Available online at http://www.journalcra.com

International Journal of Current Research
Vol. 7, Issue, 08, pp.19519-19522, August, 2015

INTERNATIONAL JOURNAL OF CURRENT RESEARCH

ISSN: 0975-833X

RESEARCH ARTICLE

ON THE NON-HOMOGENEOUS BIQUADRATIC DIOPHANTINE EQUATION WITH FIVE UNKNOWNS $x^3 - y^3 = z^3 - w^3 + 12t^4$

*,[1]Geetha, V. and [2]Gopalan, M. A.

[1]Department of Mathematics, Cauvery College for Women, Trichy-620018, Tamilnadu, India
[2]Department of Mathematics, Srimathi Indira Gandhi College, Trichy-620002, Tamilnadu, India

ARTICLE INFO

Article History:
Received 17th May, 2015
Received in revised form
15th June, 2015
Accepted 26th July, 2015
Published online 31st August, 2015

Key words:
Biquadratic with five unknowns,
Integral solutions.

ABSTRACT

The biquadratic Diophantine equation with five unknowns represented by $x^3 - y^3 = z^3 - w^3 + 12t^4$ is analysed for finding its non-zero distinct integral solutions. Introducing the linear transformations $x = u+1, y = u-1, z = v+1, w = v-1$ and employing the method of factorization different patterns of non zero distinct integer solutions of the equation under the above equation are obtained. A few interesting relations between the integral solutions are exhibited.

Citation: Geetha, V. and Gopalan, M. A. 2015. "On the non-homogeneous Biquadratic Diophantine equation with five unknowns $x^3 - y^3 = z^3 - w^3 + 12t^4$", *International Journal of Current Research*, 7, (8), 19519-19522.

INTRODUCTION

The theory of Diophantine equations offers a rich variety of fascinating problems. In particular biquadratic Diophantine equations, homogeneous and non-homogeneous have aroused the interest of numerous mathematicians since antiquity (Carmichael, 1959; Dickson, 1952). In this context one may refer (Gopalan and Sangeetha, 2010; Gopalan and Sangeetha, 2010; Gopalan and Sangeetha, 2011; Manju Somanath *et al*., 2011; Manju Somanath *et al*., 2012; Gopalan and Sivkami, 2013; Gopalan and Geetha, 2013; Sangeetha *et al*., 2014; Gopalan and Geetha, 2015; Gopalan *et al*., 2015) for various problems on the biquadratic Diophantine equations. However, often we come across homogeneous biquadratic equations and as such one may require its integral solution in its most general form. This paper concern with the non-homogeneous biquadratic equation with five unknowns for determining $x^3 - y^3 = z^3 - w^3 + 12t^4$ its infinitely many non-zero integral solutions. Also a few interesting properties among the solutions are presented.

***Corresponding author: Geetha, V.**
Department of Mathematics, Cauvery College for Women, Trichy-620018, Tamilnadu, India.

2. METHOD OF ANALYSIS

The biquadratic diophantine equation with five unknowns to be solved for getting non-zero integral solution is

$$x^3 - y^3 = z^3 - w^3 + 12t^4 \quad \ldots(1)$$

On substituting the linear transformations

$$x = u+1, y = u-1, z = v+1, w = v-1 \quad \ldots\ldots(2)$$

in (1), it leads to the equation

$$u^2 - v^2 = 2t^4 \quad \ldots\ldots.(3)$$

It is noted that the following sets of integers satisfy (1)

$(6T^2+1, 6T^2-1, 2T^2+1, 2T^2-1, 2T)$,
$(8T^3+T+1, 8T^3+T-1, 8T^3-T+1, 8T^3-T-1, 2T)$,
$(8T^4+2, 8T^4, 8T^4, 8T^4-2, 2T)$,
$(4T^3+2T+1, 4T^3+2T-1, 4T^3-2T+1, 4T^3-2T-1, 2T)$.

we present below different methods of solving (3) and thus obtain different pattern of integral solutions to (1).

Pattern I

Rewrite equation (3) as

$$u^2 = v^2 + 2t^4 \quad .....(4)$$

Assume $u(a,b) = a^2 + 2b^2$ .....(5)

Substituting (5) in (4) and employing the method of factorization, define

$$(a + i\sqrt{2}b)^2 = v + i\sqrt{2}t^2$$

Equating real and imaginary parts, we have

$$v(a,b) = a^2 - 2b^2$$
$$t^2(a,b) = 2ab$$

Choosing $a = 2^{2\alpha-1}b$, the values of $u, v \& t$ are

$$\left.\begin{array}{l} u(b) = (2^{2\alpha-2} + 2)b^2 \\ v(b) = (2^{2\alpha-2} - 2)b^2 \end{array}\right\} \quad ....(6)$$

$$t(b) = 2^{\alpha} b \quad ....(7)$$

Substituting the values of (6) in (2), we get the non-zero distinct integer solutions

$$x(b) = (2^{4\alpha-2} + 2)b^2 + 1$$
$$y(b) = (2^{4\alpha-2} + 2)b^2 - 1$$
$$z(b) = (2^{4\alpha-2} - 2)b^2 + 1$$
$$w(b) = (2^{4\alpha-2} - 2)b^2 - 1$$

Thus, these values along with (7) represent non-zero distinct integer solutions of (1).

Properties:

1. Each of the following expression is a nasty number.

i) $6\left[x(b) + y(b) + z(b) + w(b)\right]$

ii) $6\left[(x(b) + z(b))(y(b) + w(b)) + 4\right]$

iii) $6\left[x(b) - y(b) + z(b) - w(b)\right]$

iv) $6\left[x(b)y(b) + 1\right]$

v) $6\left[z(b)w(b) + 1\right]$

2. $4\left[(x(b) + y(b))(z(b) + w(b))\right] + 64b^4 = \left[(x(b) + y(b) + z(b) + w(b)\right]^2$

3. $8\left[(x(b) + y(b) + z(b) + w(b)\right] = 2^{4\alpha}\left[(x(b) + y(b) - z(b) - w(b)\right]$

4. $8\left[(t(b)\right]^2 = 2^{2\alpha}\left[(x(b) + y(b) - z(b) - w(b)\right]$

5. $4\left[(x(b)y(b)) - (z(b)w(b))\right] = \left[(x(b) + y(b)\right]^2 - \left[z(b) + w(b)\right]^2$

Pattern II

Rewrite equation (4) as

$$v^2 + 2t^4 = u^2 \times 1 \quad ....(8)$$

Write 1 as

$$1 = \frac{(1 + i2\sqrt{2})(1 - i2\sqrt{2})}{9} \quad ....(9)$$

Substitute (5) and (9) in (8) and employing the method of factorization, define

$$v + i\sqrt{2}t^2 = (a + i\sqrt{2}b)^2 \frac{(1 + i2\sqrt{2})}{3}$$

Equating real and imaginary parts, we get

$$v(a,b) = \frac{1}{3}\left[a^2 - 2b^2 - 8ab\right] \quad ......(10)$$

$$t^2(a,b) = \frac{2}{3}\left[a^2 - 2b^2 + ab\right] \quad .....(11)$$

Since our aim is to find integral solutions, substituting $a = 3A \& b = 3B$ in (5), (10)

and (11) then the values of $u, v \& t^2$ are

$$\left.\begin{array}{l} u(A,B) = 9A^2 + 18B^2 \\ v(A,B) = 3A^2 - 6B^2 - 24AB \end{array}\right\} \quad .....(12)$$

$$t^2(A,B) = 6A^2 - 12B^2 + 6AB \quad ....(13)$$

Treating (13) as quadratic in A and solving for A, we get

$$A = 6R^2 + 2S^2 \; \& \; B = 6R^2 - S^2 \quad .....(14)$$

$$t(R,S) = 18RS \quad .....(15)$$

Substituting (14) in (12) and using (2) the values of x, y, z, w are

$$\left.\begin{aligned} x(R,S) &= 54\left[18R^4 + S^4\right] + 1 \\ y(R,S) &= 54\left[18R^4 + S^4\right] - 1 \\ z(R,S) &= 54\left[-18R^4 + S^4\right] + 1 \\ w(R,S) &= 54\left[-18R^4 + S^4\right] - 1 \end{aligned}\right\} \quad \ldots\ldots.(16)$$

Thus, (16) and (15) represent the non-zero distinct integer solutions to (1).

Properties

1. $x(R,S) + y(R,S) + z(R,S) + w(R,S) \equiv 0 \pmod{216}$
2. $t(R,R-1) - 36t_{3,R} = 0$
3. $4\left[x(R,S)y(R,S)+1\right] \pm \begin{bmatrix}(x(R,S)+y(R,S)) \\ (z(R,S)+w(R,S))\end{bmatrix} \equiv 0 \pmod{2916}$
4. $4\left[x(R,S)y(R,S)+1\right] - \begin{bmatrix}(x(R,S)+y(R,S)) \\ (z(R,S)+w(R,S))\end{bmatrix} \equiv 0 \pmod{419904}$
5. $68\left[x(R,S)y(R,S)+1\right] + 19\begin{bmatrix}(x(R,S)+y(R,S)) \\ (z(R,S)+w(R,S))\end{bmatrix} \equiv 0 \pmod{419904}$
6. $54t(R^2, R(R-1) - x(R,S) + 1 \equiv 0 \pmod{27}$
7. $18y(R,S) - 54t(S^3, S) + 1 \equiv 0 \pmod{17496}$
8. $z(R,S) + w(R,S) + 108(t(R^3,R) - S^4) = 0$
9. $(x(R,S) + y(R,S))(z(R,S) + w(R,S))$ can be written as difference of two squares

10.Each of the following expression is a nasty number.

i) $18\left[x(R,S) + y(R,S) - z(R,S) - w(R,S)\right]$

ii) $648\left[z(R,S) + w(R,S) + 108t(R^3,R)\right]$

Remark 1

It is to be noted that (13) is satisfied by the following three choices of $A \,\&\, B$

i) $A = -12R^2 - S^2 \;\&\; B = 6R^2 - S^2$

ii) $A = 12R^2 + 4S_1^2 \;\&\; B = 12R^2 - 2S_1^2$

iii) $A = 12R^2 + 4S_1^2 \;\&\; B = -6R^2 + 4S_1^2$

Following the procedure as above, the corresponding integer solutions to (1) are exhibited below

Solution for Choice 1

$$\begin{aligned} x(R,S) &= 27\left[72R^4 + S^4\right] + 1 \\ y(R,S) &= 27\left[72R^4 + S^4\right] - 1 \\ z(R,S) &= 27\left[72R^4 - S^4\right] + 1 \\ w(R,S) &= 27\left[72R^4 - S^4\right] - 1 \\ t(R,S) &= 18RS \end{aligned}$$

Solution for Choice 2

$$\begin{aligned} x(R,S_1) &= 216\left[18R^4 + S_1^4\right] + 1 \\ y(R,S_1) &= 216\left[18R^4 + S_1^4\right] - 1 \\ z(R,S_1) &= 216\left[-18R^4 + S_1^4\right] + 1 \\ w(R,S_1) &= 216\left[-18R^4 + S_1^4\right] - 1 \\ t(R,S_1) &= 36RS_1 \end{aligned}$$

Solution for Choice 3

$$\begin{aligned} x(R,S_1) &= 216\left[9R^4 + 2S_1^4\right] + 1 \\ y(R,S_1) &= 216\left[9R^4 + 2S_1^4\right] - 1 \\ z(R,S_1) &= 216\left[3R^4 - S_1^4\right] + 1 \\ w(R,S_1) &= 216\left[3R^4 - S_1^4\right] - 1 \\ t(R,S_1) &= 36RS_1 \end{aligned}$$

Remark 2

In addition to (9), we have the following representations for 1

$$1 = \begin{cases} \dfrac{(1+i12\sqrt{2})(1-i12\sqrt{2})}{289} \\ \dfrac{(7+i6\sqrt{2})(7-i6\sqrt{2})}{121} \\ \dfrac{(17+i6\sqrt{2})(17-i6\sqrt{2})}{361} \end{cases}$$

Repeating the analysis presented above, we obtain the other patterns of integer solutions to (1).

Conclusion

To conclude one may consider biquadratic equation with multivariables (≥5) and search for their non-zero distinct integer solutions along with their corresponding properties.

REFERENCES

Carmichael R.D., The Theory of numbers and Diophantine analysis, Dover publications, new York, 1959.

Dickson L.E., History of theory of Numbers, Chelsa Publishing company, New Yors, 1952.

Gopalan M.A. and Geetha, V. On the homogeneous biquadratic diophantine equation with five unknowns $x^4 - y^4 = 5(z^2 - w^2)T^2$, *Global Journal of Engineering Science and Researches*, 2(6), 30-37, June 2015

Gopalan M.A., Kavitha, A. and Kiruthika, R. "Observations on the biquadratic equation with five unknowns $2(x-y)(x^3+y^3) = (1+3k^2)(X^2-Y^2)w^2$", *International Research Journal of Engineering and Technology*, 2(3), 1562-1573, June 2015.

Gopalan, M.A. and Geetha, V, "Integral solutions to ternary biquadratic equation $x^2 + 13y^2 = z^4$", *International Journal of Latest Research Science and Technology*, 2(2), 59-61, 2013.

Gopalan, M.A. and Sangeetha, G. Integral solutions of Non-homogeneous biquadratic $x^4 + x^2 + y^2 - y = z^2 + z$, Acta Ciencia Indica, Vol. XXXVII M.No.4, 799-803, 2011.

Gopalan, M.A. and Sangeetha, G. Integral solutions of Ternary Quartic equation $x^2 - y^2 + 2xy = z^4$, *Antartica J.Math*., 7(1), 95-101, 2010.

Gopalan, M.A. and Sangeetha. G., Integral solutions of non-homogeneous Quartic equation $x^4 - y^4 = \left(2\alpha^2 + 2\alpha + 1\right)\left(z^2 - w^2\right)$, impact *J.Sci.Tech*., 4(3), (July-Sep), 15-21, 2010.

Gopalan, M.A. and Sivkami, B. Integral solutions of quartic equation with four unknowns $x^3 + y^3 + z^3 = 3xyz + 2(x+y)w^3$, *Antartica J.Math*., 10(2), 151-159, 2013.

Manju Somanath, Sangeetha. G., and Gopalan, M.A., Integral solutions of non-homogeneous Quartic equation $x^4 - y^4 = \left(k^2+1\right)\left(z^2 - w^2\right)$, *Archimedes J.Math*., 1(1), 51-57, 2011.

Manju Somanath, Sangeetha. G., and Gopalan.M.A., Integral solutions of biquadratic equation with four unknowns given by $xy + \left(k^2+1\right)z^2 = 5w^2$, *Pacific-Asian Journal of Mathematics*, 6(2),185-190, July- Dec 2012.

Sangeetha, G., Manju Somanath, Gopalan, M.A. and Pushparani., Integral solutions of the homogeneous biquadratic equation with five unknowns $\left(x^3 + y^3\right)z = \left(w^2 - p^2\right)R^2$, *International conference on Mathematical methods and Computation*, Pg:221-226, Feb 2014.

****