$$\int_0^{h_{1i}} P_i(w_i/x)dw_i = \frac{\gamma}{2} = \int_{h_{2i}}^{1} P_i(w_i/x)dw_i$$

Consider the equation

$$\int_0^{h_{1i}} P_i(w_i/x)dw_i = \frac{\gamma}{2}$$

$$\frac{\int_0^{h_{1i}} \frac{\alpha_i}{w_i} \sum_{j=0}^{n-r} \binom{n-r}{j} (-1)^j / \left(y_j - \log w_i + \ln(1 - x_{(r)})\right)^{\alpha_i + 1} . dw_i}{\sum_{j=0}^{n-r} \binom{n-r}{j} (-1)^j / y_j^{\alpha_i}} = \frac{\gamma}{2}$$

$$\frac{\sum_{j=0}^{n-r} \binom{n-r}{j} (-1)^j \frac{\alpha_i \left(y_j + \ln(1 - x_{(r)})\right)}{\left(y_j + \ln(1 - x_{(r)})\right)^{\alpha_i + 1}} \int_{\left(\frac{-\ln h_{1i}}{y_j + \ln(1 - x_{(r)})}\right)}^{\infty} \frac{dd_i}{(1 + d_i)^{\alpha_i + 1}}}{\sum_{j=0}^{n-r} \binom{n-r}{j} (-1)^j / y_j^{\alpha_i}} = \frac{\gamma}{2}$$

$$\frac{\sum_{j=0}^{n-r} \binom{n-r}{j} (-1)^j \frac{1}{\left(y_j + \ln(1 - x_{(r)})\right)^{\alpha_i}} \frac{1}{\left(1 - \frac{\ln h_{1i}}{y_j + \ln(1 - x_{(r)})}\right)^{\alpha_i}}}{\sum_{j=0}^{n-r} \binom{n-r}{j} (-1)^j / y_j^{\alpha_i}} = \frac{\gamma}{2}$$

$$\frac{\sum_{j=0}^{n-r} \binom{n-r}{j} (-1)^j \frac{1}{\left(y_j - \ln h_{1i} + \ln(1 - x_{(r)})\right)^{\alpha_i}}}{\sum_{j=0}^{n-r} \binom{n-r}{j} (-1)^j / y_j^{\alpha_i}} = \frac{\gamma}{2} \qquad ..(4.3)$$

Similarly consider

$$\int_{h_{2i}}^{1} P_i(w_i|x)dw_i = \frac{\gamma}{2}$$

$$\int_0^{h_{2i}} P_i(w_i|x)dw_i = 1 - \frac{\gamma}{2}$$

Using (4.3), we can get

$$\frac{\sum_{j=0}^{n-r} \binom{n-r}{j} (-1)^j \frac{1}{\left(y_j - \ln h_{2i} + \ln(1 - x_{(r)})\right)^{\alpha_i}}}{\sum_{j=0}^{n-r} \binom{n-r}{j} (-1)^j / y_j^{\alpha_i}} = 1 - \frac{\gamma}{2} \qquad ..(4.4)$$

Solving (4.3) and (4.4), we get $(1-\gamma)100\%$ equal tail credible interval (h_{1i}, h_{2i}) for w_i^*, hence for future observation it becomes $\left(h_{1i} + x_{(r)},\ h_{2i} + x_{(r)}\right)$.

5. Bayes predictive estimator for the remaining $(n-r)$ order statistics truncated at $x_{(r)}$ and their $(1-\gamma)100\%$ equal tail credible interval

In this section we have used the method considered by Hawlader and Hossain (1995).

Let $X_{(s)i}, r+1 \leq S \leq n$ denote the failure time of the 5^{th} unit to fail in case of i^{th} double prior distribution.

The conditional pdf of $u_i = X_{(s)i} - X_{(r)}$ from a pdf truncated at $X_{(r)}$ is given by,

$$f(u_i/\theta) = \frac{\left(F(u_i)\right)^{s-r-1}\{1-F(u_i)\}^{n-s}.f(u_i)}{\beta_{(s-r,n-s+1)}}\ ,\quad 0<u_i<1$$

Using (1.1) and (1.2) we get

$$f(u_i/\theta) = \frac{\theta u_i^{\theta(s-r)-1}\left(1-u_i^{\theta}\right)^{n-s}}{\beta_{(s-r,n-s+1)}\sum_{j=s-r}^{n-r}\binom{n-r}{j}\left(1-x_{(r)}\right)^{\theta_j}\left\{1-\left(1-x_{(r)}\right)^{\theta}\right\}^{n-r-j}}\ ;$$

where $0<u_i<1-x_{(r)}$

since using the well known relation between binomial sum and incomplete beta function.

$$\int_0^{(1-x_{(r)})^{\theta}} w^{s-r-1}(1-w)^{n-s}dw = \beta_{(s-r,n-s+1)}\sum_{j=s-r}^{n-r}\binom{n-r}{j}\left(\left(1-x_{(r)}\right)^{\theta}\right)^{j}\left\{1-\left(1-x_{(r)}\right)^{\theta}\right\}^{n-r-j}$$

Which is the pdf of u_i under the range $0<u_i<1-x_{(r)}$.

For given $\underline{x}$, the conditional joint pdf of u_i and θ, is given by

$$f(x_i,\theta/x) = f(x_i/\theta).\pi_i(\theta/\underline{x})\ ,\quad 0<x_i<1-x_{(r)}$$

$$= \frac{\theta u_i^{\theta(s-r)-1}\left(1-u_i^{\theta}\right)^{n-s}.\sum_{j=0}^{n-r}\binom{n-r}{j}e^{-y_j\theta}\theta^{\alpha_i-1}(-1)^j}{\beta_{(s-r,n-s+1)}\sum_{j=0}^{n-r}\frac{(-1)^j\binom{n-r}{j}\Gamma\alpha_i}{y_j^{(\alpha_i)}}.\left(\sum_{j=s-r}^{n-r}\binom{n-r}{j}\left(1-x_{(r)}\right)^{\theta_j}\{1-(1-x_{(r)})^{\theta}\}^{n-r-j}\right)}$$

Integrating out θ, the predictive density of u_i is given by,

$$P(u_i/x) = \frac{\sum_{j=0}^{n-1}(-1)^j\int_0^{\infty}\theta^{\alpha_i}u_i^{\theta(s-r)-1}\left(1-u_i^{\theta}\right)^{n-s}e^{-y_j\theta}.d\theta}{\beta_{(s-r,n-s+1)}\sum_{j=0}^{n-r}\frac{(-1)^j\binom{n-r}{j}\Gamma\alpha_i}{y_j^{(\alpha_i)}}.\left(\sum_{j=s-r}^{n-r}\binom{n-r}{j}\left(1-x_{(r)}\right)^{\theta j}\{1-(1-x_{(r)})^{\theta}\}^{n-r-j}\right)}$$

$;0<u_i<1-x_{(r)}$

$$= \frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{1}{u_i}\int_0^{\infty}\theta^{\alpha_i}e^{\theta(s-r)\log u_i}\left(\sum_{w=0}^{n-s}\binom{n-s}{w}\left(-u_i^{\theta}\right)^{n-s-w}\right)e^{-y_j\theta}.d\theta}{\left(\frac{1}{n-r}\right)\left(\sum_{j=0}^{n-r}\frac{(-1)^j\binom{n-r}{j}\Gamma\alpha_i}{y_j^{(\alpha_i)}}\right)\left(\sum_{j=1}^{n-r}\binom{n-r}{j}\left(1-x_{(r)}\right)^{\theta_j}\right)\left(1-(1-x_{(r)})^{\theta}\right)^{n-r-j}}$$

$$= \frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\frac{1}{u_i}\sum_{w=0}^{n-s}\binom{n-s}{w}(-1)^{n-s-w}\int_0^{\infty}\theta^{\alpha_i}e^{-\theta(y_j-(n-w-r)\log u_i)}.d\theta}{\left(\frac{1}{n-r}\right)\left(\sum_{j=0}^{n-r}\frac{(-1)^j\binom{n-r}{j}\Gamma\alpha_i}{y_j^{(\alpha_i)}}\right)\left(\sum_{j=1}^{n-r}\binom{n-r}{j}\left(1-x_{(r)}\right)^{\theta_j}\right)\left(1-(1-x_{(r)})^{\theta}\right)^{n-r-j}}$$

$$= \frac{\sum_{j=0}^{n-1}(-1)^j\binom{n-r}{j}\frac{1}{u_i}\sum_{w=0}^{n-s}\frac{\binom{n-s}{w}(-1)^{n-s-w}\Gamma(\alpha_i+1)}{\{y_j-(n-w-r)\log u_i\}^{\alpha_i+1}}}{\left(\frac{1}{n-r}\right)\left(\sum_{j=0}^{n-r}\frac{(-1)^j\binom{n-r}{j}\Gamma\alpha_i}{y_j^{(\alpha_i)}}\right)\left(\sum_{j=1}^{n-r}\binom{n-r}{j}(1-x_{(r)})^{\theta_j}\right)\left(1-(1-x_{(r)})^{\theta}\right)^{n-r-j}},$$

$$0 < u_i < 1 - x_{(r)} \quad ..(5.1)$$

which is predictive density of u_i.

Under squared error loss function, Bayes predictive estimate of u_i is given as,

$$u_i^* = E(u_i/x) = \int_0^{1-x_{(r)}} u_i . P(u_i/x) du_i$$

$$= \frac{\int_0^{1-x_{(r)}} u_i . \sum_{j=0}^{n-1}(-1)^j\binom{n-r}{j}\int_0^{\infty}\theta^{\alpha_i}u_i^{\theta(s-r)-1}\left(1-u_i^{\theta}\right)^{n-s}e^{-y_j\theta}du_i . d\theta}{\beta_{(s-r,n-s+1)} . \sum_{j=0}^{n-r}\frac{(-1)^j\binom{n-r}{j}\Gamma\alpha_i}{y_j^{(\alpha_i)}} . \left(\sum_{j=s-r}^{n-r}\binom{n-r}{j}\left(1-x_{(r)}\right)^{\theta_j}\{1-(1-x_{(r)})^{\theta}\}^{n-r-j}\right)}$$

$$= \frac{\sum_{j=0}^{n-1}(-1)^j\binom{n-r}{j}\int_0^{\infty}\int_0^{1-x_{(r)}}\theta^{\alpha_i}u_i^{\theta(s-r)-1}\left(1-u_i^{\theta}\right)^{n-s}e^{-y_j\theta} . du_i . d\theta}{\beta_{(s-r,n-s+1)} . \sum_{j=0}^{n-r}\frac{(-1)^j\binom{n-r}{j}\Gamma\alpha_i}{y_j^{(\alpha_i)}} . \left(\sum_{j=s-r}^{n-r}\binom{n-r}{j}\left(1-x_{(r)}\right)^{\theta_j}\{1-(1-x_{(r)})^{\theta}\}^{n-r-j}\right)}$$

..(5.2)

For $s = r + 1$,

$$u_i^* = \frac{(n-r)(\alpha_i)\sum_{j=0}^{n-1}(-1)^j\binom{n-r}{j}\sum_{w=0}^{n-r-1}\binom{n-r-1}{w}(-1)^{n-r-w-1}\int_0^{1-x_{(r)}}\frac{1}{\{y_j-(n-w-r)\log u_i\}^{\alpha_i+1}} . du_i}{\left(\sum_{j=0}^{n-r}\frac{(-1)^j\binom{n-r}{j}}{y_j^{(\alpha_i)}}\right)\left(\sum_{j=1}^{n-r}\binom{n-r}{j}(1-x_{(r)})^{\theta_j}(1-(1-x_{(r)})^{\theta})\right)^{n-r-j}} \quad ..(5.3)$$

(1-γ) 100% equal tail credible interval (H_{1i}, H_{2i}) can be obtained from the equation:

$$\int_0^{H_{1i}} P_i(u_i/x)du_i = \frac{\gamma}{2} = \int_{H_{2i}}^{1} P_i(u_i/x)du_i \quad ..(5.4)$$

Consider the equation

$$\int_0^{H_{1i}} P_i(u_i/x)du_i = \frac{\gamma}{2}$$

$$\frac{\int_0^{H_{1i}}\frac{\alpha_i}{u_i} . \sum_{j=0}^{n-1}\left\{\frac{(-1)^j\binom{n-r}{j}\sum_{w=0}^{n-s}\binom{n-s}{w}(-1)^{n-s-w}}{\left(y_j-(n-w-r)\log u_i\right)^{\alpha_i+1}}\right\}du_i}{\beta_{(s-r,n-s+1)} . \sum_{j=0}^{n-r}\frac{(-1)^j\binom{n-r}{j}}{y_j^{(\alpha_i)}} . \left(\sum_{j=s-r}^{n-r}\binom{n-r}{j}\left(1-x_{(r)}\right)^{\theta_j}\{1-(1-x_{(r)})^{\theta}\}^{n-r-j}\right)} = \frac{\gamma}{2}$$

$$\frac{\sum_{j=0}^{n-1}(-1)^j\binom{n-r}{j}\sum_{w=0}^{n-s}\binom{n-s}{w}(-1)^{n-s-w}\left[\left(y_j-(n-w-r)\log H_{1i}\right)^{-\alpha_i}\right]}{\beta_{(s-r,n-s+1)} . \sum_{j=0}^{n-r}\frac{(-1)^j\binom{n-r}{j}}{y_j^{(\alpha_i)}} . \left(\sum_{j=s-r}^{n-r}\binom{n-r}{j}(1-x_{(r)})^{\theta_j}\{1-(1-x_{(r)})^{\theta}\}^{n-r-j}\right)} = \frac{\gamma}{2} \quad ..(5.5)$$

Similarly we solve

$$\int_{H_{2i}}^{1-x_{(r)}} P_i(u_i|x)\,du_i = \frac{\gamma}{2}$$

$$\int_0^{H_{2i}} P_i(u_i/x)\,du_i = 1-\frac{\gamma}{2}$$

$$\frac{\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\sum_{w=0}^{n-s}\binom{n-s}{w}(-1)^{n-s-w}\left\{\left(y_j-(n-w-r)\log H_{2i}\right)^{-\alpha_i}\right\}}{\beta_{(s-r,n-s+1)}\cdot\sum_{j=0}^{n-r}\frac{(-1)^j\binom{n-r}{j}}{y_j^{(\alpha_i)}}\cdot\left(\sum_{j=s-r}^{n-r}\binom{n-r}{j}(1-x_{(r)})^{\theta j}\{1-(1-x_{(r)})^{\theta}\}^{n-r-j}\right)} = 1-\frac{\gamma}{2} \qquad \ldots (5.6)$$

Solving (5.5) and (5.6) for H_{1i} and H_{2i}, we get equal tail credible interval for u_i.

Hence (1-γ) 100% equal tail credible interval for x_{s_i} becomes;

$$\left(H_{1i}+x_{(r)}, \quad H_{2i}+x_{(r)}\right)$$

Particular for S= r+1, the interval can be obtained by solving the equations,

$$\frac{(n-r)\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\sum_{w=0}^{n-r-1}\binom{n-r-1}{w}(-1)^{n-r-w-1}\left\{\left(y_j-(n-w-r)\log H_{1i}\right)^{-\alpha_i}\right\}}{\sum_{j=0}^{n-r}\frac{(-1)^j\binom{n-r}{j}}{y_j^{(\alpha_i)}}\cdot\left(\sum_{j=1}^{n-r}\binom{n-r}{j}(1-x_{(r)})^{\theta j}\{1-(1-x_{(r)})^{\theta}\}^{n-r-j}\right)} = \frac{\gamma}{2}$$

and

$$\frac{(n-r)\sum_{j=0}^{n-r}(-1)^j\binom{n-r}{j}\sum_{w=0}^{n-s}\binom{n-r-1}{w}(-1)^{n-w-r-1}\left\{\left(y_j-(n-w-r)\log H_{2i}\right)^{-\alpha_i}\right\}}{\sum_{j=0}^{n-r}\frac{(-1)^j\binom{n-r}{j}}{y_j^{(\alpha_i)}}\cdot\left(\sum_{j=1}^{n-r}\binom{n-r}{j}(1-x_{(r)})^{\theta j}\{1-(1-x_{(r)})^{\theta}\}^{n-r-j}\right)} = 1-\frac{\gamma}{2}$$

7. Simulation study

A Monte Carlo simulation study is carried out to compare the performance of the Bayes estimators under different joint priors and single prior. To generate 1000 Type-II censored samples the value of the parameter θ is considered as 0.5 and the values of the hyper parameters for all joint and single priors are considered as $a_i = 3$, $b_i = 2$, i=1,2,34 and c = 1 for Jeffery's prior, c=2 for non informativeprior, c= 3 for non informative prior and c=0 for only gamma prior. The reliability is calculated at time t = 0.6. Simulation is done for sample size (n) 20 and for different censored values (r) like20and 15. In each case Bayes estimates of θ, R(t), future observation z^* and $(r+1)^{th}$ ordered failure time $X_{(r+1)}$ are obtained. Their mean squared errors (MSE) and Bayes equal tail credible intervals are also obtained. The first, second and third values in each cell of columns third and fourth of Tables 1 to 2 denote the Bayes estimate, MSE and credible intervals.

8-A. Comparison of priors based on the MSE and credible interval of θ

From the third column of Tables 1 it is observed that the values of the MSE of the Bayes estimator of parameter θ is smaller in case of Gamma-non informative (c= 3) joint prior and then followed by Gamma-non informative (c= 2), Gamma - Jeffery's (c =1) and only gamma priors for all the values of n and r considered here . Length of its credible interval is smallest also in Gamma-non informative (c= 3) joint prior and then followed by Gamma-non informative (c= 2), only gamma and Gamma-non informative (c= 2). Also as value of c increases, MSE of θ decreases. Similarly increase in the value of r has decreasing effect on MSE of θ for any types of joint or single priors.

8-B. Comparison of priors based on the MSE and credible interval of R(t)

From the fourth column of Tables 1 it is observed that for all values of n and r considered here the values of the MSE of the Bayes estimator of R(t) is smaller in case of the joint prior Gamma-non informative (c= 3) and then followed by Gamma-non informative (c= 2) , Gamma-non informative (c= 1) only gamma priors. i.e. as the value of c increases in gamma and non informative joint prior the value of MSE of R(t) decreases.

Table 1. Bayes estimates, MSE and Credible intervals for θ and R(t) for n = 20

Joint priors	r	θ	R(t)
Gamma - Jeffery's (c=1)	10	0.56086	0.24629
		0.01731	0.00233
		(0.33533, 0.82611)	(0.15650, 0.34181)
	15	0.55484	0.24424
		0.01507	0.00206
		(0.34371, 0.81018)	(0.16043, 0.33671)
Gamma-non informative (c= 2)	10	0.53722	0.23727
		0.01420	0.00196
		(0.30364, 0.798623)	(0.15255, 0.33267)
	15	0.52955	0.23458
		0.01185	0.00167
		(0.32219, 0.97541)	(0.15117, 0.32642)
Gamma-non informative (c= 3)	10	0.51877	0.23025
		0.01280	0.00183
		(0.37824, 0.77497)	(0.15120, 0.32478)
	15	0.50421	0.22477
		0.00996	0.00148
		(0.30094, 0.74874)	(0.141486, 0.31590)
Only gamma	10	0.58367	0.25497
		0.02131	0.00282
		(0.35754, 0.85100)	(0.16598, 0.35008)
	15	0.58002	0.25373
		0.01964	0.00262
		(0.36480, 0.84066)	(0.16939, 0.34679)

Table 2. Bayes estimates and Credible intervals for Z^* and $x_{(r+1)}$ for n = 20

Joint priors	r	Z^*	$x_{(r+1)}$
Gamma - Jeffery's (c =1)	10	0.48864	0.23391
		0.00764	0.02136
		(0.21613, 0.95918)	(0.21315, 0.66199)
	15	0.68611	0.57757
		0.00939	0.01789
		(0.52027, 0.97662)	(0.52015, 1.51168)
Gamma-non informative (c= 2)	10	0.48127	0.26600
		0.00745	0.03735
		(0.21335, 0.95240)	(0.21301, 0.51712)
	15	0.681142	0.57266
		0.00921	0.01800
		(0.52021, 0.97541)	(0.52015, 1.51027)
Gamma-non informative (c= 3)	10	0.47221	0.33012
		0.00721	0.05614
		(0.21023, 0.94918)	(0.20873, 0.38294)
	15	0.67599	0.568013
		0.00910	0.01806
		(0.52018, 0.97401)	(0.52015, 1.50461)
Only gamma	10	0.49880	0.26508
		0.00742	0.02626
		(0.21808, 0.96170)	(0.21737, 0.78409)
	15	0.69095	0.58237
		0.00913	0.01790
		(0.52036, 0.97771)	(0.52016, 1.51410)

The minimum length of the credible interval of R(t) is observed for the joint prior Gamma-non informative (c= 3) and then followed by Gamma-non informative (c= 2), only gamma and Gamma-non informative (c= 1) in case of all the values of n and r considered here. i.e. as the value of c increases in gamma and non informative joint prior the value of length of credible interval of R(t) decreases.

8-C. Comparison of priors based on MSE and credible interval of future predicted value

From the third column of Tables 2 we find that MSE in case of r = 10 as well as r=15is minimum in case of Gamma-non informative (c= 3) and then followed by Only gamma prior, Gamma-non informative (c= 2) and Gamma - Jeffery's (c =1) in case of all the values of n and r considered here.
The length of the confidence interval of future predicted value becomes minimum in case of Gamma-non informative (c= 3) and then followed by Gamma-non informative (c= 2), Gamma - Jeffery's (c =1) and only gamma priors. Here we observe that as c increases in gamma and non informative joint prior the length of future predicted value decreases for r = 10 as well as 15.

8-D. Comparison based on the MSE and credible interval of next ordered failure time $X_{(r+1)}$.

From the fourth column of the Tables 2 we observed that minimum MSE is observed for Gamma- Jeffery's (c =1) joint prior then followed by only gamma, Gamma-non informative (c= 2) and Gamma-non informative (c= 3) priors. Here MSE increases as the value of c increases in the joint priors.

The minimum length of credible interval of $X_{(r+1)}$ is observed in case of Gamma-non informative (c= 3) then followed by Gamma-non informative (c= 2), Gamma-non informative (c= 1) and only gamma priors for r = 10 as well as 15.. We also observe that as c increases the length of confidence interval of $X_{(r+1)}$ decreases as c increases for r = 10 and 15 both.
Thus we observed that Gamma-non informative (c= 3) joint prior performs well compared to the other single and joint priors considered in this study for almost all the characteristics considered here except for MSE of future predicted value.

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