ISSN: 0975-833X

Available online at http://www.journalcra.com

International Journal of Current Research
Vol. 8, Issue, 03, pp. 27474-27478, March, 2016

INTERNATIONAL JOURNAL OF CURRENT RESEARCH

RESEARCH ARTICLE

OBSERVATIONS ON FUNCTIONS VIA GSΛ SETS IN TOPOLOGICAL SPACES

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ARTICLE INFO

Article History:
Received 20th December, 2015
Received in revised form
18th January, 2016
Accepted 10th February, 2016
Published online 16th March, 2016

Key words:

gsΛ irresolute functions and contra
gsΛ irresolute function.

ABSTRACT

The In the cognitive process of research on gsΛ sets we bring in a new class of functions called gsΛ irresolute function and contra gsΛ irresolute function, and observe some of their characteristics.

Citation: Vijilius Helena Raj and Pious Missier, S. 2016. "Observations on functions via gsΛ sets in Topological spaces", *International Journal of Current Research,* 8, (03), 27474-27478.

INTRODUCTION

In 1986, Maki, (1986) continued the work of Levine and Dunham on generalized closed sets and closure operators by acquainting the concept of Λ sets in topological spaces. In 2008 M. Caldas, S. Jafari and T. Noiri Lamb-gs introduced Λgeneralized closed sets (Λg, Λ-g, gΛ) and their properties. They also studied the concept of Λ closed maps. Recently, many authors investigated some new maps and their notions via Λ open sets and Λ closed sets. In 2007 M.Caldas, S.Jafari and T.Navalagi more lamb introduced the concept of Λ irresolute maps. The notion of irresolute functions weak was introduced and investigated by M. Caldas in 2000. Recently Vijilius @el familiarized a new set named gsΛ sets in topological spaces In this direction we establish a new class of function called gsΛ irresolute function and contra gsΛirresolute function. In this article we investigate some of their fundamental properties and the connections between these maps and other existing topological maps are studied. Throughout this paper (X.τ), (Y,σ) and (Z,ϖ) (or simply X, Y and Z) will always denote topological spaces on which no separation axioms are assumed unless explicitly stated. Int(A), Cl(A), λInt (A), λClA), gsΛCl(A) and gsΛ Int(A) denote the interior of A, closure of A, lambda interior of A, lambda closure of A gsΛ closure of A and gsΛ Interior of A respectively.

Preliminary Definitions

Let us recall some definitions in sequel which is useful for this paper.

Definition: 1

A topological space (X, τ) is said to be

1. (Jin Han Park *et al.,* 2002) a generalized closed if Cl(A) ⊆ U, whenever A ⊆ U and U is open in X.

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2. (Caldas *et al.,* 2008) a subset A of a space X is called Λ-closed if A = B⋂C, where B is a Λ-set and C is a closed set.
3. (Caldas *et al.,* 2008) a subset A of X is said to be a Λg closed set if Cl(A)⊆ U whenever A⊆ U , where U is Λ open in X.
4. (Missier, 2013) a subset A of X is said to be a gsΛ closed set [23] if λCl (A)⊆ U whenever A⊆ U, where U is semi open in X.

The complement of above closed sets are called its respective open sets. The gsΛ closure (respectively closure, Λ closure) of a subset A of X denoted by gsΛCl(A), (Cl(A), λClA) is the intersection of all gsΛ closed sets (closed sets, Λ closed sets) containing A.

Lemma: 2 (Jin Han Park *et al.,* 2002)

1. Every Λ-set is a Λ-closed set,
2. Every open and closed sets are Λ-closed sets.

Definition: 3

A function f: (X.τ) ⟶(Y,σ) is called

1. gsΛ closed if f(F) is Λ closed in (Y, σ) for every Λ closed set F of (X. τ),
2. (Levine and Semi, 1963) semi continuous if f^{1}(V) is semi open in (X,τfor every open set V in (Y, σ),
3. (Maki, 1989) Λ continuous if f-1 (V) is Λ open (Λ closed) in (X.τ) for every open (closed) set V in (Y, σ),
4. (Dontchev, 1996) contra continuous if f^{1} (V) is open (closed) in (X,τfor every closed (open) set V in (Y, σ),
5. (Dontchev and Noiri, 1999) contra semi continuous if f^{1} (V) is semi open (semi closed) in (X, τ for every closed (open) set V in (Y,σ),
6. (Caldas *et al.,* 2006) contra Λ continuous map if f^{1} (V) is Λ open (Λ closed) in (X,τ for every closed (open) set V in (Y, σ),
7. (Jin Han Park *et al.,* 2002) gc irresolute if the inverse images of g closed sets in(Y,σ) are g closed in (X.τ),
8. (Caldas *et al.,* 2007) Λ irresolute if the inverse image of Λ open sets in Y are Λ open in (X.τ),
9. (Missier *et al.,* 2012) gsΛ closed map (gsΛ open map) if the image of each closed set (open set) in X is gsΛ closed (gsΛ open) in Y.
10. (Missier and Vijilius, 2013) gsΛ continuous function if the inverse image f^{1} (V) of each closed set (open set) V in (Y,σ) is gsΛ closed (gsΛ open) in (X.τ).
11. (Vijilius *et al.,* 2012) M.gsΛ closed map (M.gsΛ open map) if the image of each gsΛ closed set (gsΛ open set) in X is gsΛ closed (gsΛ open) in Y

Lemma: 4 (Caldas, 2006)

1. i) A space (X. τ) is said to be ΛS-space if every Λ open subset of X is semi open in X.
2. ii) A space (X, τ)is said to be Λ-space if every Λ closed(Λ open) subset of X is closed(open) in X.

Preposition-5 (Missier and Vijilius, 2012 and 2013)

In a topological space (X.τ), the following properties hold:

1. Every closed set is gsΛ closed(gsΛ open),
2. Every open set is gsΛ closed (gsΛ open),
3. Every Λ closed (Λ open) set is gsΛ closed (gsΛ open),
4. Union (intersection) of gsΛ closed (gsΛ open) sets is not gsΛ closed(gsΛ open),
5. In T_1 space every gsΛ closed set (gsΛ open) is Λ closed (Λ open),
6. In Partition space every gsΛ closed(gsΛ open) set is g closed(g open),
7. In a door space every subset is gsΛ closed (gsΛ open), and
8. In $T_{1/2}$ space every subset is gsΛ closed (gsΛ open).

Definition: 3

1. 1.Contra gsΛ continuous function if the inverse image f^{1} (V) of each closed set (open set) V in (Y, σ) is gsΛ open (gsΛ closed) in (X.τ) .
2. gsΛ irresolute function if the inverse image f^{1} (V) of gsΛ each closed set (gsΛ open set) V in (Y,σ) is gsΛ closed (gsΛ open) in (X.τ) .

Observations on gsΛ functions

Theorem: 1

Composition of gsΛ irresolute functions is gsΛ irresolute.

Proof:

Let $f : X \to Y$ and $g : Y \to Z$ be gsΛ irresolute functions.
Let F be a gsΛ open set of (Z,ϖ). Then $g^{-1}(F)$ is a gsΛ open set in (Y,σ) as $g:(Y,\sigma) \longrightarrow (Z,\varpi)$ is a a gsΛ irresolute function and $f^{-1}g^{-1}(F)=(gof)^{-1}(F)$ is a gsΛ open set in (X,τ) as f is a a gsΛ irresolute function. Thus gof: $(X.\tau) \longrightarrow (Z,\varpi)$ is a gsΛ irresolute function.

Theorem: 2

Composition of contra gsΛ irresolute functions is gsΛ irresolute.

Proof:

Let $f : X \to Y$ and $g : Y \to Z$ be contra gsΛ irresolute functions.
Let F be a gsΛ open set of (Z,ϖ). Then $g^{-1}(F)$ is a gsΛ closed set in (Y,σ) as g: $(Y,\sigma) \longrightarrow (Z,\varpi)$ is a contra gsΛ irresolute function and $f^{-1}(g^{-1}(F))=(gof)^{-1}(F)$ is a gsΛ open set in $(X,\tau$as f:$(X.\tau) \longrightarrow (Y,\sigma)$ is a contra gsΛ irresolute. Thus gof:$(X.\tau) \longrightarrow (Z,\varpi)$ is a gsΛ irresolute function.

Theorem: 3

If f: $(X,\tau) \longrightarrow (Y,\sigma)$ contra gsΛ irresolute function and g:$(Y,\sigma) \longrightarrow (Z,\varpi)$ gsΛ irresolute function, then gof:$(X.\tau) \longrightarrow (Z,\varpi)$ is a contra gsΛ irresolute function .

Proof:

Let F be a gsΛ open set of (Z,ϖ). Then $g^{-1}(F)$ is a gsΛ open set in (Y,σ) as g:$(Y,\sigma) \longrightarrow (Z,\varpi)$ is a gsΛ irresolute function and $f^{-1}(g^{-1}(F))=(gof)^{-1}(F)$ is a gsΛ closed set in (X,τ) as f:$(X.\tau) \longrightarrow (Y,\sigma)$ is a contra gsΛ irresolute. Thus gof:$(X.\tau) \longrightarrow (Z,\varpi)$ is a contra gsΛ irresolute function.

Theorem: 4

Composition of gsΛ irresolute functions is gsΛ continuous function.

Proof:

Let f:$(X.\tau) \longrightarrow (Y,\sigma)$ and g:$(Y,\sigma) \longrightarrow (Z,\varpi)$ be gsΛ irresolute functions.
Let F be a open set of (Z,ϖ). Then F is also gsΛ open set in (Z,ϖ) [Proposition 5].Thus we have $g^{-1}(F)$ is a gsΛ open set in (Y,σ) as g:$(Y,\sigma) \longrightarrow (Z,\varpi)$ is a gsΛ irresolute function and
$f^{-1}(g^{-1}(F))=(gof)^{-1}(F)$ is a gsΛ open set in (X,τ) as f:$(X.\tau) \longrightarrow (Y,\sigma)$ is a gsΛ irresolute function. Hence gof:$(X.\tau) \longrightarrow (Z,\varpi)$ is a gsΛ continuous function.

Theorem: 5

Composition of contra gsΛ irresolute functions is gsΛ continuous function.

Proof:

Let f:$(X.\tau) \longrightarrow (Y,\sigma)$ and g:$(Y,\sigma) \longrightarrow (Z,\varpi)$ be contra gsΛ irresolute functions. Let F be a open set of (Z,ϖ). Then F is also gsΛ open set in (Z,ϖ) [Preposition 5].Thus we have $g^{-1}(F)$ is a gsΛ closed set in (Y,σ) as g:$(Y,\sigma) \longrightarrow (Z,\varpi)$ is a contra gsΛ irresolute function and $f^{-1}(g^{-1}(F))=(gof)^{-1}(F)$ is a gsΛ open set in (X,τ) as f:$(X.\tau) \longrightarrow (Y,\sigma)$ is also a contra gsΛ irresolute function. Thense gof: $(X.\tau) \longrightarrow (Z,\varpi)$ is a gsΛ continuous function.

Theorem: 6

Composition of gsΛ irresolute functions is contra gsΛ continuous function.

Proof:

Let f:$(X.\tau) \longrightarrow (Y,\sigma)$ and g:$(Y,\sigma) \longrightarrow (Z,\varpi)$ be gsΛ irresolute functions
Let F be a open set of (Z,ϖ). Then F is also gsΛ closed set in (Z,ϖ) [Preposition 5]. Thus we have $g^{-1}(F)$ is a gsΛ closed set in (Y,σ) as g:$(Y,\sigma) \longrightarrow (Z,\varpi)$ is a gsΛ irresolute function and $f^{-1}(g^{-1}(F))=(gof)^{-1}(F)$ is a gsΛ closed set in (X,τ) as f:$(X.\tau) \longrightarrow (Y,\sigma)$ is a gsΛ irresolute function. Consequently gof: $(X.\tau) \longrightarrow (Z,\varpi)$ is a contra gsΛ continuous function.

Theorem: 7

Composition of contra gsΛ irresolute functions is contra gsΛ continuous function.

Proof:

Let f:(X.τ) ⟶(Y,σ) and g:(Y,σ) ⟶ (Z,ϖ) be contra gsΛ irresolute functions
Let F be a closed set of (Z,ϖ). Then F is also gsΛ open set in (Z,ϖ) [Preposition 5]. Thus we have $g^{-1}(F)$ is a gsΛ closed set in (Y,σ) as g:(Y,σ) ⟶(Z,ϖ) is a contra gsΛ irresolute function and f^{1} $(g^{-1}(F))=(gof)^{-1}(F)$ is a gsΛ open set in (X,τ) as f:(X.τ) ⟶(Y,σ) is a contra gsΛ irresolute function. Hence gof:(X.τ) ⟶ (Z,ϖ) is a contra gsΛ continuous function

Theorem: 8

Let f:(X,τ) ⟶ (Y, σ) and g: (Y, σ) ⟶ (Z,ϖ) contra gsΛ irresolute function, then gof: (X.τ) (Z,ϖ) is a contra Λ continuous function if (X,τ) is a T_1 space.

Proof: Let f:(X.τ) ⟶(Y,σ) and g: (Y,σ) ⟶(Z,ϖ) be contra gsΛ irresolute functions.
Let F be a open set of (Z,ϖ). Then F is also gsΛ closed set in (Z,ϖ) [Preposition 5]. Thus we have $g^{-}1(F)$ is a gsΛ open set in (Y,σ) as g: (Y,σ))⟶(Z,ϖ) is a contra gsΛ irresolute function and f^{1} $g^{-1}(F) = (gof)^{-1}(F)$ is a gsΛ closed set in (X,τ) as f: (X.τ) ⟶ (Y,σ) is a contra gsΛ irresolute function. Now $(gof)^{-1}(F)$ is a Λ closed set in X, as X is a T_1 space,. Thus gof: (X,τ)⟶ (Z,ϖ) is a contra Λ continuous function.

Theorem: 9

Composition of gsΛ irresolute functions is a Λ continuous function if the domain of the composite function is a T_1 space.

Proof:

Let f:(X.τ) ⟶(Y,σ) and g:(Y,σ) ⟶(Z,ϖ) be gsΛ irresolute functions.
Let F be a closed set of (Z,ϖ). Then F is also gsΛ closed set in (Z,ϖ) [Preposition 5]. Thus we have $g^{-}1(F)$ is a gsΛ closed set in (Y,σ) as g: (Y,σ))⟶(Z,ϖ) is a gsΛ irresolute function and
f^{1} $g^{-1}(F) = (gof)^{-1}(F)$ is a gsΛ closed set in (X,τ) as f: (X.τ) ⟶ (Y,σ) is a gsΛ irresolute function. Now $(gof)^{-1}(F)$ is a Λ closed set in X, as X is a T_1 space[Preposition 5]. Thus gof: (X,τ)⟶ (Z,ϖ) is a Λ continuous function.

Theorem: 10

Composition of contra gsΛ irresolute functions is a Λ continuous function if the domain of the composite function is a T_1 space.

Proof:

Let f:(X.τ) ⟶(Y,σ) and g:(Y,σ) ⟶ (Z,ϖ) be gsΛ irresolute functions.
Let F be a closed set of (Z,ϖ) . Then F is also gsΛ closed set in (Z,ϖ) [Preposition 5]. Thus we have $g^{-}1(F)$ is a gsΛ open set in (Y,σ) as g: (Y,σ))⟶(Z,ϖ) is a contra gsΛ irresolute function and f^{1} $g^{-1}(F) = (gof)^{-1}(F)$ is a gsΛ closed set in (X,τ) as f: (X.τ) ⟶ (Y,σ) is a contra gsΛ irresolute function. Now $(gof)^{-1}(F)$ is a Λ closed set in X, as X is a T_1 space.
Thus gof: (X,τ)⟶ (Z,ϖ) is a contra Λ continuous function.

Theorem: 11

If f:(X,τ⟶ (Y,σ) is a gsΛ irresolute function and g: (Y,σ)⟶ (Z,ϖ) is a gsΛ continuous function, then gof:(X.τ) ⟶ (Z,ϖ) is a gsΛ continuous function.

Proof:

Let f : (X,τ)⟶ (Y,σ) is a gsΛ irresolute function and g: (Y,σ)⟶ (Z,ϖ) is a gsΛ continuous function. Let F be a closed set of (Z,ϖ) . Then we have $g^{-1}(F)$ is a gsΛ closed set in (Y,σ) as
g: (Y,σ))⟶(Z,ϖ) is a gsΛ continuous function and f^{1} $g^{-1}(F) = (gof)^{-1}(F)$ is a gsΛ closed set in (X,τ) as f: (X.τ) ⟶ (Y,σ) is a gsΛ irresolute function. It can be observed that gof:(X.τ) ⟶ (Z,ϖ) is a gsΛ continuous function.

Theorem: 12

If f:(X,τ) ⟶ (Y,σ) is a gsΛ irresolute function and g: (Y,σ) ⟶(Z,ϖ) is a Λ continuous function, then gof:(X.τ) ⟶ (Z,ϖ) is a gsΛ continuous function.

Proof:

Proof follows as every Λ open set is gsΛ open set.

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