



RESEARCH ARTICLE

EFFECT OF VISCOUS DISSIPATION ON FALKNER-SKIN STRETCHING AND SHRINKING WEDGE
FLOW OF A POWER – LAW FLUID WITH CONVECTIVE BOUNDARY CONDITION

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ABSTRACT

The steady, two dimensional, Falkner-Skan stretching and shrinking wedge flow of a power law fluid in the presence of viscous dissipation and convective boundary condition Using the similarity transformations, the governing equations have been transformed into a system of ordinary differential equations. These differential equations are highly nonlinear which cannot be solved analytically. Therefore, bvp4c MATLAB solver has been used for solving it. Numerical results are obtained for the skin-friction coefficient and the local Nusselt number as well as the velocity and temperature profiles for different values of the governing parameters, namely, consistency parameter, Falkner-Skin flow parameter, power law index parameter, convective parameter, wedge velocity parameter, suction/injection parameter and Eckert number.

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INTRODUCTION

Motivated by significant applications in packed bed reactor, geothermal system, extractions of crude oil, water or nuclear pollution, and so forth, the wedge flow over shaped bodies has attracted the attention of various researchers as the early formulation given by Falkner and Skan (1931). Later, Asaithambi (1998) analyzed the Falkner-Skan equation by using finite difference scheme. Postelnicu and Pop (2011) studied the two-dimensional laminar boundary layer flow of a power-law fluid past a permeable stretching wedge beneath a variable free stream. Hendi and Hussain (2012) discussed the MHD Falkner-Skan flow over a porous surface. Thus, a number of non-Newtonian fluid models have been proposed. In the literature, the vast majority of non-Newtonian fluid models are concerned with simple models like the power law and grade two or three. Eldabe and Salwa (1995) studied the non-Newtonian Casson fluid flow between two rotating cylinders under a radial magnetic field. Dash *et al.* (1996) studied the Casson fluid flow in a pipe filled with a homogeneous porous medium.

Boyd *et al.* (2007) studied the Casson and Carreau- Yasuda non-Newtonian blood models in steady and oscillatory flow using the lattice Boltzmann method. Nadeem *et al.* (2012) investigated the magneto hydrodynamic flow of a Casson fluid over an exponentially shrinking sheet. Kandasamy and Pai (2012) studied the Entrance region flow of casson fluid in a circular tube. Bahmani (2013) studied the power-law fluids velocity profile between two parallel plates. Mudassar (2013) investigated the boundary layer flow of power-law fluid over a power-law stretching surface. Bachok *et al.* (2013) investigated the heat transfer characteristics of steady two-dimensional stagnation-point flow of a copper (Cu)-water nanofluid over a permeable stretching/shrinking sheet. Mukhopadhyay (2013) studied the Boundary-layer forced convection flow of a Casson fluid past a symmetric wedge. Yacob and Ishak (2014) investigated the laminar flow of a power-law fluid over a permeable shrinking sheet of constant surface temperature and also found that the heat transfer rate at the surface increases with an increase in the Prandtl number. Rashad (2009) investigated the radiative effect on free convection flows in porous medium in the presence of pressure work and viscous dissipation. Hamid *et al.* (2010) studied the effects of the Joule heating and viscous dissipation on the magnetohydrodynamics (MHD) Marangoni convection boundary layer flow and

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concluded that the combined effects of the Joule heating and viscous dissipation have significantly influenced the surface temperature gradient. Sheikholeslami *et al.* (2011) studied the hydromagnetic flow between two horizontal plates in a rotating system, where the lower plate is a stretching sheet and the upper is a porous solid plate in the presence of viscous dissipation and concluded that the increasing Prandtl number in presence of viscous dissipation leads to temperature increasing between two plates, while in absence of viscous dissipation, the changes are inverse. Gangadhar (2012) conclude that the local skin friction coefficient increases and local Nusselt number coefficient decreases in the presence of viscous dissipation. Kumar (2013) analyzed the problem of MHD mixed convective flow of a micropolar fluid with the effect of Ohmic heating, radiation and viscous dissipation over a chemically reacting porous plate with constant heat flux. Kabir *et al.* (2013) investigated the effects of the viscous dissipation on MHD natural convection flow along a uniformly heated vertical wavy surface with heat generation. Gangadhar (2015) investigated the radiation, heat generation viscous dissipation and magnetohydrodynamic effects on the laminar boundary layer about a flat-plate in a uniform stream of fluid (Blasius flow), and about a moving plate in a quiescent ambient fluid (Sakiadis flow) both under a convective surface boundary condition. Khilap Singh and Manoj Kumar (2015) investigated the study of heat and mass transfer characteristics of the free convection on a vertical plate in porous media with variable wall temperature and concentration in a doubly stratified and viscous dissipating micropolar fluid in presence of chemical reaction, heat generation and Ohmic heating.

Makinde & Olanrewaju (2010) studied the effects of thermal buoyancy on the laminar boundary layer about a vertical plate in a uniform stream of fluid under a convective surface boundary condition. Very more recently, Olanrewaju *et al* (2011) studied the Radiation and viscous dissipation effects for the Blasius and Sakiadis flows with a convective surface boundary condition. Gangadhar (2013) investigated solet and dufour effects on hydro magnetic heat and mass transfer over a vertical plate with a convective surface boundary condition and chemical reaction. The present study investigates the steady, two dimensional, Flakner-Skan stretching and shrinking wedge flow of a power law fluid in the presence of viscous dissipation and convective boundary condition. Using the similarity transformations, the governing equations have been transformed into a set of ordinary differential equations, which are nonlinear and cannot be solved analytically, therefore, bvp4c MATLAB solver has been used for solving it. The results for velocity, microrotation and temperature functions are carried out for the wide range of important parameters namely; consistency parameter, Falkner-Skin flow parameter, power law index parameter, convective parameter, wedge velocity parameter, suction/injection parameter and Eckert number. The skin friction, the couple wall stress and the rate of heat transfer have also been computed.

Mathematical formulation

Consider a two dimensional steady viscous incompressible boundary layer flow due to non-Newtonian fluid past a porous stretching and shrinking wedge. It is assumed that free stream

velocity is of the form $\bar{u}_e(\bar{x}) = U_0 \left(\frac{\bar{x}}{L}\right)^m$. It is further assumed

that wedge velocity is of the form $\bar{u}_w(\bar{x}) = \lambda U_0 \left(\frac{\bar{x}}{L}\right)^m$.

Under the above assumptions, the partial differential equations and the corresponding boundary conditions govern the problem are given by:

Continuity equation

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0 \tag{1}$$

Linear momentum equation

$$\bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} = \bar{u}_e \frac{d\bar{u}_e}{dx} + \frac{1}{\rho} \frac{\partial}{\partial y} \left[\bar{K} \left| \frac{\partial \bar{u}}{\partial y} \right|^{n-1} \frac{\partial \bar{u}}{\partial y} \right] \tag{2}$$

Energy equation

$$\bar{u} \frac{\partial T}{\partial x} + \bar{v} \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{1}{\rho c_p} \left(\frac{\partial \bar{u}}{\partial y} \right)^2 \tag{3}$$

The boundary conditions for the velocity, Angular Velocity and temperature fields are

$$\begin{aligned} \bar{u} &= -\lambda U_0 \left(\frac{\bar{x}}{L}\right), \bar{v} = \bar{v}_w(\bar{x}), -k \frac{\partial T}{\partial y} = h_f (T_w - T) \quad \text{at} \\ \bar{y} &= 0 \\ \bar{u} &= \bar{u}_e(\bar{x}), T = T_\infty \quad \text{as } \bar{y} \rightarrow \infty \end{aligned} \tag{4}$$

Where \bar{u} and \bar{v} are the velocity components in the \bar{x} - and \bar{y} - directions, respectively, T is the fluid temperature inside boundary layer, ρ is the fluid density, c_p is the specific heat, $\alpha = k / \rho c_p$ is the thermal diffusivity, $\bar{v}_w > 0$ is the suction velocity while $\bar{v}_w < 0$ is the injection velocity, k is the thermal conductivity, h_f is the convective heat transfer coefficient, T_∞ is the free stream temperature, and n is the index in the power-law variation of a non-Newtonian fluid. It was pointed out by Postelnicu and Pop (2011) that Equation (2) governs the flow of a shear-thinning or pseudoplastic fluid for the case $n < 1$ and a shear-thickening or dilatants fluid for the case $n > 1$, \bar{K} is consistency of the fluid, T_w is the wall temperature and k is the thermal conductivity.

We assumed the Reynold’s model for the variation of consistency with temperature be (Szeri, 1998; Massoudi & Phuoc, 2004)

$$\bar{K}(\theta) = K_0 \exp(-M\theta) \dots\dots\dots(5)$$

Here K_0 is the ambient fluid dynamic consistency; M is a consistency variation parameter.

We now introduce the following dimensionless variables to reduce the number of independent variables and the number of equations,

$$x = \frac{\bar{x}}{L}, y = \frac{\bar{y} \text{Re}^{\frac{1}{n+1}}}{L}, u = \frac{\bar{u}}{U_0}, v = \frac{\bar{v} \text{Re}^{\frac{1}{n+1}}}{U_0}, u_e = \frac{\bar{u}_e}{U_0}, \theta = \frac{T - T_\infty}{T_w - T_\infty} \dots\dots (6)$$

Where $\text{Re} = \frac{U_0^{2-n} L^n}{\nu}$ is the generalized Reynolds number based the characteristic length L .

The stream function $\psi(x, y)$ defined $u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$

satisfy the continuity equation (1) automatically. We have

$$\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} = mx^{2m-1} + \frac{K_0 \exp(-M\theta)}{\rho \nu} \left[n \left(\frac{\partial^2 \psi}{\partial y^2} \right)^{n-1} \frac{\partial^3 \psi}{\partial y^3} - M \frac{\partial \theta}{\partial y} \left(\frac{\partial^2 \psi}{\partial y^2} \right)^n \right] \dots\dots(7)$$

$$\frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} = \frac{\alpha \text{Re}^{n+1}}{LU_0} \frac{\partial^2 \theta}{\partial y^2} + \frac{U_0^2 \text{Re}^{n+1}}{\rho c_p L^2} \left(\frac{\partial^2 \psi}{\partial y^2} \right)^2 \dots\dots(8)$$

$$\frac{\partial \psi}{\partial y} = -\lambda x^m, \frac{\partial \psi}{\partial x} = -v_w(x) \text{Re}^{\frac{1}{n+1}}, \frac{\partial \theta}{\partial y} = -\frac{Lh_f}{k} \text{Re}^{\frac{1}{n+1}} (1-\theta) \text{ at } y=0 \dots(9)$$

$$\frac{\partial \psi}{\partial y} \rightarrow x^m, \theta \rightarrow 0 \text{ as } y \rightarrow \infty \dots\dots(10)$$

In order to find similarity transformations, we consider following simplified form of the one-parameter group (Uddin *et al.*, 2012; Mutlag *et al.*, 2012; Na, 1979)

$$x = e^{-\varepsilon c_1} x^*, y = e^{-\varepsilon c_2} y^*, \psi = e^{-\varepsilon c_3} \psi^*, \theta = e^{-\varepsilon c_4} \theta^*, h_f = e^{-\varepsilon c_5} h_f^* \dots\dots(11)$$

$$v_w = e^{-\varepsilon c_6} v_w^*, c_p = e^{-\varepsilon c_7} c_p^*$$

where $c_1, c_2, c_3, \dots, c_7$ are constant and ε is the parameter of the group. Substituting Equation (10) into the Equations (7)-(9), we will then obtain the following relationship among c 's

$$c_1 = c_3 \left(\frac{n+1}{2mn-m+1} \right), c_2 = c_5 = c_3 \left(\frac{-mn+2m-1}{2mn-m+1} \right), c_4 = 0 \dots\dots(12)$$

$$c_6 = c_3 \left(\frac{2mn-m-n}{2mn-m+1} \right), c_7 = c_3 \left(\frac{3m-3mn+n-1}{2mn-m+1} \right)$$

Note that $\theta^* = \theta$, i.e., θ is invariants. The characteristic equations are as follows:

$$\frac{dx}{\left(\frac{n+1}{2mn-m+1} \right) x} = \frac{dy}{\left(\frac{mn-2m+1}{2mn-m+1} \right) y} = \frac{d\psi}{\psi} = \frac{dD_1}{\left(\frac{-mn+2m-1}{2mn-m+1} \right) D_1}$$

$$= \frac{dv_w}{\left(\frac{2mn-m-n}{2mn-m+1} \right) v_w} = \frac{dc_p}{\left(\frac{3m-3mn+n-1}{2mn-m+1} \right) c_p} \dots\dots\dots(13)$$

Solving the above equations, the following similarity transformations are obtained

$$\eta = x^{\left(\frac{-nm+2m-1}{n+1} \right)} y, \psi = x^{\left(\frac{2mn-m+1}{n+1} \right)} f(\eta), \theta = \theta(\eta), h_f = x^{\frac{-nm+2m-1}{n+1}} (h_f)_0 \dots(14)$$

$$v_w = x^{\frac{nm-2m-n}{n+1}} (v_w)_0, c_p = x^{\frac{3m-3mn+n-1}{n+1}} (c_p)_0$$

where $(h_f)_0, (v_w)_0, (c_p)_0$ are constant convective heat transfer coefficient, velocity of suction/injection and the specific heat. Using Equation (14) in Equations (7)-(10), we obtain following nonlinear system of ordinary differential equations

$$f'''(f'')^{n-1} - \frac{M}{n} \theta'(f'')^n + \exp(M\theta) \left(\frac{2mn-m+1}{n(n+1)} f f'' - \frac{m}{n} f'^2 + \frac{m}{n} \right) = 0 \dots\dots\dots(15)$$

$$\theta'' + \frac{\text{Pr}_m (2mn-m+1)}{n+1} f \theta' + \text{Pr}_m \text{Ec} f'^2 = 0 \dots\dots(16)$$

The boundary conditions become,

$$f(0) = -\frac{n+1}{2mn-m+1} f_w, f'(0) = -\lambda, \theta'(0) = -Bi(1-\theta(0))$$

$$f'(\infty) = 1, \theta(\infty) = 0 \dots\dots\dots(17)$$

Where $\text{Pr}_m = \frac{LU_0 \text{Re}^{\frac{-2}{n+1}}}{\alpha}$ is the modified Prandtl number (for power law fluids), $Bi = \frac{(h_f)_0 L \text{Re}^{\frac{-1}{n+1}}}{k}$ is the dimensionless convective parameter, $\text{Ec} = \frac{U_0^2 \text{Re}^{\frac{2}{n+1}}}{\rho (c_p)_0 L^2}$ is the Eckert number and

$$f_w = \frac{(n+1)(v_w)_0 \text{Re}^{\frac{1}{n+1}}}{U_0}$$

the suction/injection parameter respectively. Note that all parameters are free from x which confirms the true. Here primes denote differentiation with respect to η .

It is to be noted that for $M = \text{Ec} = 0, n = 1$ our problem reduces to Jiji (2009) in this case the Equations (15)-(16) are

$$f''' + \frac{m+1}{2} f f'' - m f'^2 + m = 0 \dots\dots\dots(18)$$

$$\theta'' + \frac{\text{Pr}_m(m+1)}{2} f\theta' = 0 \quad (19)$$

Note that when $m = M = \text{Ec} = 0$, $n = 1$ then Equation (18) conforms to Equation (2,15) in Ishak and Bachok (2009) and the system of ordinary differential Equations (15)-(16) is the same as that obtained by Aziz (2009) when $m = M = \text{Ec} = 0$, $n = 1$. Expressions for the quantities of physical interests, the skin friction factor and the rate of heat transfer can be found from the following definitions:

$$C_{f\bar{x}} = \frac{-K}{\rho u_e^2(\bar{x})} \left(\frac{\partial \bar{u}}{\partial \bar{y}} \right)_{\bar{y}=0}^n, Nu_{\bar{x}} = \frac{-\bar{x}}{T_w - T_\infty} \left(\frac{\partial T}{\partial \bar{y}} \right)_{\bar{y}=0} \quad (20)$$

Using (2.6) into (2.20) we get,

$$\left(\text{Re}_{\bar{x}} \right)^{\frac{1}{1+n}} C_{f\bar{x}} = \left[f''(0) \right]^n, \left(\text{Re}_{\bar{x}} \right)^{\frac{-1}{1+n}} Nu_{\bar{x}} = -\theta'(0) \quad (21)$$

where $\text{Re}_{\bar{x}} = \frac{\bar{u}_e^{2-n} \bar{x}^n}{\nu}$ is the local Reynolds number.

Solution of the problem

For solving Eqs. 15 – 17, a step by step integration method i.e. Runge–Kutta method has been applied. For carrying in the numerical integration, the equations are reduced to a set of first order differential equation. For performing this we make the following substitutions:

$$\begin{aligned} y_1 &= f, y_2 = f', y_3 = f'', y_4 = \theta, y_5 = \theta' \\ y_3' &= \frac{1}{y_3^{n-1}} \left[\frac{M}{n} y_3 y_3^n - \exp(My_4) \left(\frac{2mn-m+1}{n(n+1)} y_1 y_3 - \frac{m}{n} y_2^2 + \frac{m}{n} \right) \right] \\ y_5' &= - \left[\frac{\text{Pr}_m(2mn-m+1)}{n+1} y_1 y_5 + \text{Pr}_m \text{Ec} y_3^2 \right] \\ y_1(0) &= - \frac{n+1}{2mn-m+1} f_w, y_2(0) = -\lambda, y_5(0) = -Bi(1-y_4(0)) \\ y_2(\infty) &= 1, y_4(\infty) = 0 \end{aligned}$$

In order to carry out the step by step integration of Eqs. Refspseqn 15-17, Gills procedures as given in Ralston and Wilf (1960) have been used. To start the integration it is necessary to provide all the values of y_1, y_2, y_3, y_4 at $\eta = 0$ from which point, the forward integration has been carried out but from the boundary conditions it is seen that the values of y_3, y_5 are not known. So we are to provide such values of y_3, y_5 along with the known values of the other function at $\eta = 0$ as would satisfy the boundary conditions as $\eta \rightarrow \infty$ ($\eta = 10$) to a prescribed accuracy after step by step integrations are performed. Since the values of y_3, y_5 which are supplied are merely rough values, some corrections have to be made in these values in order that the boundary conditions to $\eta \rightarrow \infty$ are satisfied. These corrections in the values of y_3, y_5 are

taken care of by a self-iterative procedure which can for convenience be called ‘‘Corrective procedure’’. This procedure has been taken care of by the software which has been used to implement R–K method with shooting technique.

As regards the error, local error for the 4th order R–K method is $O(h^5)$; the global error would be $O(h^4)$. The method is computationally more efficient than the other methods. In our work, the step size $h = 0.01$. Therefore, the accuracy of computation and the convergence criteria are evident. By reducing the step size better result is not expected due to more computational steps vis-a`-vis accumulation of error.

RESULTS AND DISCUSSION

The governing equations (13) - (14) subject to the boundary conditions (15) are integrated as described in section 3. In order to get a clear insight of the physical problem, the velocity and temperature have been discussed by assigning numerical values to the parameters encountered in the problem. Figures 1 & 2 shows the effect λ on the non-dimensional velocity and temperature profiles. We observe that the velocity decreases whereas temperature increases with the increases the values of λ . These findings are similar to the results reported by Mutlag *et al.* (2012). Figures 3 & 4 illustrate the effect of consistency variation parameter (M) on the velocity and temperature respectively. We observed that the velocity increases where as temperature decreases with increasing M. The variation of the velocity and temperature profiles with the Falkner-skan power law parameter (m) is shown in Figures 5 & 6 respectively. It is observed that the velocity increases but temperature reduces with an increasing m . Figures 7 & 8 illustrate the effect of suction/injection parameter (f_w) on the velocity and temperature respectively. It is observed that the velocity and temperature decreases with increasing f_w . Moreover, the velocity and thermal boundary layer thickness decreases, these results are similar to the findings by Mutlag *et al.* (2012). Figures 9 & 10 illustrate the effects of the power law index parameter (n) on the velocity and temperature respectively. It is observed that velocity increases but temperature of the fluid reduces with a rising the values of n . Figures 11 & 12 illustrate the effect of convective parameter (Bi) on the velocity and temperature respectively. We observed that the velocity and temperature increases consequently momentum and thermal boundary layer thickness increases with increasing Bi. These findings are similar to the results reported by Gangadhar (2015). The variation of the temperature with the Eckert number (Ec) and Prandtl number (Pr) is shown in Figures 13 & 14 respectively. It is observed that the temperature reduces as well as thermal boundary layer thickness decreases with an increasing Ec or Pr. Physically, Pr=0.67 is Argon-30°C, Pr=0.76 is Corbon Dioxide at 30°C, Pr=2.4 is water at 70°C, Pr = 5.1 is water at 30°C. Figure 15 shows the effects of n, M and Bi on skin friction. From Figure 15 it is seen that the skin friction decreases with an increase n and increases with an increase m or Bi. The effect of n, M and Bi on local Nusselt number is shown in fig.16. It is found that the local Nusselt number enhances with an increase in the parameters M and Bi whereas local Nusselt number reduces with increase n .

Table 1. Comparison for the values of $f''(0)$ when $fw=M=\lambda=0$, $m=n=1$

$f''(0)$			
Present study 1.232588	Mutlag <i>et al.</i> (2012) 1.232587	Postelnicu and Pop (2011) 1.23259	Ishak <i>et al.</i> (2007) 1.2326

Table 2. Comparison for the values of $f''(0)$ for the values of m when $fw=M=\lambda=0$, $n=1$

$f''(0)$			
m	Present study	Mutlag <i>et al.</i> (2012)	Postelnicu and Pop (2011)
0	0.332057	0.33205	0.3206
0.111	0.511691	0.51169	0.5120
0.333	0.757137	0.75713	0.7575
1.0	1.232588	1.23258	1.2326

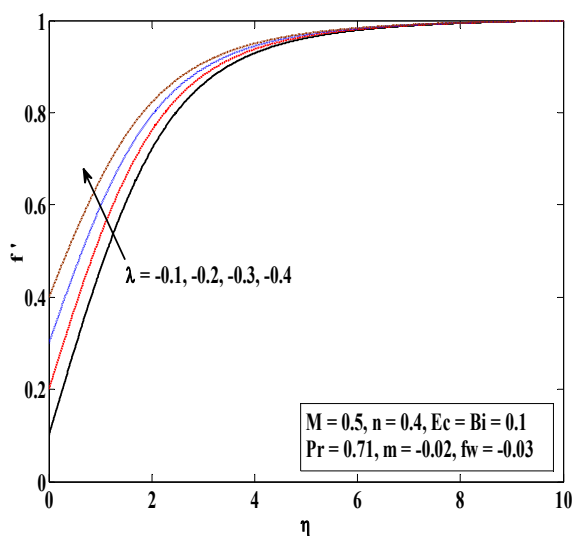


Fig.1. Velocity for various values of λ

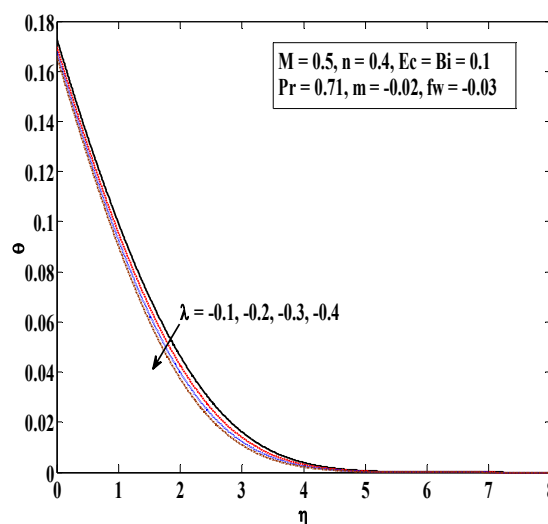


Fig.2. Temperature for various values of λ

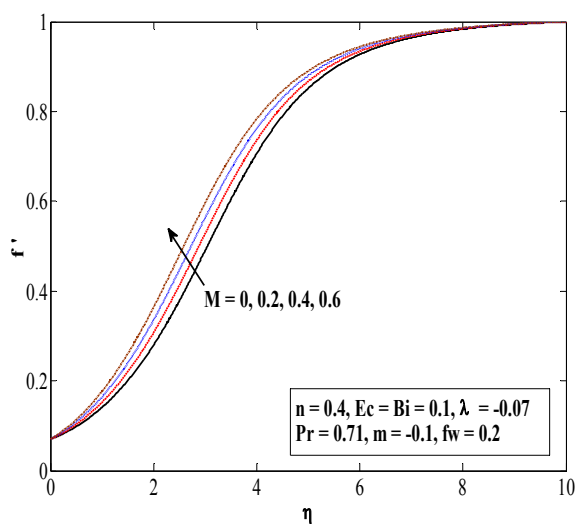


Fig.3. Velocity for various values of M

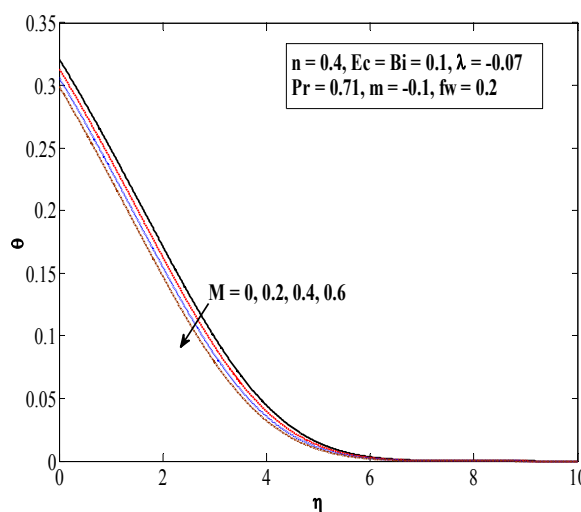


Fig.4. Temperature for various values of M

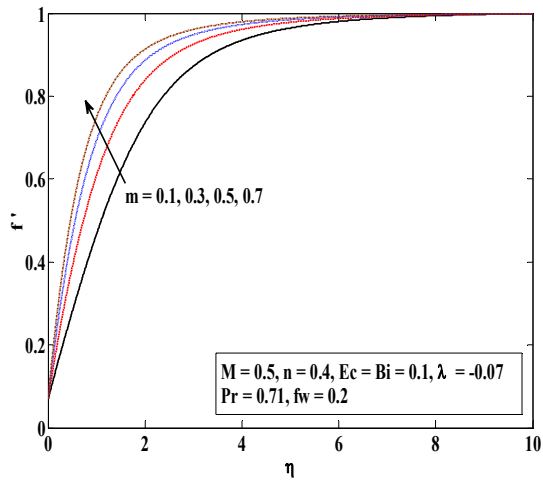


Fig.5. Velocity for various values of m

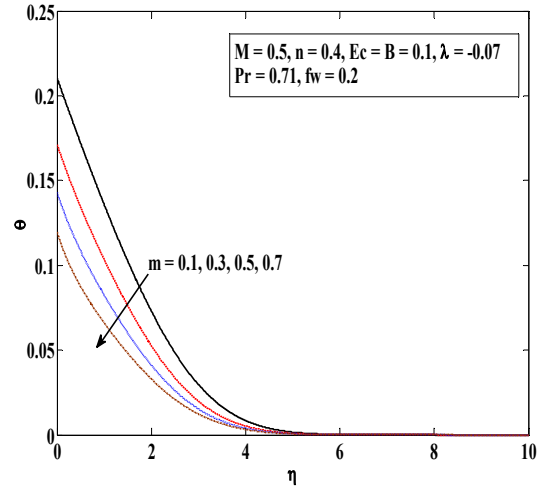


Fig.6. Temperature for various values of m

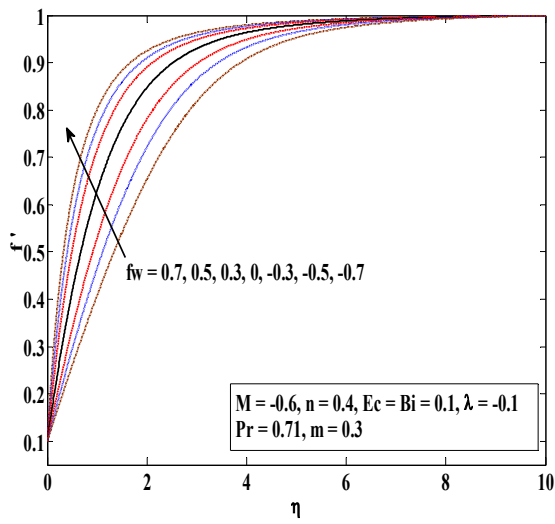


Fig.7. Velocity for different values of f_w

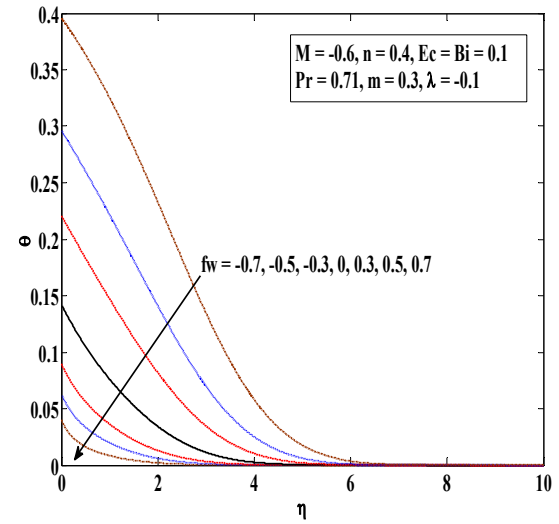


Fig.8. Temperature for various values of f_w

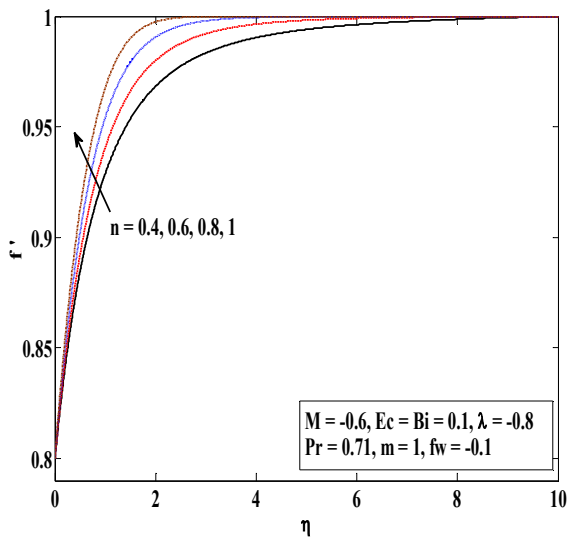


Fig.9. Velocity for different values of n

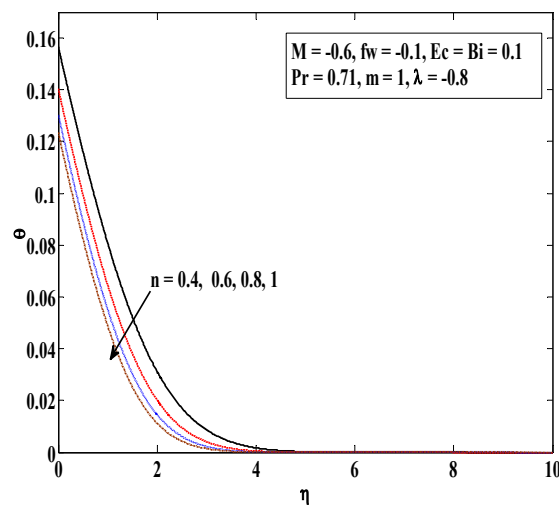


Fig.10. Temperature for various values of n

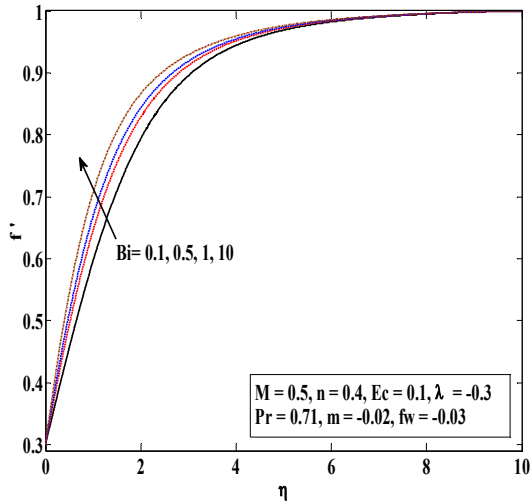


Fig.11. Velocity for different values of Bi

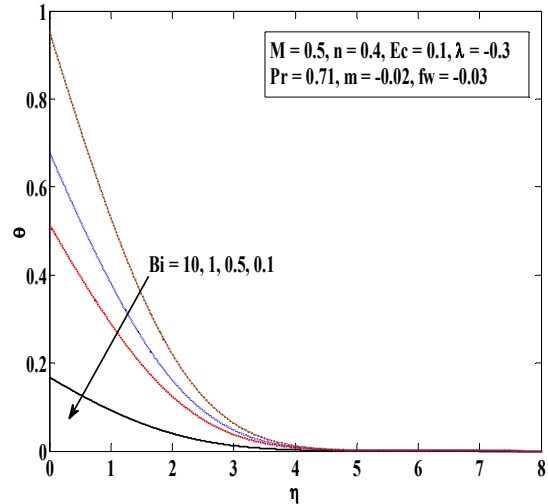


Fig.12. Temperature for various values of Bi

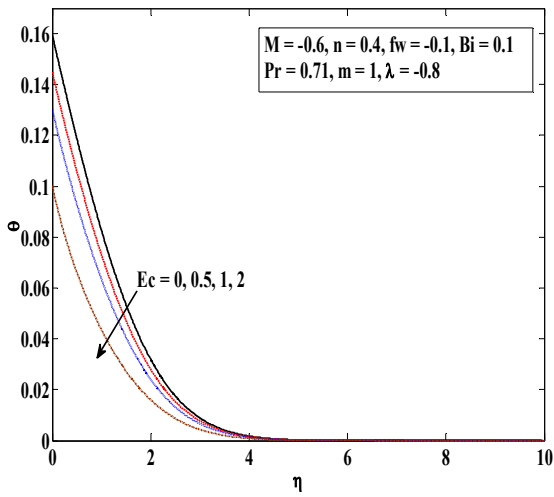


Fig.13. Temperature for various values of Ec

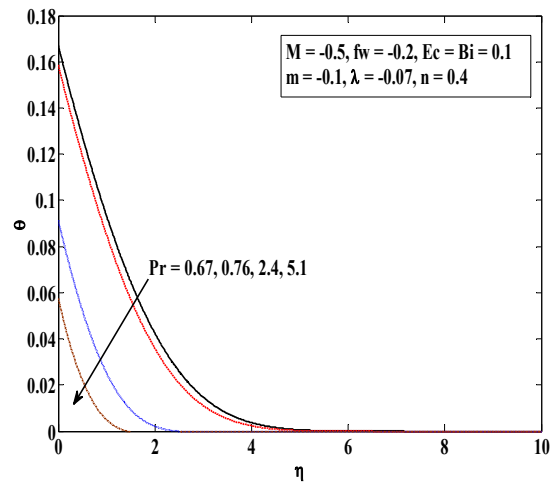


Fig.14. Temperature for various values of Pr

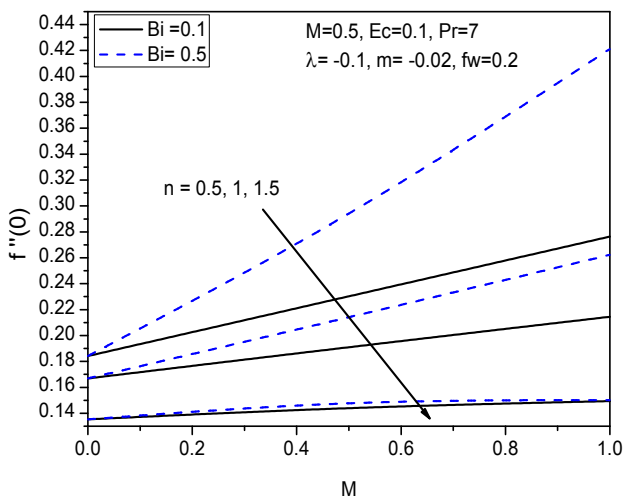


Fig.15. Local Skin friction for various values of n, M and Bi

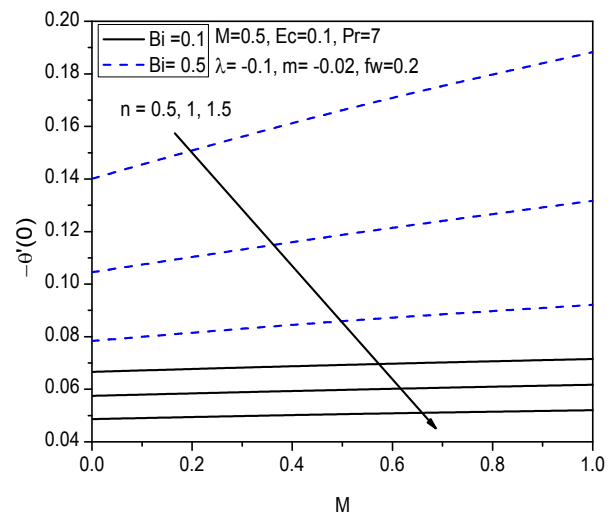


Fig.16. Local Nusselt number for various values of n, M and Bi

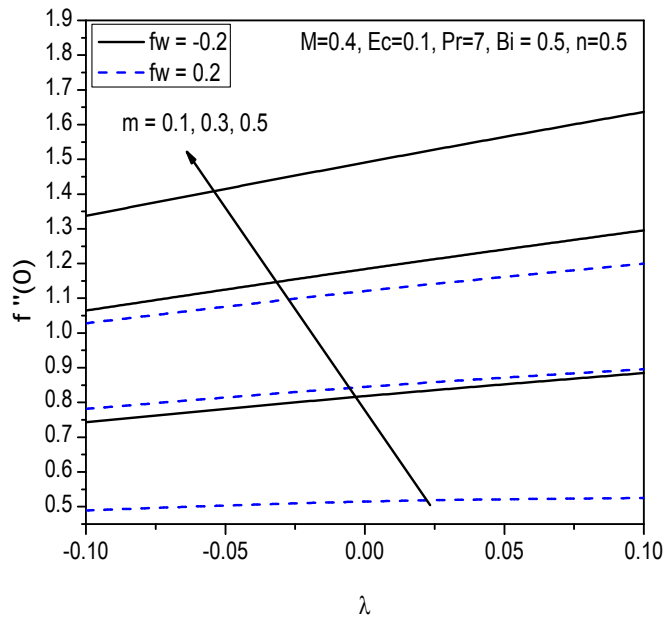


Fig.17. Local Skin friction for various values of m , λ and f_w

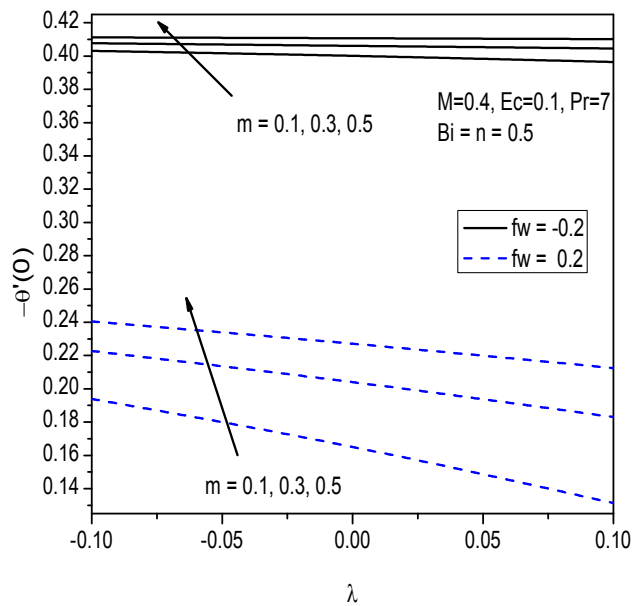


Fig.18. Local Nusselt number for various values of m , λ and f_w

The variation of m , λ and f_w on skin friction is shown in Figure 17. It is observed that the skin friction increases with an increase m , λ , whereas skin friction decreases with increases f_w . The effect of m , λ and f_w on local Nusselt number is shown in Fig.18. It is found that the local Nusselt number enhances with an increase in the parameters m whereas local Nusselt number reduces with increase λ or f_w . Table 1 & 2 that the present results perfect agreement to the previously published data.

Conclusion

In the present prater, the steady, two dimensional, Flakner-Skan stretching and shrinking wedge flow of a power law fluid in the presence of viscous dissipation and convective boundary condition. The governing equations are approximated to a system of non-linear ordinary differential equations by similarity transformation. Numerical calculations are carried out for various values of the dimensionless parameters of the problem. It has been found that

1. The velocity increases whereas temperature reduces with an increase in the Flakner-Skan power law parameter.
2. The influence of viscous dissipation reduces the temperature.
3. The momentum and thermal boundary layer thickness increases with a rising the values of convective parameter.
4. The skin friction decreases with an increase the power law index parameter and increases with increases the values of convective parameter.
5. The local Nusselt number enhances with an increase in the convective parameter whereas local Nusselt number reduces with increase the power law index parameter.

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